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# Analysis on some other models of transmission lines for CRLH meta-materials

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**Abstract**: The theme of this paper is to present and analyze two novel circuit models which can be used in the realization of composite right hand and left hand transmission line meta-materials (CRLH MTM). Along with this, the conventional circuit models which are being used in literature very extensively are also presented in two different cases namely lossy and lossless case. The main purpose of this analysis is to investigate the circuit models which give a non-linear dispersion relation or phase constant, with a maximum or minimum as a function of frequency while maintaining the low attenuation constant. It is observed that the new proposed circuit models are meeting the requirement of having no attenuation constant while having the dispersion relationship with maximum or minimum. This is proved with the mathematical expressions, giving an opportunity to explore these circuits in fabrication for practical applications.

**Keywords**: Transmission lines, CRLH TL, propagation constant, phase constant, attenuation constant and dispersion.

## I. INTRODUCTION

Since the introduction of transmission line (TL) approach of meta-materials (MTMs) by Caloz et.al [1-3], Eleftheriades et.al [4-6], and Oliner [7] there has been an extensive research in this area covering wide applications in electromagnetic engineering. The TL theory for meta-materials is well developed as of now [8]. Due to non-resonant, low loss, broad bandwidth and planar configuration of one dimensional TL MTMs [9], novel devices have been developed [10] and shown to exhibit superior performances in microwave integrated circuits domain [11]. MTMs are treated as homogeneous materials and are modeled as one-dimensional (1D) transmission lines (TLs) whose propagation direction can be in any direction in the material, forward or backward direction. The conventional right hand TL theory [12] , is the starting point in analyzing the CRLH MTMs[8] .

The CRLH MTMs have been analyzed mostly with lossless model. Due to the difference in the resonant frequencies in series and shunt branches of CRLH TL MTMS, there is a gap in the dispersion relation in which it is treated as unbalanced CRLH MTM. If both the frequencies are same then it is termed as balanced CRLH MTM. This paper it is aimed at obtaining circuits with no such gap in the dispersion relation while maintaining little attenuation constant or no (or zero) attenuation constant. In the next section, properties of right hand, left hand and CRLH MTM are briefly discussed. In Section III, the generic circuit model for TL is analyzed for propagation constant. In Section IV, a loaded transmission line similar to Miller type of loading circuit is analyzed. In section V, discussion of results and Section VI, gives the summary of the investigation from this analysis.

## II. RH, LH AND CRLH TL MODEL

The transmission line equivalent circuit model for transverse electromagnetic wave propagation consisting of inductors and capacitors (if there is no loss in the line) is in the form of a ladder network as shown in Fig.1. Resistance is incorporated in to the circuit to represent the loss. Depending on the location of the capacitor and inductor in the ladder network, these structures in periodic configurations are treated as forward wave supporting structure (Z contains inductors only and Y contains capacitors only) or backward wave supporting structure(Y contains inductors only and Z contains capacitor only).

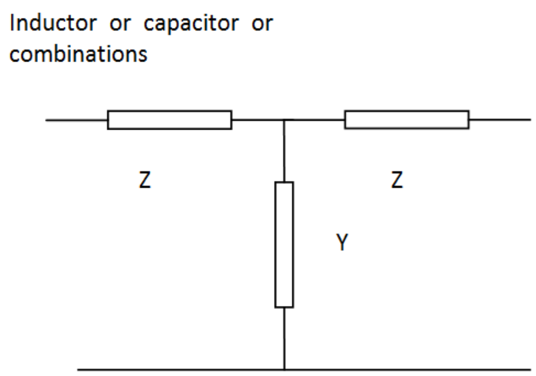


Fig 1.

In forward wave supporting structure, the phase velocity and group velocity are in same phase (for incident wave and reflected wave) and in backward wave supporting structures the phase velocity and group velocity are out of phase or anti-parallel. The CRLH material is capable of supporting both forward and backward waves (Z and Y contains combinations of inductors and capacitors). From this characteristic, it is obvious that there is a band where wave propagation is not possible and the transitions from forward to backward are abrupt. In order to make this transmission smooth, the modification in the circuit model is imperative and those circuit models are presented in the next section.

## III. ANALYSIS WITH GENERIC MODEL

A generic model for transmission line is considered from Fig.1.The series branch impedance is represented with Z

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while shunt branch admittance is represented with Y. $Z = a + bs + 1/cs$ represents a resistance of a ohms, inductor of b henries and c farads of capacitance in series. In a similar manner, f is in mho, g in farads and h in henries making $Y = f + gs + 1/hs$ as the total admittance of the shunt branch. The propagation constant and characteristic impedance of the model can be obtained from $\gamma = \alpha + j\beta = \sqrt{ZY}$ and $Z_0 = \sqrt{Z/Y}$ where $\alpha$ represents attenuation constant and $\beta$ represents the phase constant [12]. The propagation constants for four possible circuit configurations are presented in Table1. The four combinations are as follows. The series branch has components in series while shunt branch has components in shunt, series branch has components in series while shunt branch has series, series branch has components in parallel while shunt branch has components in parallel and finally series branch has components in parallel while shunt branch also has components in parallel. Two of these circuits are the dual of other circuits.

$$A = \sqrt{(a^2 + \left(b\omega - \frac{1}{c\omega}\right)^2}$$

$$B = \sqrt{f^2 + \left(g\omega - \frac{1}{h\omega}\right)^2}$$

$$\theta_1 = \frac{1}{2} tan^{-1}(\frac{1}{a}(b\omega - \frac{1}{c\omega}))$$

$$\theta_2 = \frac{1}{2} tan^{-1}(\frac{1}{f}(g\omega - \frac{1}{h\omega}))$$

| S.No | Circuit type | Propagation Constant for general case | Dispersion equation for lossless case |
|---|---|---|---|
| 1 | $Z = a + bs + \frac{1}{cs}$ (in Ω) <br> $Y = f + gs + \frac{1}{hs}$ (in ℧) | $\alpha = AB\cos(\theta_1 + \theta_2)$ <br> $\beta = AB\sin(\theta_1 + \theta_2)$ | $\beta = \frac{1}{\omega\sqrt{ch}}\sqrt{(1 - bc\omega^2)(1 - gh\omega^2)}$ <br> $\alpha = 0$ |
| 2. | $Z = a + bs + \frac{1}{cs}$ (in Ω) <br> $Y^{-1} = f + gs + \frac{1}{hs}$ (in Ω) | $\alpha = (A/B)\cos(\theta_1 - \theta_2)$ <br> $\beta = (A/B)\sin(\theta_1 - \theta_2)$ | $\beta = \sqrt{\frac{h\,(bc\omega^2 - 1)}{c\,(gh\omega^2 - 1)}}$ <br> $\alpha = 0$ |
| 3. | $Z^{-1} = a + bs + \frac{1}{cs}$ (in ℧) <br> $Y^{-1} = f + gs + \frac{1}{hs}$ (in Ω) | $\alpha = (B/A)\cos(\theta_1 - \theta_2)$ <br> $\beta = (B/A)\sin(\theta_1 - \theta_2)$ | $\beta = \frac{\omega\sqrt{ch}}{\sqrt{(1 - bc\omega^2)(1 - gh\omega^2)}}$ <br> $\alpha = 0$ |
| 4. | $Z^{-1} = a + bs + \frac{1}{cs}$ (in ℧) <br> $Y = f + gs + \frac{1}{hs}$ (in ℧) | $\alpha = (1/AB)\cos(\theta_1 - \theta_2)$ <br> $\beta = (1/AB)\sin(\theta_2 - \theta_1)$ | $\beta = \sqrt{\frac{c\,(gh\omega^2 - 1)}{h\,(bc\omega^2 - 1)}}$ <br> $\alpha = 0$ |

TABLE.1

## IV. LOADED TRANSMISSION LINE

In order to come up with the simple but novel composite right hand left hand meta-material structure, the generic model of transmission line that is discussed in previous section is loaded in Miller form as shown in Fig.2.This circuit can be converted in to the ladder configuration using Δ-Y transformation. From the standard ladder circuit configuration properties, the propagation constant is obtained as

$$\gamma = \alpha + j\beta = \sqrt{\frac{Z_1 Z_3}{2Z_1Z_2 + Z_2Z_3 + Z_1^2}}$$

where α represents attenuation constant and β represents the phase constant.

Here $Z_3$ has been taken as a loading element.

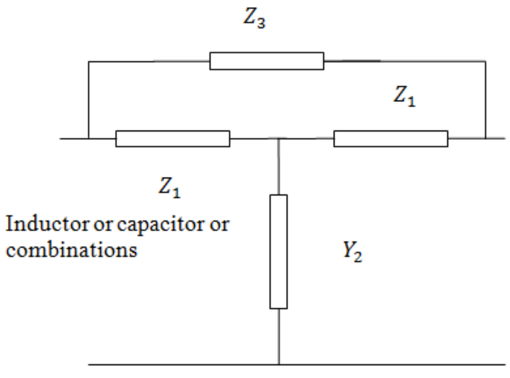


Fig.2

$Z_3$ has been considered as resistive element, capacitive element and inductive element and the corresponding dispersion relations are presented in Table.2.

In order to verify the behavior of this wave model in circuit theory, the gain (considered as unit cell) has been calculated as the below expression where $Z_0$ represents the source impedance and load impedance. This formula is used to find the phase of the transfer function.

$$\frac{V_o}{V_{in}} = Y_0 \frac{Y_1^2 + Y_3(2Y_1 + Y_2)}{(Y_0 + Y_1 + 2Y_3)(Y_0(2Y_1 + Y_2) + Y_1Y_2}$$

| S.No | Circuit Type | Propagation Constant |
|---|---|---|
| 1 | $Z_1 = j\omega L Z_2 = \frac{1}{j\omega C}$ <br> $Z_3 = R_3$ | $\alpha = \frac{\sqrt{\omega L R_3}}{\sqrt[4]{\left(\frac{2L}{c} - \omega^2 L^2\right)^2 + \left(\frac{R_3}{\omega C}\right)^2}} \cos\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{R_3}{\omega C}\frac{1}{\frac{2L}{c} - \omega^2 L^2}\right)\right)$ <br> $\beta = \frac{\sqrt{\omega L R_3}}{\sqrt[4]{\left(\frac{2L}{c} - \omega^2 L^2\right)^2 + \left(\frac{R_3}{\omega C}\right)^2}} \sin\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{R_3}{\omega C}\frac{1}{\frac{2L}{c} - \omega^2 L^2}\right)\right)$ |
| 2 | $Z_1 = \frac{1}{j\omega C}$ <br> $Z_2 = j\omega L$ <br> $Z_3 = R_3$ | $\alpha = \sqrt{\frac{R_3}{\omega C}} \frac{1}{\sqrt[4]{\left(\frac{2L}{c} - \frac{1}{\omega^2 c^2}\right)^2 + (R_3\omega L)^2}} \cos\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{R_3\omega L}{\frac{2L}{c} - \frac{1}{\omega^2 c^2}}\right)\right)$ <br> $\beta = \sqrt{\frac{R_3}{\omega C}} \frac{1}{\sqrt[4]{\left(\frac{2L}{c} - \frac{1}{\omega^2 c^2}\right)^2 + (R_3\omega L)^2}} \sin\left(\frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{R_3\omega L}{\frac{2L}{c} - \frac{1}{\omega^2 c^2}}\right)\right)$ |
| 3 | $Z_1 = j\omega L$ <br> $Z_2 = \frac{1}{j\omega C}$ <br> $Z_3 = \frac{1}{j\omega C_3}$ | $\gamma = \sqrt{\frac{L/C_3}{\frac{2L}{c} - \frac{1}{\omega^2 c c_3} - \omega^2 L^2}}$ |
| 4 | $Z_1 = j\omega L$ <br> $Z_2 = \frac{1}{j\omega C}$ <br> $Z_3 = j\omega L_3$ | $\gamma = j\omega\sqrt{\frac{L_3 L}{\frac{2L}{c} + \frac{L_3}{c} - \omega^2 L^2}}$ |
| 5 | $Z_1 = \frac{1}{j\omega C}$ <br> $Z_2 = j\omega L$ <br> $Z_3 = \frac{1}{j\omega C_3}$ | $\gamma = \frac{j}{\omega\sqrt{C C_3}} \frac{1}{\sqrt{\frac{2L}{c} + \frac{L_3}{c} - \omega^2 L^2}}$ |
| 6 | $Z_1 = \frac{1}{j\omega C}$ <br> $Z_2 = j\omega L$ <br> $Z_3 = j\omega L_3$ | $\gamma = \sqrt{\frac{L_3/C}{\frac{2L}{c} - \frac{1}{\omega^2 c^2} - \omega^2 L L_3}}$ |

TABLE 2

## V. RESULTS

*A. Lossless and Lossy Case for generic model.*

MATLAB code is generated to simulate all the expressions presented in tables. Fig.3 represents the characteristics of conventional circuit for CRLH meta-material. For this $Z = 0.1 + 0.1\,s + 1/0.1s$ and $Y = 0 + 0.1\,s + 1/hs$ with h=.01(red trace), h=0.1(black) and h=0.2(blue) (All graphs are with same values).

The phase constant in the graph is indicating the nature of supporting the forward and back ward waves. Fig.6 is the dual of this case.

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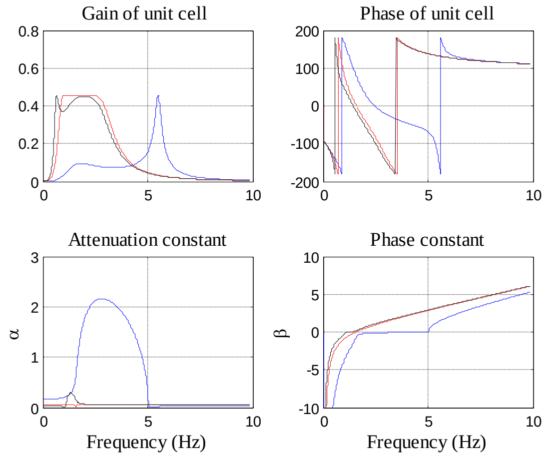


Figure 3

The other model which is overlooked in the literature (due to its band reject nature) is the series combination elements in both the series branch and shunt branchFig.4 represents this case. After analyzing this circuit in the lossy case, it is observed that this circuit topology also has the capability to support the forward wave and backward waves. Hence it is confirmed that this circuit model can also be used in the CRLH meta-material realization in the lossy case. Fig.5 is the dual of this case.

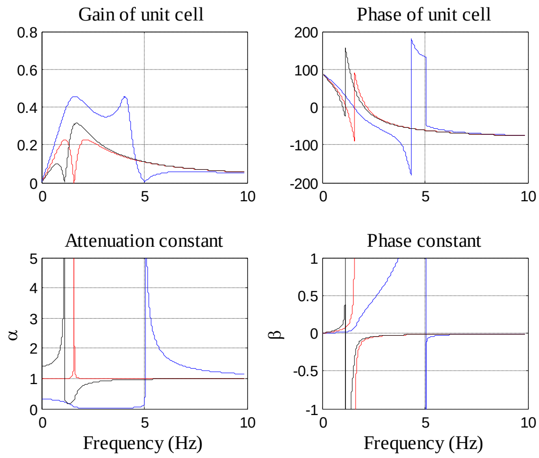


Figure 4

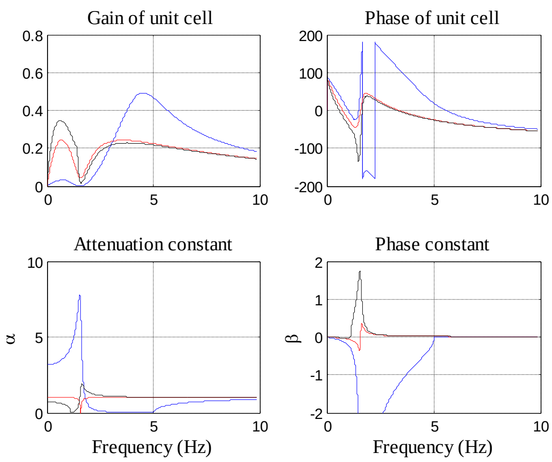


Figure 5

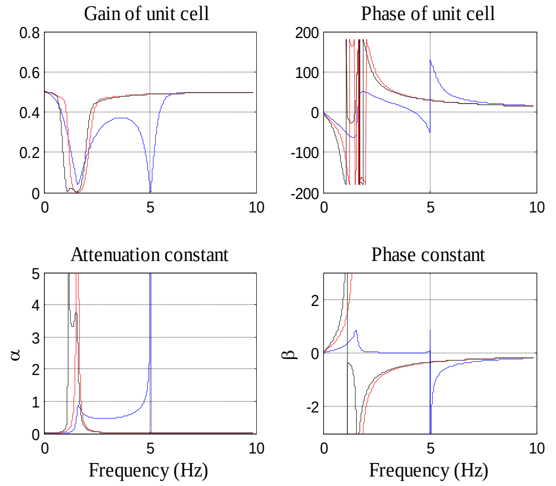


Figure 6

### *B. Miller type loading*

From the analysis of Miller type loading circuits, it is observed that the non-linear dispersion relation can be obtained very easily with capacitive and inductive loadings for forward wave supporting and backward wave supporting structures respectively. This is in fact a very positive result which can be converted in to the microstrip realizations easily. Fig.7 represents the inductive loading for forward supporting wave structure.

Fig.10 represents the capacitive loading for backward wave. Fig.8 represents the characteristics for capacitive loading for forward structure while Fig.9 represents the inductive load for backward wave supporting structure. The variation in the phase constant for this case can also be verified from the formula given in the Table.2. From the analysis it is observed that it is not possible to get a structure that supports both forward and backward waves by loading the forward structure with inductor and backward structure with capacitor.

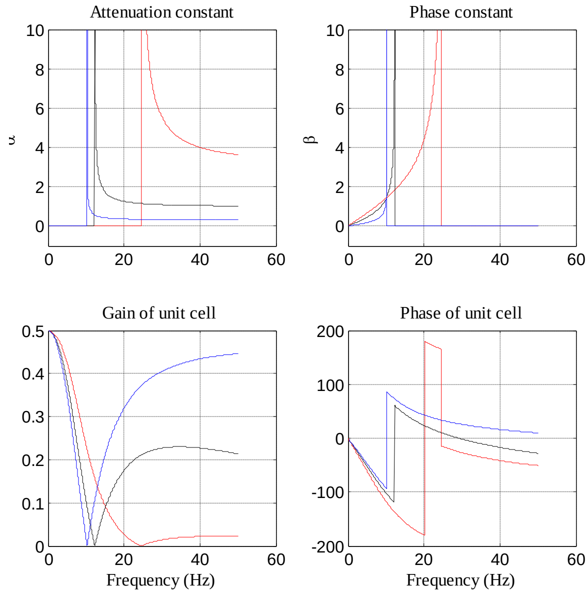


Fig 7. L=0.01, C=0.05 with L3=0.1(blue), 0.01(black) and 0.001(red)

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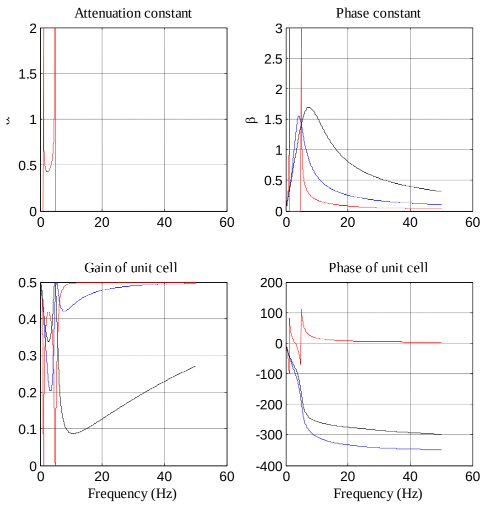


Fig.8 L=0.1, C=0.02 with C3=0.1(blue), 0.01(black) and 0.001(red)

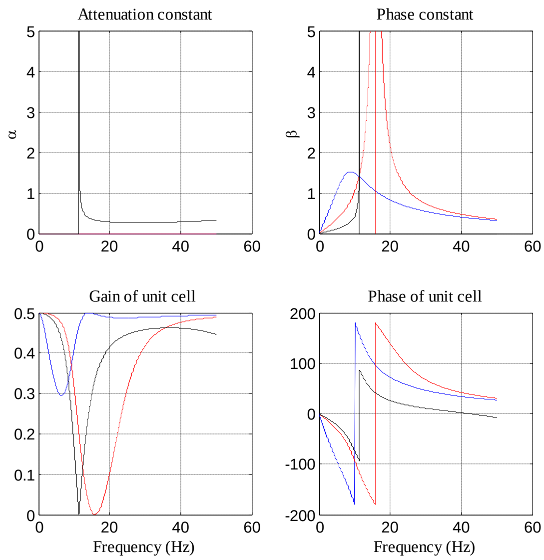


Fig 9 L=0.01, C=0.01 with L3=0.1(blue), 0.01(black) and 0.001(red)

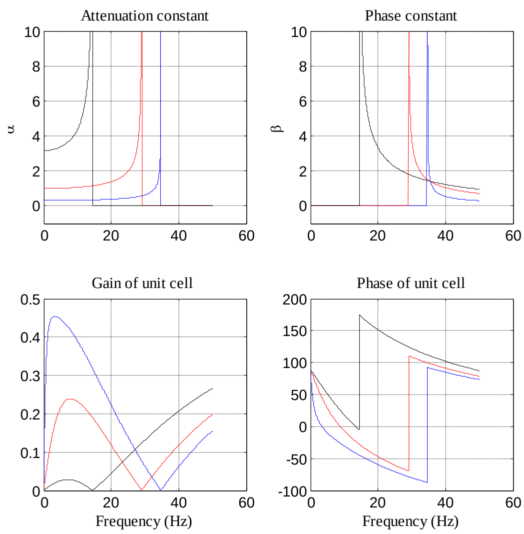


Fig 10 L=0.001, C=0.01 with C3=0.1(blue), 0.01(black) and 0.001(red)

## VI. CONCLUSION

From this analysis, it is concluded that there has to be some component of resistance in the circuit model in order to have smooth transmission from forward to backward. It is also observed that not only conventional CRLH model can support the forward and backward, other circuit models can also perform the required functionality. Miller type loading is a new type of circuit model in which case it is acting like a composite right hand left hand medium without any loss. These circuits can be further analyzed to get the practical applications to remove certain constrains in the realization of these circuits on microstrip version.

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