

An Algorithm for Computing the Best-Performing Path in a Computer Network

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Abstract: This paper addresses the problem of transmitting a given amount of data along a single path from the sending end node to the receiving end node in a directed network so that the performance of transmission is best. The performance of the transmission along a path is described in terms of an expression defined as performance reliability (PR). PR is a function of each link's reliability, capacity, and delay. A path is said to be a best-performing path (BPP) if the performance reliability of this path is maximum among all the available paths between the two specified nodes. The algorithm developed for selecting the best-performing path uses Dijkstra's shortest path algorithm. The proposed algorithm is more suitable for store-and-forward data transmission. The best-performing path selection is an important precomputation in developing routing protocols considering reliability, capacity and delay.

Key Words: *path routing, best-performing path, performance reliability, data transmission.*

1. Introduction

A computer network is a set of service centers (switches, computers, routers, etc.) linked to each other by communication links. This network is modeled by a graph $G(N, E)$ where N is the set of nodes representing service centers and E is the set of edges representing the communication links. Each communication link is assigned a 3-tuple reliability, capacity and delay; and a specified amount of flow to be transmitted from a source node 's' to a target node 't' [1-4]. The performance of transmission between the two specified nodes depends upon the path selected [11]. This paper addresses the problem of optimum path selection between the two specified nodes, so as to achieve the optimum performance of transmission. Such an optimum path selected on the basis of the proposed method is defined as a 'best-

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performing path'. Applications such as video conferencing, hotline communication etc. benefit especially from a path of high reliability and minimum lag time [2,6,7,12]. The selection of the best path proposed here is a useful pre-computation for the development of routing protocol.

In the literature, while considering reliability as a measure of the performance of computer communication networks (CCN), only connectivity of the links had been considered sufficient [9] and successful transmission had been interpreted as the existence of at least one path being available between source node 's' and target node 't'. However, in practice performance of computer communication network (CCN) depends largely upon three parameters: link connectivity, capacity of links, and delay encountered during the transmission of data. If data transmission capacity and transmission delay are ignored, then any amount of data can always be transmitted from 's' to 't' whenever a path is available. However, this is not a realistic assumption [13]. Data transmission capacity of a network link is always finite and transmission is always non-zero. When complete data is transmitted through the sequence of links between nodes 's' and 't', the performance of this data flow depends upon the sequence of the links[16]. Link selection for the data path depends upon optimum values of reliability, capacity and delay. Therefore, determining the best link combination in a path or sequence requires an appropriate model that can reflect the relationship among the three parameters of the performance considered in this paper.

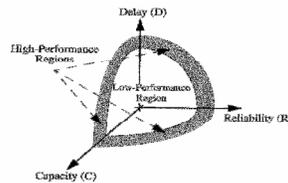


Fig. 1: 3-D Service Performance Envelope [8]

McCabe [8] stressed 3-tuple (reliability, capacity, delay) service performance requirement for computer communication networks (CCN). He proposes integration of these three by introducing a 3-D Service Performance Envelope (SPE), as shown in Figure 1. But the McCabe's SPE is not sufficient for the mathematical operations and computations. This paper attempts to quantify this performance requirement of data flow in terms of a single expression. We propose the concept of *performance reliability* so as to integrate the 3-tuple of reliability, capacity and delay into one mathematical expression. Performance Reliability can generalize the functional relationship for most reliable and quickest data transmission.

The solution provided in this paper for the path-selection problem is more general than others available [5-7]. The method in this paper is an improvement over the method proposed by Tragdous [6], which assumes equal values of the reliability for all links of the network. The proposed algorithm is applicable even in case of unequal values of link reliability, capacity and delay. This paper assumes transmission of data on the basis of store-and-forward process

as in datagram transmission [10]. Hence, the capacity of the data transmission link is always fully utilized. Whereas Tragoudos [6] considers the data flow with the minimum link-capacity among all the links of a path. The proposed algorithm is *simultaneously* selects a data path with maximum reliability and minimum delay time. The best-performing path selection algorithm is based on Dijkstra's shortest distance algorithm [14,15] with performance reliability as the link cost.

2. Acronyms, Notation, and Assumptions

Acronyms:

s-t	source to target node
PR	Performance Reliability
BPP	Best-Performing Path
CCN	Computer Communication Network

Notation:

$G(N,E)$	Directed graph of a computer network with the set of nodes (vertices) N , the set of edges E
n	Total number of nodes in the network
(u,v)	edge connecting node 'u' to node 'v'
(p,c,d)	3-tuple of reliability, capacity and delay
$p(u,v)$	Reliability of the edge (u,v)
$c(u,v)$	Capacity of the edge (u,v)
σ	Size of data to be transmitted from the sending end to the receiving end
$\lceil \sigma/c(u,v) \rceil$	Transmission time required to transfer σ flow through edge (u,v)
$d(u,v)$	Delay on edge (u,v) in addition to the transmission time $\lceil \sigma/c(u,v) \rceil$
$T(u,v)$	Minimum transmission time required to transmit σ units of data from node u to node v via an edge (u,v)
$P(v_1,v_2,\dots,v_k)$	Path with node set (v_1,v_2,\dots,v_k) ; where v_1 is starting node and v_k is terminating node
$T(P,\sigma)$	Minimum transmission time required to transmit σ units of data from node v_1 to node v_k along a path $P(v_1,v_2,\dots,v_k)$
$pr(u,v)$	Performance reliability of link (u,v)
$R(P)$	Reliability of path P
$PR(P,\sigma)$	Performance reliability of path P when σ units of data is flowing
$w(u,v)$	Cost of link (u,v) ; $w(u,v) = -\ln pr(u,v)$

Assumptions:

Following assumptions are made on the data network under consideration:

1. Each node is perfectly reliable.
2. The capacity of link is a non-negative integer value.
3. Reliabilities and capacities of different links are statistically independent.

4. The network graph is directed and has no parallel/duplicate links.
5. Data is transmitted in the network on the basis of a store-and-forward and first-in-first-out process.
6. Splitting of the transmitted data is not permitted.

3. Preliminaries

In this paper, we are interested in both reliability and speed of data transmission between pairs of nodes in a computer network. The network is modeled by a linear graph. The set of nodes corresponds to computer centers and the set of edges corresponds to communication links between centers. Let σ be the data to be transmitted via an edge (u,v) . The probability of fault-free data transmission via edge (u,v) is the reliability $p(u,v)$. The minimum transmission time required to transmit σ units of data from node u to node v via an edge (u,v) is

$$T(u, v) = d(u, v) + \left\lceil \frac{\sigma}{c(u, v)} \right\rceil \quad (1)$$

The minimum transmission time required to transmit σ units of data from node v_1 to node v_k along a path $P(v_1, v_2, \dots, v_k)$

$$T(P, \sigma) = \sum_{i=1}^{k-1} d(v_i, v_{i+1}) + \sigma \sum_{i=1}^{k-1} \left\lceil \frac{1}{c(v_i, v_{i+1})} \right\rceil \quad (2)$$

(2) is valid for data transmission via a store-and-forward process as in datagram networks. For virtual circuit networks, the term $\sigma \sum_{i=1}^{k-1} \left\lceil \frac{1}{c(v_i, v_{i+1})} \right\rceil$ in (2) should be replaced

with $\sigma \left\lceil \frac{1}{\min_{i=1}^{k-1} c(v_i, v_{i+1})} \right\rceil$, where $\min_{i=1}^{k-1} c(v_i, v_{i+1})$ is called the capacity of path $P(v_1, v_2, \dots, v_k)$.

Therefore, for virtual circuit networks, (2) can be rewritten as follows:

$$T(P, \sigma) = \sum_{i=1}^{k-1} d(v_i, v_{i+1}) + \sigma \left\lceil \frac{1}{\min_{i=1}^{k-1} c(v_i, v_{i+1})} \right\rceil \quad (3)$$

The reliability of data transmission along path $P(v_1, v_2, \dots, v_k)$ is:

$$R(P) = \prod_{i=1}^{k-1} p(v_i, v_{i+1}) \quad (4)$$

Let P be an s-t path and σ be the number of units of data to be transmitted from s to t . Then, path P may be defined as follows:

Definition 3.1: Quickest path

P is defined as the quickest s-t path to transmit σ units of data from s to t if $T(P, \sigma)$ is minimum among all s-t paths.

Definition 3.2: Most reliable path

P is defined as the most reliable s-t path to transmit σ units of data from s to t if $R(P)$ is maximum among all s-t paths.

Definition 3.3: Most reliable quickest path

P is defined as the most reliable quickest s-t path to transmit σ units of data from s to t if $R(P)$ is maximum among the quickest s-t paths.

Definition 3.4: Quickest most reliable path:

P is defined as the quickest most reliable s - t path to transmit σ units of data from s to t if $T(P,\sigma)$ is minimum among the most reliable s - t paths.

The topic of s - t path enumeration has been widely studied [2-7,10,12]. The quickest path determination problem was studied in [3,4,10] without considering failure of links or nodes. Jain and Gopal [5] evaluate the most reliable path without considering link characteristics other than reliability. With increasing applications of data networks in daily life, users are demanding highly reliable and fast responding networks. So, there is an urgent need in data transmission to consider both reliability and time required for data transfer. Much work has already been done [2,6,7] in path selection on the basis of these two requirements. Guoliang [2] proposed two algorithms for finding the most reliable quickest path and the quickest most reliable path. However, Guoliang's method does not compute both the most reliable and the quickest path simultaneously. A single algorithm that can compute a maximum reliability and minimum transmission time path is important for developing the protocols. Section 4 proposes an algorithm by introducing the concept of *best-performing path*. This algorithm for computing the *best-performing path* is better than the previously proposed algorithms [6,7].

4. Best-Performing Path (BPP)

The requirement of reliability is becoming important in data networks. Therefore, the performance must consider reliability in addition to channel capacity and transmission delay. In this section a functional relationship among the 3-tuple of reliability, capacity and delay is introduced using the concept of Performance Reliability (PR).

4.1 Performance Reliability

Definition 4.1: Performance Reliability of Edge (u,v)

Let data σ be required to be transmitted via an edge (u,v) and reliability, capacity and delay tuple $(p(u,v),c(u,v),d(u,v))$ of edge (u,v) . The minimum transmission time required to transmit σ units of data from node 'u' to node 'v' via an edge (u,v) is $T(u,v)$. Then, the *performance reliability* $pr(u,v)$ of edge (u,v) is:

$$pr(u,v) = p(u,v) \cdot \exp(-T(u,v)) \quad (5)$$

Definition 4.2: Performance Reliability of Path P

If there are k edges in a path $P(v_1, v_2, \dots, v_k)$, and $R(P)$ is the reliability of path P , then the performance reliability of the transmission of data σ along path P is defined as:

$$PR(P, \sigma) = R(P) \cdot \exp(-T(P, \sigma)) \quad (6)$$

4.2 Best-Performing Data Path

Performance reliability (PR) is indicative of the performance of a data network. As a measure of the performance of data path, $PR(P,\sigma)$, can incorporate a 3-tuple for all links in a path. We present the following criterion for selecting a best-performing path using $PR(P,\sigma)$:

For a given amount of data σ , the best performing path is the one for which performance reliability $PR(P,\sigma)$ is maximum among all the existing s - t paths.

5. Algorithm for Computing BPP

Computation of the best-performing path depends upon the amount of data to be transmitted. In this section, ALGORITHM (BPP) is proposed for the computation of the best-performing path. ALGORITHM (BPP) is an adoption of Dijkstra's shortest path algorithm [14, 15]. The proposed algorithm is applicable for a data network in which 3-tuples (p,c,d) for each edge are known. The weight of edge (u,v) , $w(u,v)$, is $-\ln pr(u,v)$. Input to the algorithm consists of a weighted directed graph $G(N,E)$ and source vertex s in G . The weights of the edges are given by a weight function $w: E \rightarrow [0, \infty]$; therefore $w(u,v) = -\ln pr(u,v)$ is the non-negative cost of transmitting data from vertex u to vertex v .

ALGORITHM (BPP) works by keeping, for each vertex v , the cost $d[v]$ of the least cost path found so far between s and v . Initially, this value is 0 for source vertex s ($d[s]=0$), and infinity for all other vertices ($d[v]=\infty$ for every v in V , except s). On completion of the algorithm, $d[v]$ is the cost of the least weight path from s to v or ∞ if no such path exists. The execution of lines 11-13 is called the relaxation of edge (u,v) . The algorithm is organized so that each edge (u,v) is relaxed only once, when $d[u]$ has reached its final value.

ALGORITHM (BPP) maintains two sets of vertices S and Q . Set S contains all vertices for which the value $d[v]$ is already the cost of the least weight path and set Q contains all other vertices. Initially, S is empty, Q is identical to N , and in each step one vertex is moved from Q to S . This vertex is chosen as the vertex with lowest value of $d[u]$. When a vertex u is moved to S , ALGORITHM (BPP) relaxes every outgoing edge (u,v) .

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ALGORITHM      (BPP)
1  function BPP( $G, w, s$ )
2    for each vertex  $v$  in  $N$ 
3       $d[v] := \text{infinity}$ 
4       $\text{previous}[v] := \text{undefined}$ 
5     $d[s] := 0$ 
6     $S := \{ s \}$ 
7     $Q := N - \{s\}$ 
8    While  $Q$  is not an empty set
9       $u := \text{Extract\_Min}(Q)$ 
10      $S := S \text{ union } \{u\}$ 
11      $Q := Q - \{u\}$ 
12     for each edge  $(u,v)$  outgoing from  $u$ 
13       if  $d[v] > d[u] + w(u,v)$ 
14          $d[v] := d[u] + w(u,v)$ 
15          $\text{previous}[v] := u$ 
16   End\_while
17   End\_BPP

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In the ALGORITHM (BPP), $u = \text{Extract_Min}(Q)$ searches for the vertex u in the vertex set

Q that has the least $d[u]$ value. The time complexity of ALGORITHM (BPP) is $O(|N|^2)$. With a binary heap, the algorithm requires $O((E + N)\log N)$ time. A Fibonacci heap improves the time complexity to $O(E + N \log N)$.

6. Illustration

The data network, shown in Figure 2, illustrates the best-performing path algorithm. In Table 1 shows the values for the 3-tuple (p, c and d) for each edge. Table 2 shows the edge cost $w(u,v)$, computed for different values of data flow. In Figure 2, nodes ‘0’ and ‘5’ are distinguished as source ‘s’ and destination ‘t’ nodes, respectively. Results of the BPP Algorithm for finding best-performing paths for 7 data values are displayed in Table 3.

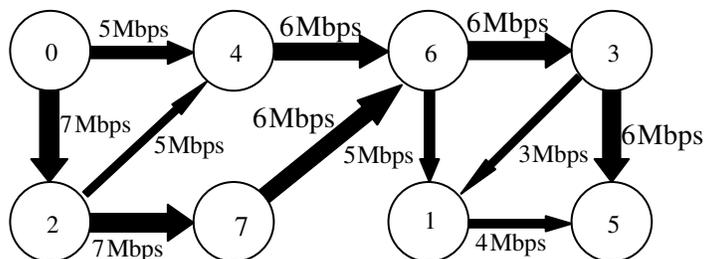


Fig. 2: Data Communication Network

Table 1: Values of 3-Tuples (p, c, d) for Each Edge (u,v) of the Network Shown in Fig. 2

Edge (u,v)	Reliability p(u,v)	Capacity c(u,v) in Mbps	Delay d(u,v) in s
(0, 2)	0.7	7	0.80
(0, 4)	0.7	5	0.50
(1, 5)	0.7	4	0.95
(2, 4)	0.8	5	1.00
(2, 7)	0.9	7	1.50
(3, 1)	0.8	3	0.70
(3, 5)	0.7	6	0.76
(4, 6)	0.6	6	0.80
(6, 1)	0.9	5	0.85
(6, 3)	0.7	6	0.75
(7, 6)	0.8	6	0.90

One can observe in Table 3 that the best-performing path depends upon the amount of data transmitted. The best-performing path can be calculated for a range of data values.

Therefore, a path-routing scheme can be determined in advance for different units of data to be transmitted.

Table 2: $w(u,v)$, Edge Cost Calculated for Different Units of Data

Edge (u,v)	$\sigma=3$ Mb	$\sigma=5$ Mb	$\sigma=6$ Mb	$\sigma=10$ Mb	$\sigma=15$ Mb	$\sigma=20$ Mb	$\sigma=50$ Mb	$\sigma=100$ Mb
(0,2)	2.15	2.15	2.15	3.15	4.15	4.15	9.15	16.15
(0,4)	1.85	1.85	2.85	2.85	3.85	4.85	10.85	20.85
(1,5)	2.30	3.30	3.30	4.30	5.30	6.30	14.30	26.30
(2,4)	2.22	2.22	3.22	3.22	4.22	5.22	11.22	21.22
(2,7)	1.60	2.60	2.60	3.60	4.60	4.60	9.60	16.60
(3,1)	1.92	2.92	2.92	4.92	5.92	7.92	17.92	34.92
(3,5)	2.11	2.11	2.11	3.11	4.11	5.11	10.11	18.11
(4,6)	2.01	2.31	2.31	3.31	4.31	5.31	10.31	18.31
(6,1)	1.95	1.95	2.95	2.95	3.95	4.95	10.95	20.95
(6,3)	2.10	2.10	2.10	3.10	4.10	5.10	10.10	18.10
(7,6)	2.12	2.12	2.12	3.12	4.12	5.12	10.12	18.12

Table 3: Best-Performing Paths Computed Using ALGORITHM (BPP) in Network Shown in Fig. 2 (between the nodes '0' and '5')

σ , Data Units Transmitted (in Mb)	Best-Performing Path Computed
3	P(0,2,7,6,3,5)
5	P(0,4,6,3,5)
6	P(0,4,6,3,5)
10	P(0,4,6,3,5)
15	P(0,4,6,3,5)
20	P(0,4,6,3,5)
50	P(0,2,7,6,3,5)
100	P(0,4,6,1,5)

7. Conclusion

This paper proposes a new algorithm using Dijkstra's algorithm for computing the best-performing path between two specified nodes. The expression used for the link costs is able to incorporate a 3-tuple in expressing the performance of complete path. The best path is defined as the path with maximum value of the product of link costs. Selection of the best path depends on the amount of data to be transmitted. This work is suited for datagram data

delivery systems. Work is in progress for extending the best path transmission for virtual circuit networks.

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