

Note: All questions are compulsory. Each question carries 5 marks. Carrying of mobile phone during examinations will be treated as case of unfair means.

1. A scaffolding system consists of three beams and six ropes as shown in **Fig. 1**. Each of the top ropes A and B can carry a load of W_1 , each of the middle ropes C and D can carry a load of W_2 , and each of the bottom ropes E and F can carry a load of W_3 . If the loads acting on beams 1, 2, and 3 are x_1 , x_2 , and x_3 , respectively, as shown, formulate the problem of finding the maximum load ($x_1 + x_2 + x_3$) that can be supported by the system. Assume that the weights of the beams 1, 2, and 3 are w_1 , w_2 , and w_3 , respectively, and the weights of the ropes are negligible. (5)

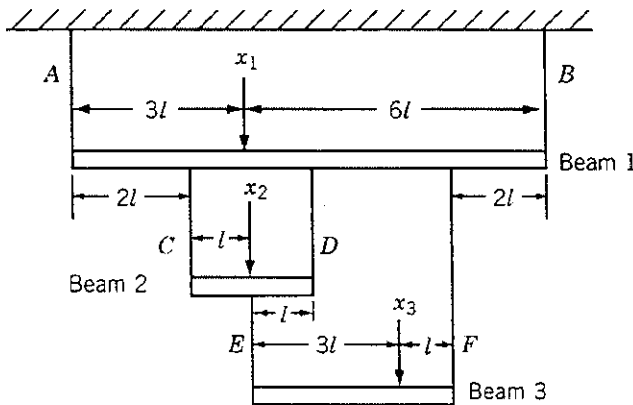


Fig 1

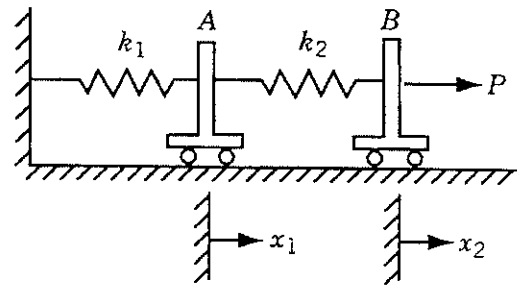


Fig. 2

2. **Figure 2** shows two frictionless rigid bodies (carts) A and B connected by two linear elastic springs having spring constants k_1 and k_2 . The springs are at their natural positions when the applied force P is zero. Find the displacements x_1 and x_2 under the force P by using the principle of minimum potential energy. (5)
3. Find the second-order Taylor's series approximation of the function, $f(x_1, x_2, x_3) = x_2^2 x_3 + x_1 e^{x_3}$ about the point $\mathbf{X}^* = [0, -1, 2]^T$. (5)
4. Find the dimensions of a box of largest volume that can be inscribed in a sphere of unit radius. (5)
5. A beam of uniform rectangular cross section is to be cut from a log having a circular cross section of diameter $2a$. The beam has to be used as a cantilever beam (the length is fixed) to carry a concentrated load at the free end. Find the dimensions of the beam that correspond to the maximum tensile (bending) stress carrying capacity. (5)