LLR BASED ANALYSIS OF QUASI-CYCLIC AND RANDOM LDPC CODES COMBINED WITH MULTI-USER MIMO SYSTEMS

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CERTIFICATE

This is to certify that the work titled **"LLR BASED ANALYSIS OF QUASI-CYCLIC AND RANDOM LDPC CODES COMBINED WITH MULTI-USER MIMO SYSTEMS"** submitted by "**Pallavi Gupta**" in partial fulfillment for the award of degree of M. Tech at Jaypee University of Information Technology, Waknaghat has been carried out under my supervision. This work has not been submitted partially or wholly to any other University or Institute for the award of this or any other degree or diploma.

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ABSTRACT

Multiple input multiple output (MIMO) system is one of the most promising wireless technologies that can achieve wide coverage, high throughput and improved reliability**.** It employs multiple antennas at both transmitter and receiver sides in order to get high spatial diversity gain which in turn helps to mitigate fading and provides improved system reliability. MIMO systems also support high data transmission rate through spatial multiplexing thus results in higher system capacity.

To further improve reliability and performance of MIMO systems, coding gain benefit can be added by combining MIMO systems with error correcting codes such as LDPC codes. Low density parity check (LDPC) codes can achieve capacity near Shannon's limit with good error correcting performance. In this thesis, we combined LDPC codes with orthogonal space time block codes (OSTBC) in MIMO systems. The performance of LDPC codes depends on the structure of the parity check matrix and decoding algorithms. In this thesis, bit-error rate (BER) performance of different LDPC codes like Quasi Cyclic (QC) and random which differs in their parity check matrix structures is analyzed. For low complexity decoding and better BER performance, log domain sum product algorithm (log-SPA) is used which has messages in the form of log-likelihood ratios (LLR). Performance of different LDPC codes with multi-user (MU-MIMO) system is also analyzed. Further, the approximate BER expressions are derived for above mentioned system with different antenna configurations.

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LIST OF ABBREVIATIONS

MIMO	Multiple Input Multiple Output
LDPC	Low Density Parity-Check
SPA	Sum Product Algorithm
LLR	Log-Likelihood Ratio
BER	Bit Error Rate
SISO	Single Input Single Output
MISO	Multiple Input Single Output
SIMO	Single Input Multiple Output
ZF	Zero Forcing
MMSE	Minimum Mean Square Error
OSTBC	Orthogonal Space Time Block Code
MU-MIMO	Multiuser Multiple Input Multiple Output
AWGN	Additive White Gaussian Noise
MAC	Multiple Access Channel
BC	Broadcast Channel
DPC	Dirty Paper Coding
DE	Density Evolution
SNR	Signal to Noise Ratio

Chapter 1

INTRODUCTION

1.1 Description of the Problem

Wireless network offer video and voice transmission in addition to data transmission. In wireless channel, there are many paths between the transmitter and receiver. Due to various paths we receive different versions of the transmitted signal at the receiver which results in fading which is the time variation of the channel due to small-scale effect of multi-path, as well as large-scale effect like pass loss by distance, attenuation and shadowing by obstacles. In wireless communication the transmitter and receiver communicate in air resulting in interference [1]. One of the major challenges of wireless system is limited availability of radio frequency spectrum. Due to these channel characteristics of wireless networks it becomes difficult to transmit video and voice reliably. So, to increase data rate, capacity and improve transmission reliability, MIMO systems are introduced [2].

Traditional communication system consists of three components: transmitter, channel and receiver as shown in Fig. 1.1. The transmitter transmits a signal through the channel. Channel is noisy and thus the signal is distorted. At the receiver, distorted signal is received. Signal distortion causes errors which are undesirable. In general, a certain level of distortion may be acceptable but it is necessary to design a system in which receiver can correct errors and bring it to the acceptable limit. This can be done through error correcting coding scheme. An error correcting coding scheme consists of an encoder-decoder pair. Channel encoder adds redundancy to the transmitted data and channel decoder exploits this redundancy in order to find and correct errors which are generated due to noise in channel [3].

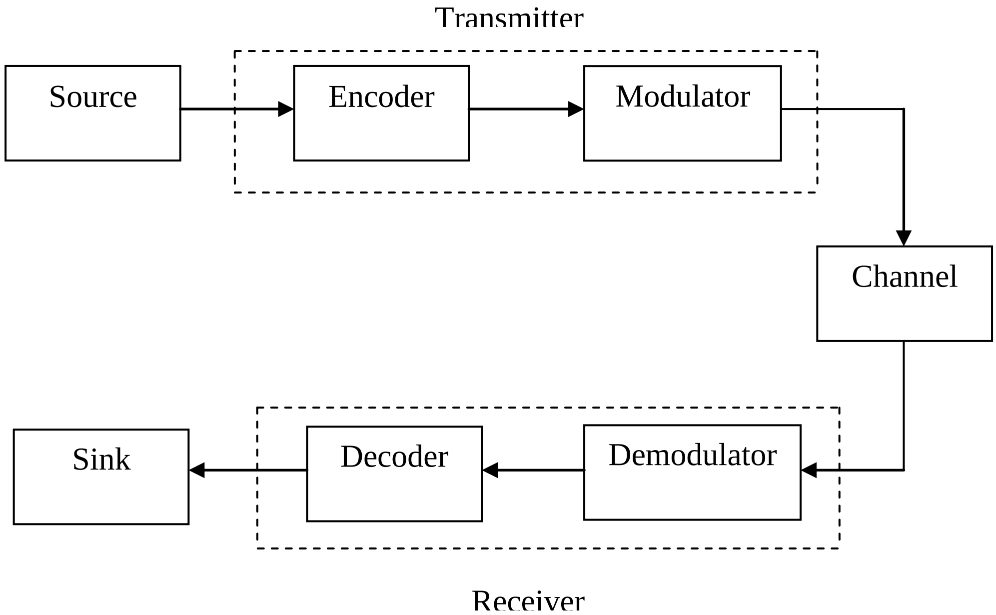

Fig. 1.1: Block diagram of digital communication system

In this thesis MIMO systems are coded using LDPC codes to further improve the system reliability.

1.2 Motivation

MIMO technology has aroused interest because of its possible applications in mobile communication, digital television, wireless local area networks and metropolitan area networks. MIMO provides better system performance as compared to SISO systems under the same transmission conditions. MIMO system improves channel capacity, provides spatial diversity which improves system robustness and reliability and also reduces inter-symbol interference [4].

In order to make MIMO systems more reliable, channel coding is done using error correction codes such as LDPC codes [5]. LDPC codes are a class of linear block codes which are considered the best error correction code to allow data transmission close to Shannon limit. The error performance of LDPC codes does not always exhibit an error floor due to the good Hamming distance spectra of these codes. Decoding of LDPC codes is very simple and it is iterative in nature and is graph based, thus parallel decoders can be implemented. LDPC codes use available bandwidth efficiently.

1.3 Objective

The objectives of this thesis are following.

- To analyze the BER performance of random LDPC coded MIMO-OSTBC system.
- To analyze the BER performance of QC-LDPC coded MIMO-OSTBC system and compare it with random LDPC coded MIMO system for different antenna configurations and with different codeword lengths.
- To analyze the BER performance of LDPC coded MU-MIMO system.

1.4 Thesis organization

In this thesis, work is organised into five chapters mentioned below

In Chapter 2, MIMO systems are discussed. The benefits of MIMO systems, system model, channel model, detection schemes for MIMO systems are discussed in detail. Further the study is done for MU-MIMO systems with its channel model and transmission methods. BER performance curve of this system is also shown.

Chapter 3 provides knowledge of LDPC codes which contains basics of LDPC codes, types of LDPC codes, LDPC encoding using Gauss Jordan elimination method and efficient encoding based on approximate lower triangulation, LDPC decoding using log domain sum product algorithm. Parameters for designing and optimizing LDPC codes are also discussed.

Chapter 4 focuses on the types of LDPC codes on the basis of construction i.e. random and QC-LDPC codes with a comparison of BER performance of both types of code.

Chapter 5 gives the system model for LDPC coded MIMO system and LDPC coded MU-MIMO system with all the simulation results. Further, the approximate BER expressions are derived for above mentioned system with different antenna configurations.

Finally, Chapter 6 concludes the thesis and provides suggestions for future work in this area.

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Chapter 2

MIMO SYSTEMS

2.1 MIMO Wireless Communication

Communication in wireless channels is affected by multi-path fading. Multipath is the arrival of the transmitted signal at an intended receiver through differing time delays, frequency shifts and/or differing angles due to the scattering of electromagnetic waves in the environment. Consequently, the power of the received signal fluctuates in time, frequency and /or space through the random superposition of the multipath components. This random fluctuation in signal power is known as fading and can severely affect the quality and reliability of wireless communication. Additionally, the limited power and scarce frequency bandwidth make the task of designing high data rate, high reliability wireless communication systems extremely challenging. To overcome these challenges, MIMO technology is used. MIMO technology is one of the most promising wireless technologies that can achieve wide coverage, high throughput and improved reliability by employing multiple antennas at both transmitter and receiver sides [2].

2.1.1 Benefits of MIMO technology

The benefits of MIMO technology that help in achieving these significant performance gains are array gain, spatial diversity gain, spatial multiplexing gain and interference reduction [6]. To define space time systems, different antenna configurations are shown in Fig. 2.1. We have four different configurations: SISO, SIMO, MISO and MIMO. SISO systems use one transmit as well as one receive antenna. SIMO systems use one transmit antenna and multiple receive antennas (M_R). MISO systems use multiple transmit antennas (M_T) and one receive antenna. MIMO systems have multiple transmit antennas (M_T) and multiple receive antennas (M_R).

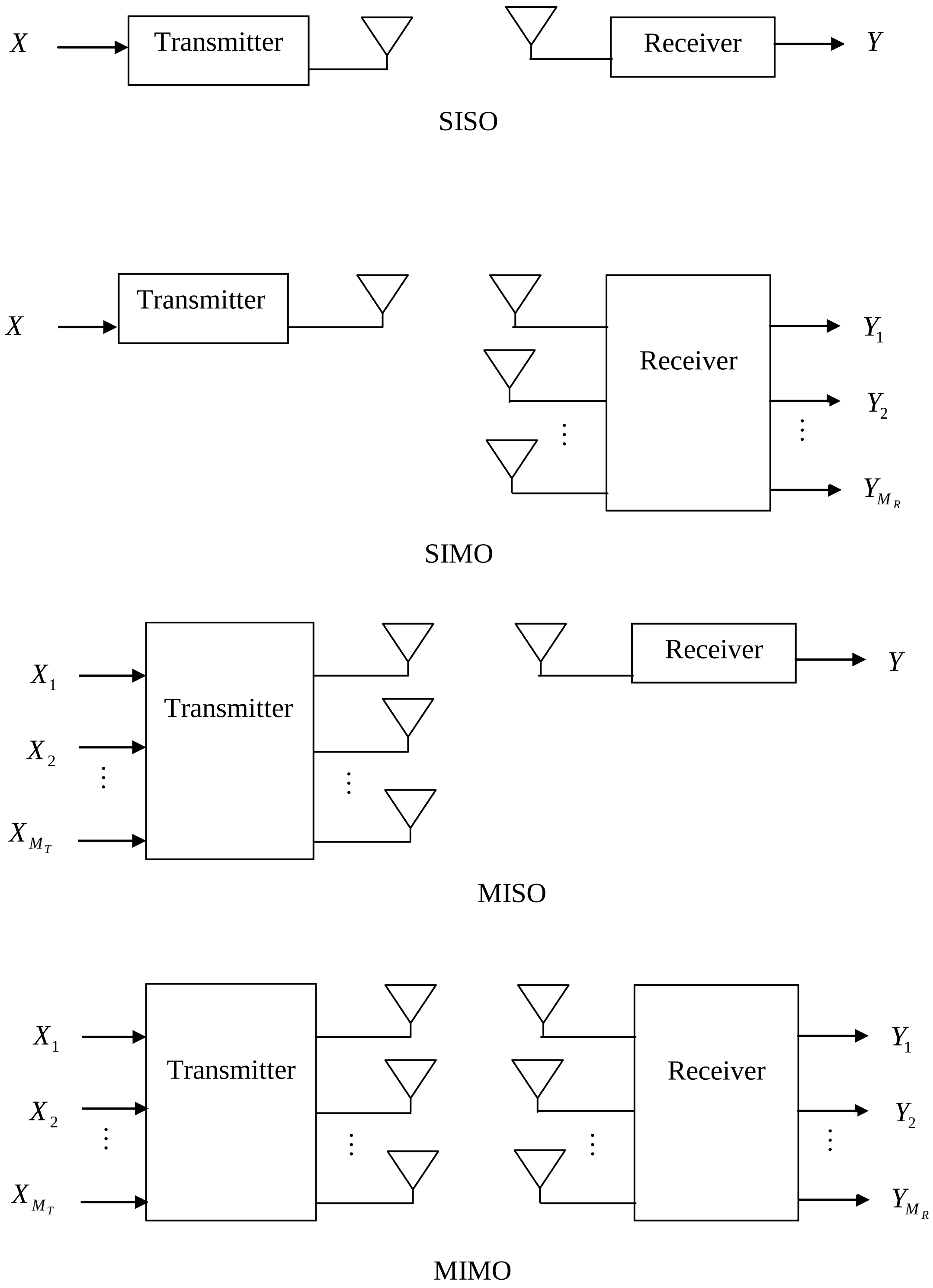


Fig. 2.1: Different antenna configurations in wireless systems

2.1.1.1 Array gain

Array gain is the increase in SNR at the receiver that results from coherent combining effect of the wireless signals at transmitter, receiver or both. In MISO case, we have transmitter array gain if the channel is known to the multiple antennas at the transmitter. Depending on the channel coefficients, the transmitter adjusts the weights for coherent combining at the receiver with single antenna. In SIMO case, we have receiver array gain if the channel is known to the multiple antennas at the receiver and is unknown to the transmitter antenna. The receiver adjusts the weights of incoming signals for coherent combining at the output and thus SNR at the receiver is increased which results in improve in range and coverage of a wireless network.

2.1.1.2 Diversity Gain

Diversity is a technique which is used to combat fading. Fading as discussed before is the random fluctuation in the signal strength or power in multipath environment. Diversity provides multiple copies of the transmitted signal in space, time or frequency. These multiple copies are ideally independent and will fade independently. Thus the probability that one of the multiple copies is not in deep fade increases, thus the reliability of transmission is improved. Diversity order is equal to the number of independent paths. If we have M_T transmit antennas and M_R receive antennas then diversity order will be $M_T M_R$.

2.1.1.3 Spatial multiplexing gain

Spatial multiplexing means transmission of multiple independent data streams within the bandwidth of operation. In $M_R \times M_T$ MIMO channel, data stream is split into M_R data streams, modulated and transmitted simultaneously from M_T transmit antennas. At the receiver, these data streams are combined to recover the original data stream. Thus data rate of the system increases. Spatial multiplexing gain is given by $\min\{ M_T, M_R \}$.

2.1.1.4 Interference reduction

Interference exists because of multiple users sharing same time and frequency resources. In MIMO systems, interference can be relieved by exploiting spatial dimension for increasing the separation between the users. Spatial dimension can also be exploited for interference avoidance. In interference avoidance, the signal energy is directed towards the intended user and thus interference to other users is minimized.

2.1.2 MIMO system model

Fig. 2.2 shows the basic building blocks that comprise a MIMO communication system [2].

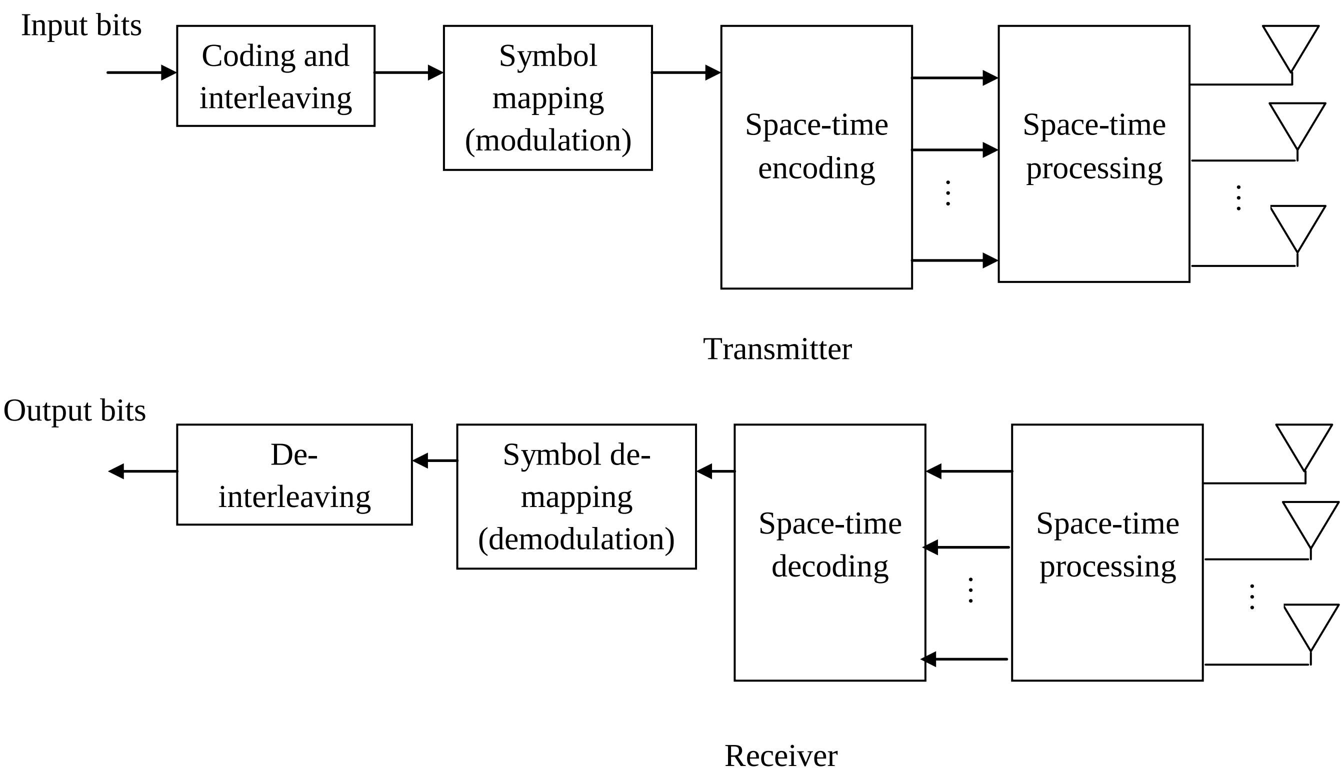


Fig. 2.2: Block diagram of MIMO communication system

Initially, the binary input signals are encoded and interleaved. Interleaving is reordering of encoded data to protect it against burst errors. These interleaved codeword is mapped into data symbols such as BPSK, QPSK, QAM, etc. by the symbol mapper. These data symbols are input to space time encoder which produces one or more spatial data streams as its output. Then we have space time pre-coding block that map spatial data streams to the transmit antennas. The signals launched from the transmit antennas propagate through the channel and arrive at the receive antenna array. The receiver collects the signals at the output of each receive antenna element and reverses the transmitter operations in order to decode the data.

2. 2 MIMO Channel Model

Consider a MIMO system as shown in Fig. 2.3 with M_T transmit antennas and transmitted vector $X = [X_1, X_2, ...X_{M_T}]^T$, each transmitted signal goes through the wireless channel and is received at M_R receive antennas and received vector is $Y = [Y_1, Y_2, ...Y_{M_R}]^T$ [6].

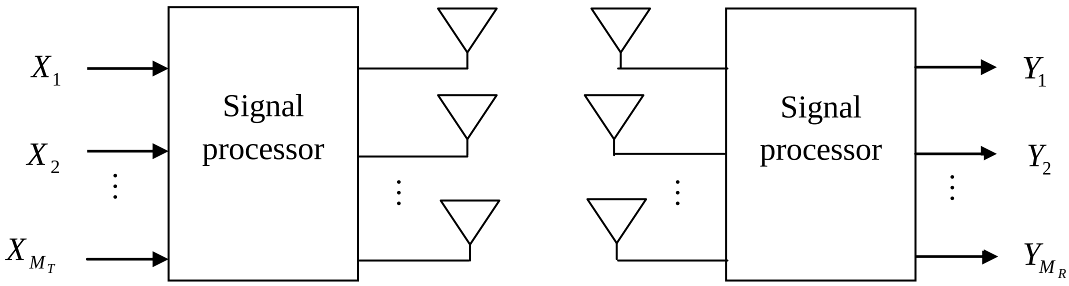


Fig. 2.3: MIMO channel model

The channel is considered to be a Gaussian channel. If the channel is unknown at the transmitter, we assume that the signals transmitted from each antenna have equal power E_X / M_T. The covariance matrix for the transmitted signal is given by

$$\Re_{XX} = \frac{E_X}{M_T} I_{M_T} \tag{2.1}$$

where E_X is the transmitted power and I_{M_T} is the $M_T \times M_T$ identity matrix.

The transmitted bandwidth is so narrow that its frequency response can be considered flat (i.e. the channel is memory less). Channel matrix H is a complex matrix of order $M_R \times M_T$ as shown below

$$\mathrm{H} = \begin{bmatrix} h_{11} & \cdots & h_{1M_R} \\ \vdots & \ddots & \vdots \\ h_{M_R 1} & \cdots & h_{M_R M_T} \end{bmatrix} \tag{2.2}$$

where h_{ij} is the fading coefficient or the complex path gain from transmit antenna *i* to receive antenna *j*. It is assumed that the received power for each received antenna is equal to the total transmitted power E_X . Consequently, the signal attenuation, antenna gains are ignored. Thus for a deterministic channel, the normalization constraint for the elements of H can be obtained as

$$\sum_{i=1}^{M_T} \left| h_{ij} \right|^2 = M_T,\ j = 1,2,...,M_R \tag{2.3}$$

If we take random channel instead of deterministic channel, then the normalization will apply to the expected value of (2.3). It is assumed that the channel matrix is unknown to the transmitter and is known to the receiver. For the transmitter to know the channel, the information from the receiver is feedback to the transmitter.

The channel output is a linear superposition of the faded versions of the inputs perturbed by noise. The noise at the receiver is $N_0 = [N_{01}, N_{02},...N_{0M_T}]^T$ which is independent and identically distributed (i.i.d.) circularly symmetric, complex additive white Gaussian noise (AWGN) vector with zero-mean and its covariance matrix is given as

$$\Re_{N_0 N_0} = \mathrm{E}(N_0 N_0^H) \tag{2.4}$$

If there is no correlation between elements of N_0 , the covariance matrix of the receiver noise is given as

$$\Re_{N_0 N_0} = \sigma_n^2 I_{M_R} \tag{2.5}$$

Since, we assumed that total received power per antenna is equal to total transmitted power, the received SNR (γ) can be written as

$$\gamma = \frac{E_X}{\sigma_n^2} \tag{2.6}$$

The channel output vector Y is given by

$$Y = \mathrm{H}X + N_0 \tag{2.7}$$

The covariance matrix of the received signal can be defined as

$$\Re_{YY} = \mathrm{E}(YY^H) \tag{2.8}$$

$$\Re_{YY} = \mathrm{H}\Re_{XX}\mathrm{H}^H \tag{2.9}$$

Now we will discuss channel model for 2×2 MIMO system [7] which is shown in Fig. 2.4.

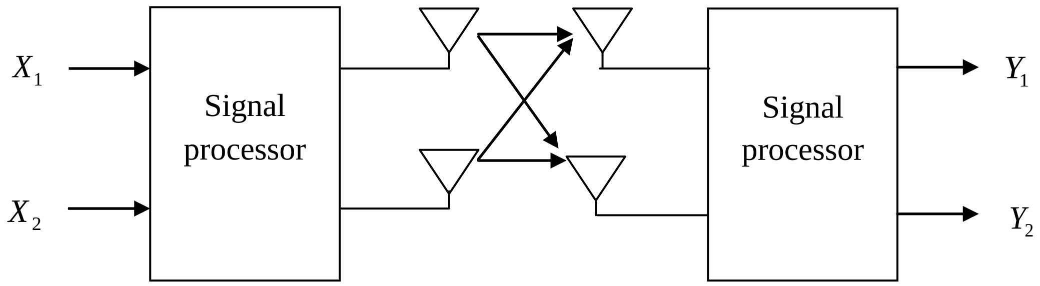


Fig. 2.4: 2×2 MIMO channel model

For this, two transmitted signals are taken X_1 and X_2. X_1 is the transmitted signal for antenna 1 and X_2 is the transmitted signal for antenna 2. Thus we get two received signals Y_1 and Y_2. Y_1 at time t and Y_2 at $t + T$. Received signal at antenna 1 is given as

$$Y_1 = Y(t) = h_{11}X_1 + h_{21}X_2 + N_{01} \tag{2.10}$$

$$Y_2 = Y(t+T) = h_{12}X_1 + h_{22}X_2 + N_{02} \tag{2.11}$$

Above equations can be written in matrix form as

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{21} \\ h_{12} & h_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} N_{01} \\ N_{02} \end{bmatrix} \tag{2.12}$$

2.3 MIMO Channel Capacity

Channel capacity as defined by Shannon [8] is the maximum data rate that can be transmitted through the channel and data is received with negligible errors. If the transmitted and received data is random in nature then the channel capacity refers to maximum mutual information between them which can be mathematically expressed as

$$\overline{C} = \max \, I(X;Y) \tag{2.13}$$

Shannon derived normalized capacity for band limited white Gaussian channel. Normalized capacity means capacity per unit bandwidth and is given as

$$\overline{C} = \log_2(1+\bar{\gamma}) \tag{2.14}$$

where $\bar{\gamma}$ is the average received SNR.

2.3.1 Capacity of a deterministic MIMO channel

For a MIMO system, received signal vector is given as

$$Y = \mathrm{H}X + N_0 \tag{2.15}$$

Consider transmitted symbol vector $X \in C^{M_T \times 1}$ then

$$Y = \sqrt{\frac{E_X}{M_T}}\mathrm{H}X + N_0 \tag{2.16}$$

Covariance matrix of the received signal is calculated as

$$\Re_{YY} = \mathrm{E}(YY^H)$$

$$= \mathrm{E}\left\{\left(\sqrt{\frac{E_X}{M_T}}\mathrm{H}X + N_0\right)\left(\sqrt{\frac{E_X}{M_T}}\mathrm{H}^H X^H + {N_0}^H\right)\right\}$$

$$= \mathrm{E}\left\{\left(\frac{E_X}{M_T}\mathrm{H}XX^H\mathrm{H}^H + N_0 N_0{}^H\right)\right\}$$

$$= \frac{E_X}{M_T}\mathrm{E}\left\{\left(\mathrm{H}XX^H\mathrm{H}^H + N_0 N_0{}^H\right)\right\} \tag{2.17}$$

Mutual information can be expressed as

$$I(X;Y) = \log_2 \det\left(I_{M_R}\ \frac{E_X}{M_T\sigma_n^2}\mathrm{H}\Re_{XX}\mathrm{H}^H\right) \tag{2.18}$$

Thus the channel capacity of deterministic MIMO channel is expressed as

$$\overline{C} = \max_{trace\Re_{XX}=M_T} \log_2 \det\left(I_{M_R}\ \frac{E_X}{M_T\sigma_n^2}\mathrm{H}\Re_{XX}\mathrm{H}^H\right) \tag{2.19}$$

If H is unknown at the transmitter,

$$\Re_{XX} = I_{M_T} \tag{2.20}$$

$$\overline{C} = \log_2 \det\left(I_{M_R}\ \frac{E_X}{M_T\sigma_n^2}\mathrm{H}\mathrm{H}^H\right) \tag{2.21}$$

Using the eigen value decomposition,

$$\mathrm{H}\mathrm{H}^H = U \wedge U^H \tag{2.22}$$

where U is unitary matrix $UU^H = I_{M_R}$ and $\wedge$ is a diagonal matrix of order $M_R \times M_R$. Thus (2.21) can be modified as

$$\overline{C} = \log_2 \det\left(I_{M_R}\ \frac{E_X}{M_T\sigma_n^2}\wedge\right) \tag{2.23}$$

$$\overline{C} = \sum_{i=1}^{r_\mathrm{H}} \log_2 \det\left(I_{M_R}\ \frac{E_X}{M_T\sigma_n^2}\alpha_i\right) \tag{2.24}$$

where r_{H} is the rank of H which is given as

$$r_{\mathrm{H}} = \min(M_T, M_R) \tag{2.25}$$

It can be seen from (2.24), that a MIMO channel is converted into r_{H} virtual SISO channels. The transmit power for each channel is $\frac{E_X}{M_T}$ I and channel gain α_i for i^{th} SISO path.

If H is full rank (i.e. $M_T = M_R$, $r_{\mathrm{H}} = M_T = M_R$) and total channel gain is fixed (i.e. $\|\mathrm{H}\|_F = \sum_{i=1}^{r_{\mathrm{H}}} \alpha_i = \xi$), the channel capacity is maximized when the singular values for H are same for all parallel channels. Therefore,

$$\alpha_i = \frac{\xi}{M_T} \text{ for all } i\text{'s} \tag{2.26}$$

The above equation shows that the MIMO capacity is maximized when the channel is orthogonal

$$\mathrm{H}\mathrm{H}^H = \mathrm{H}^H\mathrm{H}\frac{\xi}{M_T} I_{M_T} \tag{2.27}$$

This increases the capacity to M_T or M_R times to that of each parallel channel, which is given as

$$\overline{C} = M_T \log_2\left(1 + \frac{\xi E_X}{M_T \sigma_n^2}\right) \tag{2.28}$$

For the case of SIMO channel, consider the channel gain to be *h*, thus r_{H} =1 and $\alpha_i = \|h\|_F^2$. Consequently, regardless of availability of channel information at the transmitter side, the channel capacity is expressed as

$$\overline{C}_{SIMO} = \log_2\left(1 + \frac{E_X}{\sigma_n^2}\|h\|_F^2\right) \tag{2.29}$$

If $|h_i|^2 = 1$ for i=1, 2,..., M_R and $\|h\|_F^2 = M_R$, capacity can be rewritten as

$$\overline{C}_{SIMO} = \log_2\left(1 + \frac{E_X}{\sigma_n^2} M_R\right) \tag{2.30}$$

From (2.30), it is observed that the channel capacity increases logarithmically with antennas.

For the case of MISO channel, channel capacity is expressed as

$$\overline{C}_{MISO} = \log_2\left(1 + \frac{E_X}{M_T \sigma_n^2}\|h\|_F^2\right) \tag{2.31}$$

If $|h_i|^2 = 1$ for i=1, 2,...,M_T and $\|h\|_F^2 = M_T$, capacity can be rewritten as

$$\overline{C}_{SIMO} = \log_2\left(1 + \frac{E_X}{\sigma_n^2}\right) \tag{2.32}$$

2.3.2 Capacity of a random MIMO channel

In general MIMO channels change randomly, thus channel capacity is expressed in its time average form. It is assumed that random channel is an ergodic process. Ergodic channel capacity is given as

$$\begin{aligned}\hat{C} &= \mathrm{E}\left(\overline{C}(\mathrm{H})\right) \\ &= \mathrm{E}\left\{\max_{trace(\Re_{xx}) = M_T} \log_2 \det\left(I_{M_R} + \frac{E_X}{M_T \sigma_n^2}\mathrm{H}\Re_{XX}\mathrm{H}^H\right)\right\}\end{aligned} \tag{2.33}$$

Ergodic channel capacity for open loop and closed loop is defined as

$$\hat{C}_{OL} = \mathrm{E}\left\{\sum_{i=1}^{r_{\mathrm{H}}} \log_2 \det\left(1 + \frac{E_X}{M_T \sigma_n^2}\alpha_i\right)\right\} \tag{2.34}$$

$$\hat{C}_{CL} = \mathrm{E}\left\{ \max_{\sum_{i=1}^{r_{\mathrm{H}}} \mathrm{P}_i = M_T} \sum_{i=1}^{r_{\mathrm{H}}} \log_2 \det\left(1 + \frac{E_X}{M_T \sigma_n^2} \alpha_i \mathrm{P}_i\right)\right\} \tag{2.35}$$

In case of closed loop, channel information on the transmitter side is also taken. In (2.35), P_i is the power transmitted by i^{th} antenna. Simplifying (2.35), we get

$$\hat{C}_{CL} = \mathrm{E}\left\{ \sum_{i=1}^{r_{\mathrm{H}}} \log_2 \det\left(1 + \frac{E_X}{M_T \sigma_n^2} \alpha_i \mathrm{P}_i^{opt}\right)\right\} \tag{2.36}$$

where P_i^{opt} is the optimal power.

2.4 Linear Detection Schemes for MIMO Systems

At the receiver, we get the superposition of transmitted signals. Linear detectors are used at the receiver to recover the desired signal from multiple transmitted signals.

2.4.1 Zero forcing (ZF) detector

In ZF detector [9], the signal from each transmit antenna is considered as the desired signal and other signals are considered as interferers. The amplitude of interferers are set to zero by inverting the channel response. When the channel matrix of the MIMO system is a square matrix (i.e. the number of rows of the matrix equals to number of column of the matrix) and non-singular then inverse of the channel matrix is taken to recover the desired signal.

$$\hat{X} = \mathrm{H}^{-1} Y \tag{2.37}$$

where $\hat{X}$ is the detected signal.

When the channel matrix is not a square matrix and it is a tall matrix (i.e the number of rows of the matrix is greater than the number of column of the matrix), then matrix inverse cannot be calculated. In this case pseudo inverse of the channel matrix is calculated and thus we get the desired signal as

$$\hat{X} = (\mathrm{H}^H\mathrm{H})^{-1}\mathrm{H}^H Y \tag{2.38}$$

Substituting the value of Y in (2.38) from (2.15), we get

$$\begin{aligned}\hat{X} &= \mathrm{H}^{-1}(\mathrm{H}X) + \mathrm{H}^{-1}N_0 \\ &= \psi X + \tilde{N}_0\end{aligned} \tag{2.39}$$

ψ is the residual interference and $\tilde{N}_0$ is the correlated noise at ZF detector. ZF focuses on cancelling interference at the expense of noise enhancement. This problem can be solved by using MMSE.

2.4.2 Minimum mean square error (MMSE) detector

MMSE detector [9] is designed to suppress noise enhancement and at the same time remove interference.

$$\hat{X} = (\mathrm{H}^H\mathrm{H} + \sigma_n^2 I)^{-1}\mathrm{H}^H Y \tag{2.40}$$

where I is the identity matrix. It provides better bit error rate than ZF detector, however the performance of MMSE approaches the performance of ZF as SNR tends to infinity.

2.5 Space Time Block Code (STBC)

The goal of space-time coding is to achieve the maximum diversity of $M_T M_R$, the highest possible throughput and the maximum coding gain [1].

2.5.1 Alamouti space time code

Alamouti developed a complex orthogonal space time block code for two transmit antennas. In the Alamouti encoder [10], two consecutive symbols X_1 and X_2 are encoded with the help of a space time codeword matrix as shown in (2.41).

$$X = \begin{bmatrix} X_1 & -X_2^* \\ X_2 & X_1^* \end{bmatrix} \tag{2.41}$$

Alamouti codes are called space time code because in this the signal is transmitted from two transmit antennas and in two time periods as shown in Fig. (2.5)

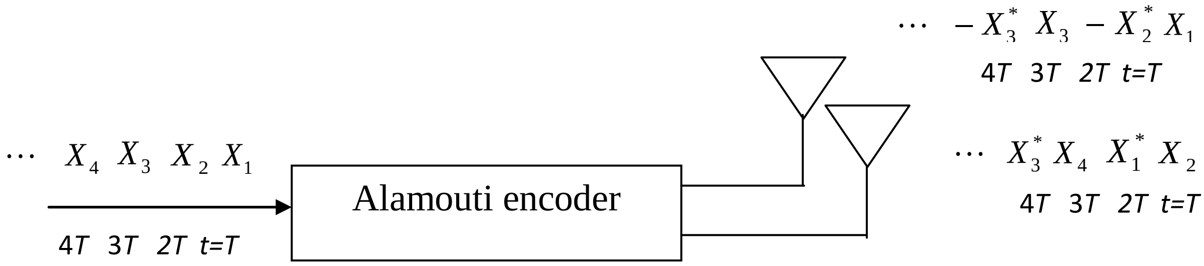


Fig. 2.5: Alamouti encoder

In first time instant, X_1 is transmitted from transmit antenna 1 and X_2 is transmitted from transmit antenna 2. In second time instant, $-X_2^*$ is transmitted from transmit antenna 1 and X_1^* is transmitted from transmit antenna 2. This is a full rate code.

2.5.2 Orthogonal space time code (OSTBC)

The received signals at the i^{th} receive antenna for the first and second time slots are given by [11]

$$Y_i(1) = \frac{1}{\sqrt{M_T}}(\mathrm{H}_{1,i}X_1 + \mathrm{H}_{2,i}X_2) + N_0(1) \tag{2.42}$$

$$Y_i(2) = \frac{1}{\sqrt{M_T}}(-\mathrm{H}_{1,i}X_1 + \mathrm{H}_{2,i}X_2) + N_0(2) \tag{2.43}$$

2. 6 MU-MIMO

Channel capacity of single user $M_R \times M_T$ MIMO system is proportional to min $\{M_T, M_R\}$. Most of the communication systems deal with multiple users who share the same radio resources. In downlink MU-MIMO systems, the base station simultaneously transmit signals to several mobile stations using the same frequency band; in this case a user receives signals of other users also with its desired signal. The signals of other users are undesired and lead to interference which degrades system performance.

2. 6.1 Mathematical model of MU- MIMO

In MU- MIMO [12], assume the number of independent users be U, number of antennas at the mobile station to be M_T and the number of antennas at the base station to be M_R. We have two channels here uplink and downlink. Uplink channel is known as multiple access channel (MAC) and downlink channel is known as broadcast channel (BC) [13].

First we will consider the multiple access channels as shown in Fig. 2.6.

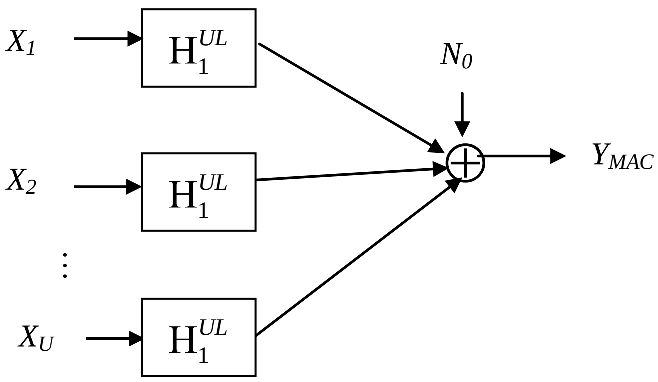


Fig. 2.6: Uplink multiuser channel for multiuser MIMO system: MAC

Let X_u be the transmit signal vector of order $M_T \times 1$ where, u = 1, 2, ..., U and Y_{MAC} be the received signal vector of order $M_R \times 1$ at the base station which is given as

$$Y_{MAC} = \mathrm{H}_1^{UL} X_1 + \mathrm{H}_2^{UL} X_2 + ... + \mathrm{H}_U^{UL} X_U + N_0 \tag{2.44}$$

In t

he matrix format it can be represented as

$$Y_{MAC} = \begin{bmatrix} \mathrm{H}_1^{UL} & \mathrm{H}_2^{UL} & \cdots & \mathrm{H}_U^{UL} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_U \end{bmatrix} + N_0$$

$$= \mathrm{H}_u^{UL} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_U \end{bmatrix} + N_0 \tag{2.45}$$

where H_u^{UL} is the channel gain of order $M_R \times M_T$ between u user MS and BS and N_0 is the zero mean circularly symmetric complex Gaussian noise vector.

Now we will consider broadcast channel which is shown in Fig. 2.7.

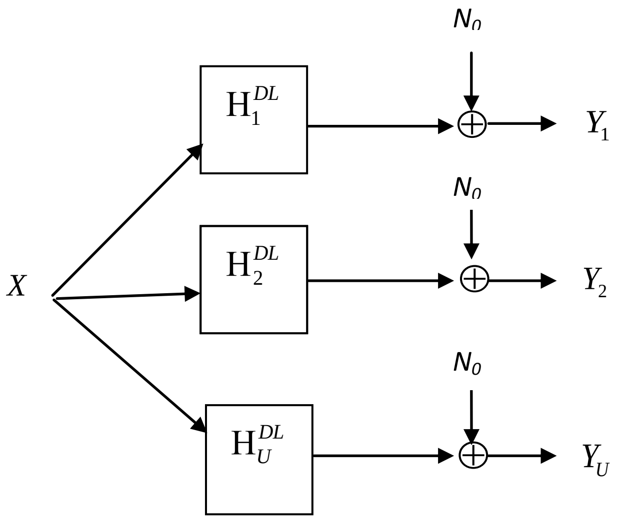


Fig. 2.7: Downlink multiuser channel for multiuser MIMO system: BC

Let X be the transmit signal vector from BS of order $M_T \times 1$ and Y_U be the received signal vector of order $M_R \times 1$ where, u = 1, 2, ... , U which can be represented as

$$Y_u = \mathrm{H}_u^{DL} X + N_{0u} \tag{2.46}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_U \end{bmatrix} = \begin{bmatrix} \mathrm{H}_1^{DL} \\ \mathrm{H}_2^{DL} \\ \vdots \\ \mathrm{H}_U^{DL} \end{bmatrix} X + \begin{bmatrix} N_{01} \\ N_{02} \\ \vdots \\ N_{0U} \end{bmatrix} \tag{2.47}$$

2.6.2 Transmission method for broadcast channel

There are various transmission methods such as channel inversion, block diagonalization, dirty paper coding, Tomlinshon-Harashima precoding. In this thesis, we fill focus on channel inversion method which is described below.

Assume $M_{MS} = 1$ for all users and $U = M_{BS}$. Let u^{th} user signal be $\hat{X}_u$ and channel matrix between BS and u^{th} user be H_U^{DL} of order [10] . The received signal of u^{th} user be given as

$$Y_u = \mathrm{H}_u^{UL} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \vdots \\ \hat{X}_U \end{bmatrix} + N_0 \tag{2.48}$$

The received signal for all users is expressed as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_U \end{bmatrix} = \begin{bmatrix} \mathrm{H}_1^{UL} \\ \mathrm{H}_2^{UL} \\ \vdots \\ \mathrm{H}_U^{UL} \end{bmatrix} \begin{bmatrix} \hat{X}_1 \\ \hat{X}_2 \\ \vdots \\ \hat{X}_U \end{bmatrix} + \begin{bmatrix} N_{01} \\ N_{02} \\ \vdots \\ N_{0U} \end{bmatrix} \tag{2.49}$$

The received signal at each user terminal in (2.49) is a scalar quantity; each user is equipped with one antenna. Therefore, interference due to other signals cannot be cancelled. To remove this interference, pre-coding techniques are used. In MU-MIMO, channel inversion is used as ZF is used in MIMO systems. The difference between ZF and channel inversion is that H is replaced by H^{DL}.

2.6.3 Channel capacity of MU-MIMO systems

Based on the Mathematical model of MU- MIMO system, channel capacity of MAC and BC is discussed in AWGN channel [10].

First the capacity of MAC is discussed. Let P_u denote the power of the u^{th} user in the MU-MIMO system with U users and R_u denote the data rate of the u^{th} user in the MU-MIMO system with U users [14]. Capacity region of MAC with U = 2 and $M_T = 1$ is shown in Fig. 2.8.

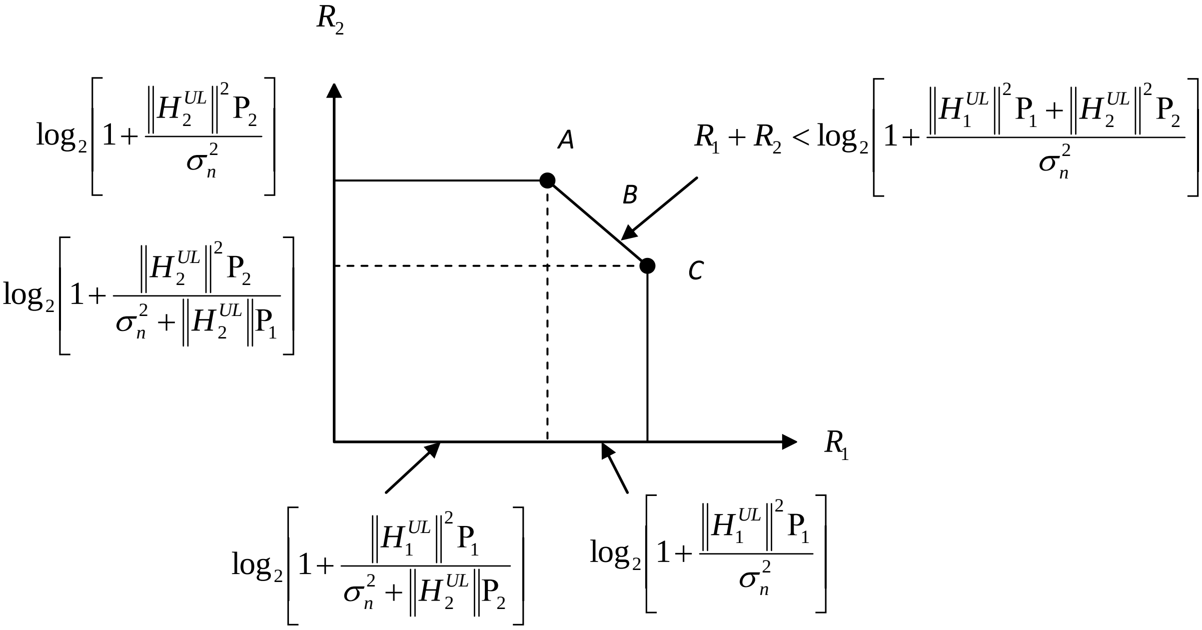


Fig. 2.8: Capacity region of MAC with U = 2 and $M_T = 1$

The capacity region is expression as

$$R_1 \le \log_2\left[1+\left\|H_1^{UL}\right\|^2 \mathrm{P}_1\right] \tag{2.50}$$

$$R_2 \le \log_2\left[1+\left\|H_2^{UL}\right\|^2 \mathrm{P}_2\right] \tag{2.51}$$

$$R_1 + R_2 \le \log_2\left[1 + \left\|H_1^{UL}\right\|^2 \mathrm{P}_1 + \left\|H_2^{UL}\right\|^2 \mathrm{P}_2\right] \tag{2.52}$$

The received signal is expressed as

$$Y_{MAC} = H_1^{UL} X_1 + H_2^{UL} X_2 + \begin{bmatrix} N_{0_1} \\ N_{0_2} \end{bmatrix} \tag{2.53}$$

$$Y_{MAC} = \begin{bmatrix} H_1^{UL} & H_2^{UL} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} N_{0_1} \\ N_{0_2} \end{bmatrix} \tag{2.54}$$

In order to achieve the point *A* in Fig. 2.8, we assume that X_1 interferes with the signal from user 2 and we detect X_1. Once X_1 is detected correctly, it can be cancelled from the received signal. This is possible only when data rate is less than the corresponding channel capacity

$$R_1 \le \log_2\left[\frac{1 + \left\|H_1^{UL}\right\|^2 \mathrm{P}_1}{1 + \left\|H_2^{UL}\right\|^2 \mathrm{P}_2}\right] \tag{2.55}$$

$$\hat{Y}_{MAC} = Y_{MAC} - H_1^{UL} X_1 = H_2^{UL} X_2 + \begin{bmatrix} N_{0_1} \\ N_{0_2} \end{bmatrix} \tag{2.56}$$

$\hat{Y}_{MAC}$ is an interference free signal. X_2 is detected from $\hat{Y}_{MAC}$ to achieve rate of $R_2 \le \log_2\left[1 + \left\|H_2^{UL}\right\|^2 \mathrm{P}_2\right]$.The point *C* can be achieved in the other way around. Data rate at any other point on the line *AC*, say point *B* can be achieved by time-sharing or rate-splitting between the multiple access schemes in point *A* and point *C*. Sum rate capacity of the MAC channel is proportional to $\min(M_R, U.M_T)$ [14].

The capacity region of broadcast channel is unknown since it belongs to the category of non-degraded vector channels. However, by using the DPC strategy in a duality between broadcast channels and MAC has been established in [15].

2.7 BER Performance of the System

BER is the ratio of number of bits in error to the total number of bits [16]. BER performance can be analyzed by analyzing BER at a given signal to noise ratio (SNR). The errors are received bits that have different values than the transmitted bits. SNR is a measure, that compare desired signal level with the noise level and it is given as

$$SNR = 10\log\frac{E_S}{N_0} \tag{2.57}$$

where E_S is the signal energy. The higher the SNR, the less will be the noise. Thus by increasing SNR, number of bits in error decreases and therefore BER performance is improved.

2.8 Simulation Results

Design parameters for the simulation results are shown in Table 2.1.

Table 2.1 Simulation Parameters for MIMO systems

Parameters	Values
Modulation	BPSK
Channel Model Used	Rayleigh fading channel
Channel Fading	Rayleigh Independent and identically distributed
Channel Estimation	Ideal

Fig. 2.9 shows BER performances for 2×2 MIMO system in which we obtain BER of 3×10^{-2} around 17 dB, 10 dB, 6 dB in SISO, MIMO and OSTBC-MIMO respectively. Thus, it shows that we get best BER performance with OSTBC. Fig. 2.10 shows BER

performances of MIMO system with increased diversity order. On comparing results of 2×2 and 4×4, it can be concluded that we get better BER performance with 4×4 configuration because of its higher diversity order.

Fig. 2.11 shows BER performance comparison of 2×2 MIMO with 2×2 MU-MIMO systems with 4 users. Transmission method used here is channel inversion. It can be seen from this Fig.2.11 that MIMO achieves better performance than MU-MIMO because with increase in number of users, interference increases, which worsens the BER performance.

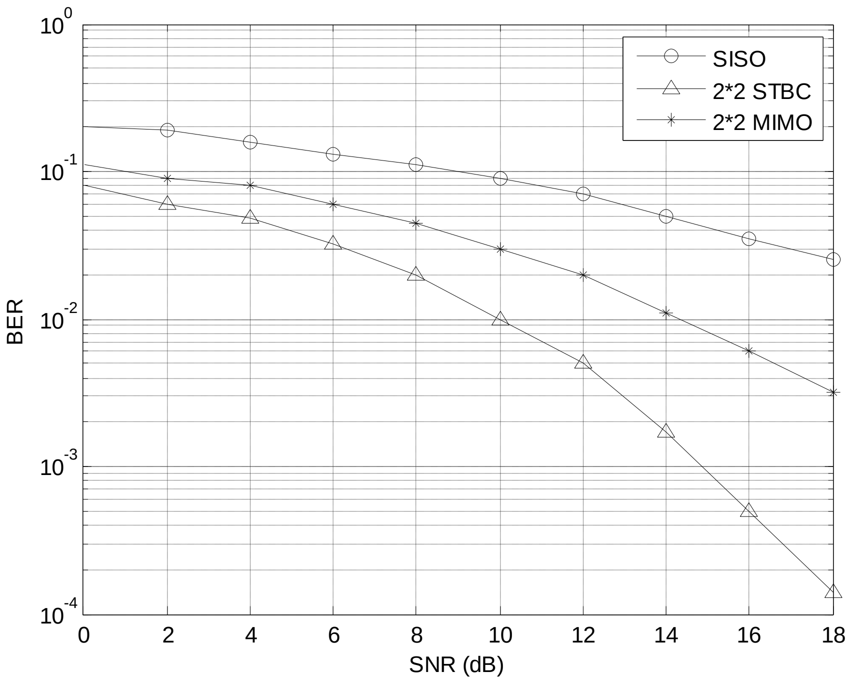


Fig. 2.9: BER performance of SISO, 2×2 MIMO and 2×2 STBC systems

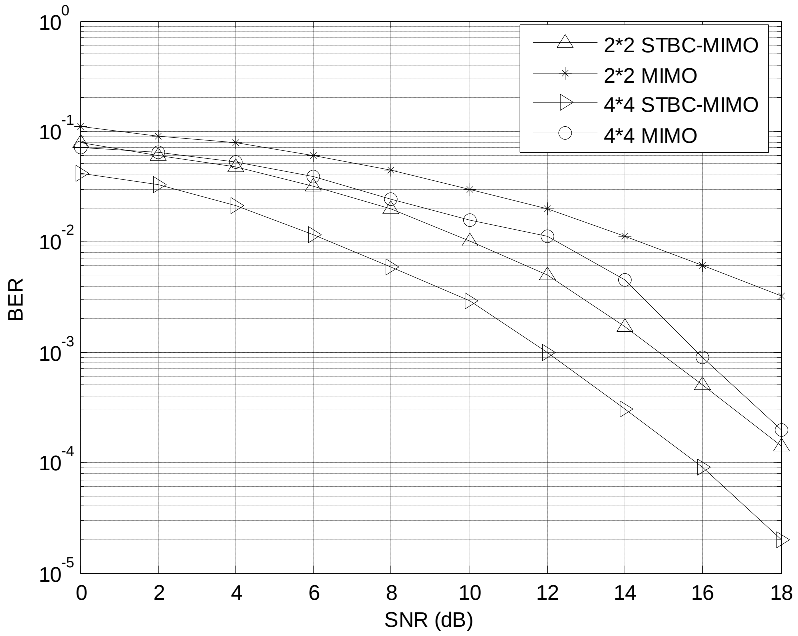


Fig. 2.10: BER performance comparison of 4×4 MIMO and 4×4 MIMO-STBC systems with 2×2 MIMO and 2×2 MIMO-STBC systems

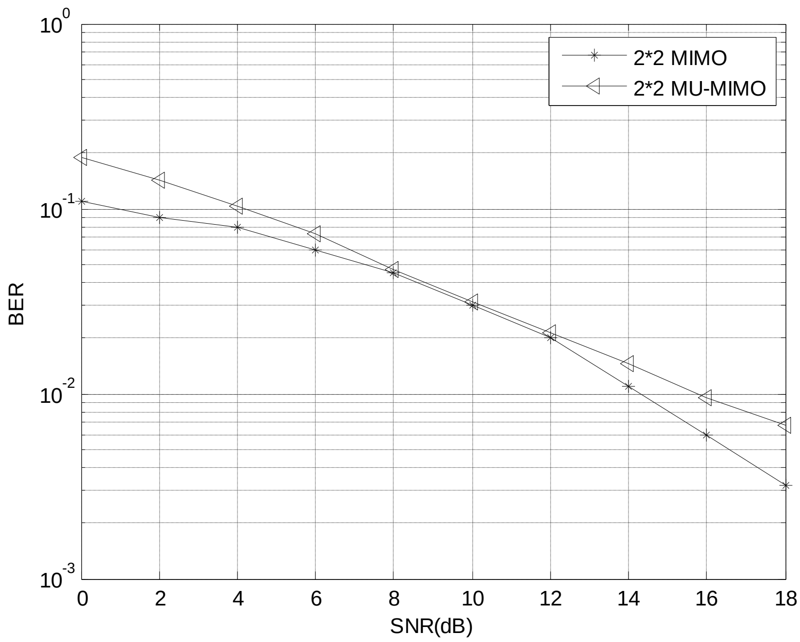


Fig. 2.11: BER performance comparison of 2×2 MIMO and 2×2 MU-MIMO system

Chapter 3

LOW DENSITY PARITY CHECK (LDPC) CODES

Low-density parity-check (LDPC) codes are forward error-correction codes [17]. They were proposed by Robert Gallager in 1962 in his PhD thesis at MIT. However, they remained neglected for over 35 years since they could not attain the hardware required for the encoding process. In the mean time the field of forward error correction codes [18] was dominated by highly structured algebraic block and convolutional codes [5]. With the enhancement in technology and development of relevant theories such as Turbo codes, LDPC codes were rediscovered by Mackay and Neal in 1996 [19].

3.1 Basics of LDPC Codes

LDPC codes are a class of linear block codes. In this section, overview of linear block codes is given and thus LDPC codes are discussed.

3.1.1 Linear block codes

The error correcting codes are obtained by adding additional bits to the original message to be transmitted. These bits are redundant and used to detect and correct errors on the received data. In linear block codes, the original message is divided into blocks of fixed lengths of K bits such as $m = [m_1, m_2, ..., m_K]$ i.e., one of 2^K possible messages [5]. The encoder takes this message and generates a codeword $C = [C_1, C_2, ..., C_N]$ of length N. Here, $N > K$ and $M = N - K$ is the redundancy. The amount of information sent per codeword is measured by the code rate $R = K/N$. A linear code is characterized by the fact that the sum of any subset of codeword is always equal to a codeword from the code space.

In encoding process the encoder stores all possible combination of codeword but for large K, it is inefficient. By using linear generator matrix, linear block codes reduce the complexity of encoding. Assume a linear block code $C(N,K)$ and $[g_1, g_2, ..., g_K]$ be K linearly dependent vectors. Each codeword is a linear combination of these vectors:

$$C = m_1 g_1 + m_2 g_2 + ... + m_K g_K \tag{3.1}$$

All vector and matrix operations are done in modulo-2 arithmetic. The generator matrix is given as

$$G = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_K \end{bmatrix} \tag{3.2}$$

For a given message vector *m*, the corresponding codeword can be obtained by multiplying it with the generator matrix G as

$$C = mG \tag{3.3}$$

$$C = [m_1, m_2, ...m_K] \times \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_K \end{bmatrix} = m_1 g_1 + m_2 g_2 + ... + m_K g_K \tag{3.4}$$

Dual space of the code C is used to derive the parity check matrix H i.e., $GH^T = 0$. The parity check matrix H is given as

$$H = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{N-K} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N-K,1} & h_{N-K,2} & \cdots & h_{N-K,N} \end{bmatrix} \tag{3.5}$$

It can also be verified that the parity-check equations can be obtained from the parity check matrix H i.e., $CH^T = 0$. Hence, this matrix completely specifies the given block code.

The construction of parity check matrix can be done in two forms, systematic and non-systematic. Systematic form is shown in Fig. 3.1. In systematic form $G = [I_K \mid P]$ and H is derived as $H = [-P^T \mid I_M]$, where I_K is $K \times K$ identity matrix and P is a random matrix with $M \times N$ dimensions.

K message bits	(N-K) parity check bits

Fig. 3.1: Systematic form of a codeword of a block code

3.1.2 LDPC codes

LDPC codes are block codes with parity-check matrices that consist of sparse parity check matrix that contains small number of ones. Because of this sparseness of H, decoding complexity increases only linearly with the code length [17]. The difference between LDPC codes and classical block codes is in their decoding procedure. Classical block codes are generally decoded with ML like decoding algorithms and so are usually short and designed algebraically to make this task less complex. LDPC codes [20] however are decoded iteratively using a graphical representation of their parity-check matrix and so are designed with the properties of H as a focus. LDPC codes can be denoted as (N, K) or (N, w_c, w_r) where N is the length of the codeword, K is the length of the message bits, w_c is the column weight (i.e., the number of non zero elements in a column of the parity-check matrix) and w_r is the row weight (i.e., the number of non zero elements in a row of the parity-check matrix). The block diagram of LDPC system is shown in Fig. 3.2.

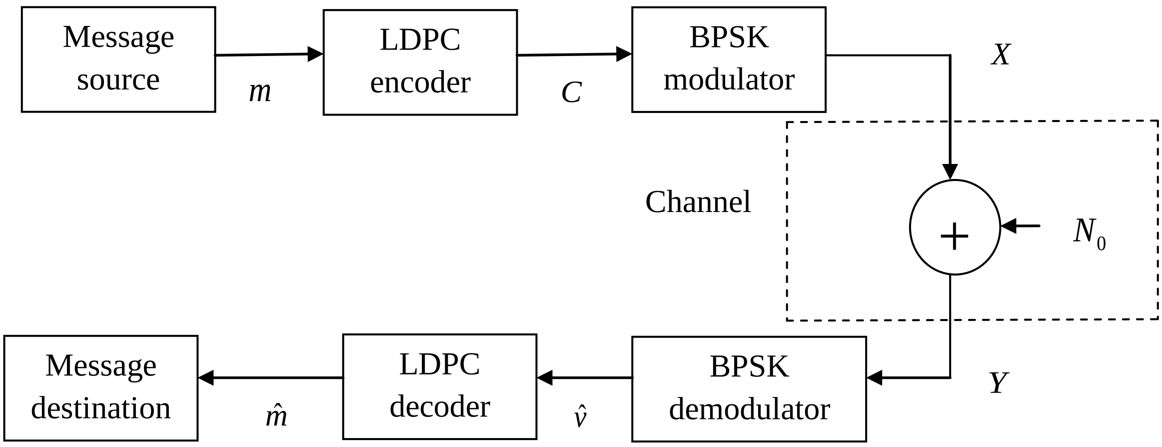


Fig. 3.2: LDPC system overview

where,

- $m \Rightarrow$ Message
- $C \Rightarrow$ Codeword
- $X \Rightarrow$ Modulated signal
- $N_0 \Rightarrow$ AWGN noise
- $Y \Rightarrow$ Received signal
- $\hat{v} \Rightarrow$ Estimated codeword
- $\hat{m} \Rightarrow$ Estimated message

Message source

The message source is the data of the end-user. In terms of mobile communications, the message source would be the voice information transmitted by the end-user.

LDPC encoder

Encoder is implemented at the transmitter side. Encoding is done with the help of generator matrix as discussed in section 3.2.

BPSK modulator

The BPSK (Binary Phase Shift Keying) modulator maps the input binary signals, to an analog signal for transmission. BPSK signal is represented by 0 and 1, where 0 corresponds to $-\sqrt{E_b}$ and 1 corresponds to $\sqrt{E_b}$. E_b is the energy per bit of the baseband signal.

Channel

The channel is the medium of which information is transmitted from the transmitter to the receiver. This channel can be wired or wireless. In mobile communication, wireless channels are used. When the information is transmitted from the transmitter to the receiver, noise gets added to the information. In this thesis, the channel is modelled as an AWGN channel. AWGN means Additive White Gaussian Noise. Additive means this noise is added to any intrinsic noise. White suggests that this noise has uniform power across frequency band, it is analogous to white light which has uniform emissions at all frequencies in the visible spectrum. This noise follows zero mean normal distribution therefore known as Gaussian.

BPSK demodulator

BPSK demodulator involves two steps. First is the translation of the received signal back to the baseband signal with the recovery of band limited message waveform. Second is the regeneration of the binary message bit from the band limited message waveform.

LDPC decoder

Decoder is implemented at the receiver side. For decoding LDPC codes, we have different decoding algorithms. In this thesis, we implemented SPA due to its lesser complexity and better BER performance as compared to other mentioned decoders. To further reduce the decoder complexity, SPA algorithm can be implemented in log domain. This algorithm is discussed in section 3.3.

Message destination

The end-user receiving the data is known as message destination. In the simulations, we do not show message destination instead we compare the estimated message with the transmitted message in order to calculate erroneous bits in the transmitted signal [21].

3.1.3 Tanner graph

A Tanner graph is a bipartite graph with "variable" nodes corresponding to the columns and "check" nodes corresponding to the rows of the parity check matrix [22]. A cycle (loop) in a Tanner graph is a sequence of connected vertices which starts and ends at the same vertex in the graph and which contains other vertices no more than once. The length of a cycle is the number of edges it contains. Since Tanner graphs are bipartite, every cycle will have even length. Minimum length of the cycle in the Tanner graph is known as the girth of the cycle.

Example: Let

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{3.6}$$

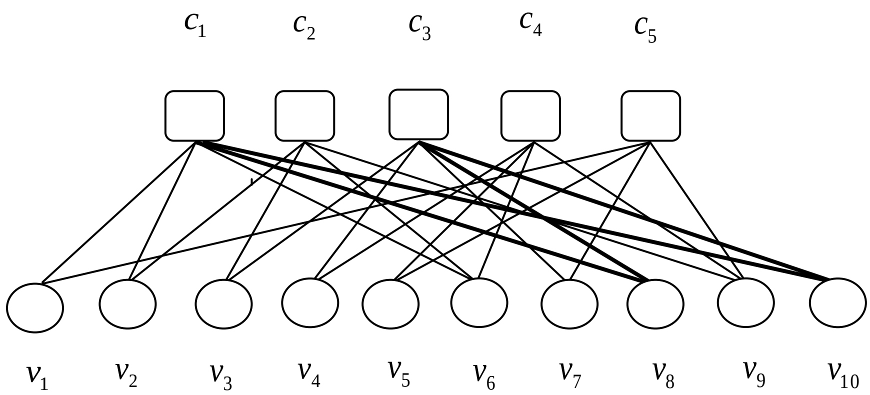


Fig. 3.3: Tanner graph corresponding to parity check matrix of (3.6)

The tanner graph corresponding to the parity check matrix is shown in Fig. 3.3.The path with black bold lines i.e., $c_1 \rightarrow v_8 \rightarrow c_3 \rightarrow v_{10} \rightarrow c_1$ is a cycle of length 4. It is the smallest cycle of this graph thus it is also called the girth of this graph. In LDPC codes, decoding is done through iterative decoding algorithm. In this decoding algorithm, the messages are iteratively passed from variable to check nodes and vice versa. These messages are assumed to be statistically independent. But when there exists a cycle, messages generated from one node will be passed again to that node and thus messages will not be independent. So, cycles should be of small length or graph should have high girth value.

3.1.4 Types of LDPC codes

LDPC codes can be classified as regular LDPC codes and irregular LDPC codes as discussed below

3.1.4.1 Regular LDPC codes

Regular LDPC code [20] is defined by $M \times N$ parity check matrix with fixed number of '1's per column (w_c) and fixed number of '1's per row (w_r). Both w_c and w_r should be small numbers compared to number of columns of H i.e., the code length N and number of rows in H. To attain high girth value no two rows share more than one column for which they both have a '1' in that column. Code rate of such a code is $R = 1 - w_c / w_r$

Example of such a parity check matrix with w_c =2 and w_r = 3 is

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \tag{3.7}$$

3.1.4.2 Irregular LDPC codes

Irregular LDPC code [23] is defined by $M \times N$ parity check matrix with variable number of '1's per column (w_c) and variable number of '1's per row (w_r).The fraction of edges connected to the check nodes and variable nodes are given by two polynomials which are its degree distribution. Degree distribution polynomials are denoted by $\gamma(x)$ and $\rho(x)$.

$$\gamma(x) = \sum_{i=1}^{d_v} \gamma_i x^{i-1} \tag{3.8}$$

where γ_i corresponds to the fraction of edges connected to variable nodes and d_v denotes the maximum variable node degree.

$$\rho(x) = \sum_{i=1}^{d_c} \rho_i x^{i-1} \tag{3.9}$$

where ρ_i corresponds to the fraction of edges connected to check nodes and d_c denotes the maximum variable node degree.

3.2 Encoding of LDPC Codes

LDPC codes have high encoding complexity and encoding delay. The encoding of LDPC code is done in the same way as that of block codes. LDPC codes are designed starting by constructing sparse parity check matrix and then this matrix is used to generate codeword.

3.2.1 Encoding based on Gauss-Jordan elimination [17]

Take a codeword C and a parity check matrix H of the order $M \times N$ .Now partition this codeword into m message bits m and p parity bits as

$$C = [m \mid p] \tag{3.10}$$

Convert H into systematic format after Gauss-Jordan elimination and partition it into two matrices A of order $M \times N\text{-}M$ and B of order $M \times M$.

$$H = [A \mid B] \tag{3.11}$$

Since CH^T=0 for all codeword, we have

$$Am^T + Bp^T = 0 \tag{3.12}$$

From the above equation we can compute the check bits as

$$p^T = B^{-1}Am^T \tag{3.13}$$

Using these check bits and message bits, codeword can be generated.

3.2.2 Efficient encoding based on approximate lower triangulation [24]

In Gauss-Jordan elimination method the encoding complexity is $O(N^2)$.To reduce the complexity to $O(N)$, we use this method. In this method parity check matrix H can be put into lower triangular form using row and column permutations as

$$H = \begin{bmatrix} A & B & T \\ C & D & E \end{bmatrix} \tag{3.14}$$

where T is a lower triangular matrix i.e., it has ones on the diagonal from left to right and all entries above the diagonal are zero. Size of the matrices *A, B, T, C, D, E* are shown in the Fig. 3.4.

$$H = \begin{bmatrix} A & B & T \\ C & D & E \end{bmatrix} =$$

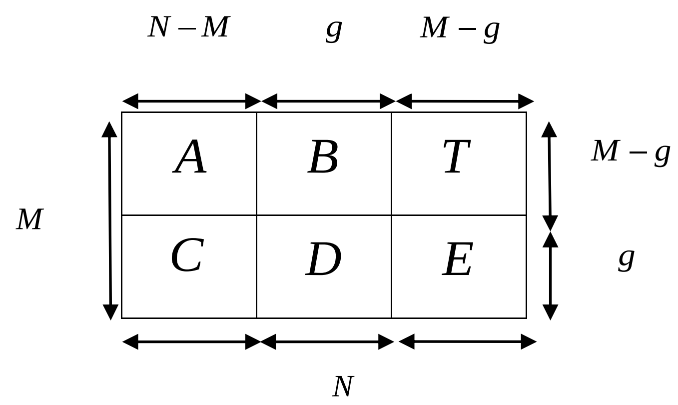


Fig. 3.4: Parity check matrix in lower triangular form

Apply Gauss-Jordan elimination to clear E in H. This can be done by multiplying H with

$$\begin{bmatrix} I_{M-g} & 0 \\ -ET^{-1} & I_g \end{bmatrix}$$

to give

$$H = \begin{bmatrix} A & B & T \\ \hat{C} & \hat{D} & E \end{bmatrix} \tag{3.15}$$

where $\hat{C} = -ET^{-1}A + C$ and $\hat{D} = -ET^{-1}B + D$.

Now partition codeword into *m* message bits *m*, p_1 and p_2 parity bits as

$$C = [m, p_1, p_2] \tag{3.16}$$

where, $p_1 = [p_{1_1}, p_{1_2}, ..., p_{1_g}]$ and $p_1 = [p_{2_1}, p_{2_2}, ..., p_{2_{M-g}}]$

Since CH^T =0 for all codeword, we have

$$Am + Bp_1 + Tp_2 = 0 \tag{3.17}$$

$$\hat{C}m + \hat{D}p_1 + 0\,p_2 = 0 \tag{3.18}$$

p_1 can be calculated from (3.18), it only depends on message bits since *E* is 0. If $\hat{D}$ is invertible p_1 can be found from

$$p_1 = \hat{D}^{-1}\hat{C}m \tag{3.19}$$

If $\hat{D}$ is not invertible, it is made invertible by column permutations. Using the value of p_1 and substituting it in (3.17), we can calculate p_2 .

$$p_2 = -T^{-1}(Am + Bp_1) \tag{3.20}$$

Substituting the values in (3.16), we can obtain the codeword.

3.3 Decoding of LDPC Codes

The process of decoding tries to recover the transmitted codeword *C* from the received codeword $\hat{v}$ using the parity check matrix *H*. LDPC codes can be decoded by using different decoding algorithms such as bit–flipping [25], min-sum (MS) [26] and iterative belief propagation or SPA. SPA has lesser complexity and better BER performance as compared to other mentioned decoders. SPA is a message passing algorithm which is shown in Fig. 3.5.

In SPA, messages are iteratively passed from variable to check node and vice versa [27]. Initially each variable node send messages to all the check nodes connected to it and in next iteration check nodes will send messages back to variable nodes as shown in Fig. 3.5. Note that only the extrinsic information i.e., information other than it already has, is sent back. SPA can be implemented in probability domain as well as log domain. SPA in probability domain requires message multiplications whereas log domain SPA i.e., log-SPA is based upon LLR which requires message additions [28].

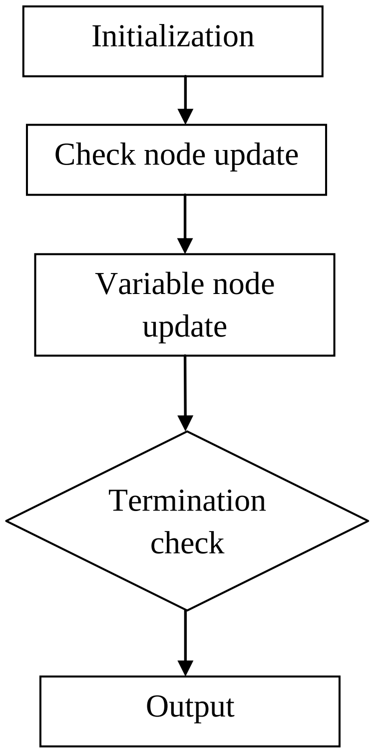
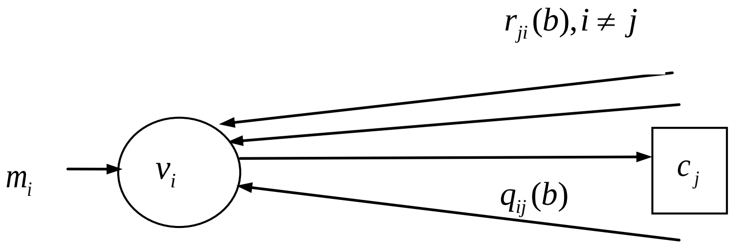
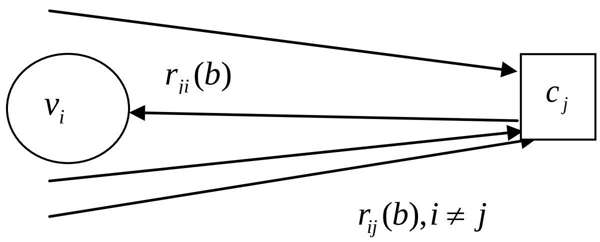


Fig. 3.5: Flow diagram of iterative message passing decoding algorithm



a) Computation of $q_{ij}(b)$



b) Computation of $r_{ij}(b)$

Fig. 3.6: Iterations in SPA decoding algorithm

There are two types of probabilities that express the relation between a variable and an event *E*. These probabilities are a priori and a posteriori probabilities. The input bit probabilities are called the a priori probabilities for the received bits because they were known in advance before running the LDPC decoder. The bit probabilities returned by the decoder are called the a posteriori probabilities. A priori probability *m* with respect to *E*, which is the probability that *m* is equal to a, and is denoted by

$$P_E^{priori}(m = a) = P(m = a) \tag{3.21}$$

$$P_E^{post}(m = a) = P(m = a \mid E) \tag{3.22}$$

where,

$$P(m = a \mid E) = \frac{1}{P(E)} P(E \mid m = a) P(m = a) \tag{3.33}$$

$P(m = a \mid E)$ is proportional to extrinsic probability which is given as

$$P_E^{ext}(m=a) = uP(E \mid m=a) \tag{3.34}$$

where, u is a constant that normalize the probability sum to 1. Relation between all the probabilities is as follows

$$P_E^{post}(m=a) = P_E^{priori}(m=a)P_E^{ext}(m=a) \tag{3.35}$$

We use LLR because our concern is binary case. In LLR, probability of variable m is expressed in terms of a real number. Thus LLR of m is defined as

$$LLR(m) = \log\frac{P(m=1)}{P(m=0)} = \log\frac{p}{1-p} \tag{3.36}$$

where, p is the probability that m = 1. Thus the probability that m = 0 is given by 1-p. Thus (3.35) can be written as

$$LLR_E^{post}(m=a) = LLR_E^{priori}(m=a)LLR_E^{ext}(m=a) \tag{3.37}$$

If $p \geq 0.5$, $LLR(m)$is positive and if $p < 0.5$, $LLR(m)$is negative. In this algorithm, the a-priori information is taken from channel and extrinsic information comes from other nodes.

Notations used in Log-SPA are :

$q_{ij}(b)$ = Probability that variable node v_i whose bit value is b, satisfies all check equations except c_j.,

$r_{ji}(b)$ = Probability that check equation c_j is satisfied if variable node v_i has value b,

$Q_i(b)$ = Probability that variable node v_i has value b where b is 0 or 1,

v_j = variable nodes connected to check node c_j,

$v_{j/i}$ = variable nodes connected to check node c_j except v_i,

c_j = check nodes connected to variable node v_j,

$c_{i/j}$ = check nodes connected to variable node v_i except c_j,

The decoding steps are given below:

1) Initialization: Values from the received vector Y are given to the variable nodes. The initial probability is given by

$$L(m_i) = \ln \frac{P(m_i = 1 | Y_i)}{P(m_i = 0 | Y_i)} \tag{3.38}$$

In AWGN channel,

$$L(q_{ij}) = L(m_i) = \frac{2Y_i}{\sigma_{N_0}^2} \tag{3.39}$$

where $\sigma_{N_0}^2$ is the noise variance. Messages to check nodes and variable nodes, and check nodes LLRs are initialized to zero by sending values on variable nodes to connected check nodes.

2) Check node update: Based on variable node messages, LLR and check to variable node messages are calculated.

$$L(r_{ji}) = \ln\left(\frac{r_{ji}(0)}{r_{ji}(1)}\right) \tag{3.40}$$

3) Variable node update: $L(q_{ij})$ is calculated.

$$L(q_{ij}) = \ln\left(\frac{q_{ij}(0)}{q_{ij}(1)}\right) \tag{3.41}$$

$$L(q_{ij}) = L(u_i) + \sum_{j' \in c_i / j} L(r_{j'i}) \tag{3.42}$$

$L(Q_i)$ is updated.

$$L(Q_i) = \ln\left(\frac{Q_i(0)}{Q_i(1)}\right) \tag{3.43}$$

$$L(Q_i) = L(u_i) + \sum_{j \in c_i} L(r_{ji}) \tag{3.44}$$

where $L(r_{ji}) = \sum_{i' \in V_j / i} \boxplus L(q_{i'j})$

$\boxplus$ is the box-operator defined by

$$L(x_1) \boxplus L(x_2) = \text{sgn}[L(x_1)].\text{sgn}[L(x_2)]\min\{|L(x_1)|, |L(x_2)|\} \quad (3.45)$$

4) Decision:Values of variable nodes are decided as 0 or 1 by $L(Q_i)$

$$\hat{v} = \begin{cases} 1\ ; if\ L(Q_i\ ) < 0 \\ 0\ ; \quad\quad else \end{cases} \quad (3.46)$$

If $\hat{v}H^T = 0$, decoding algorithm halts and $\hat{v}$ is considered as valid result. Otherwise, the algorithm repeats from step 2. Algorithm is terminated if algorithm reaches maximum number of iterations.

Flowchart for sum product decoding algorithm is shown in Fig. 3.7.

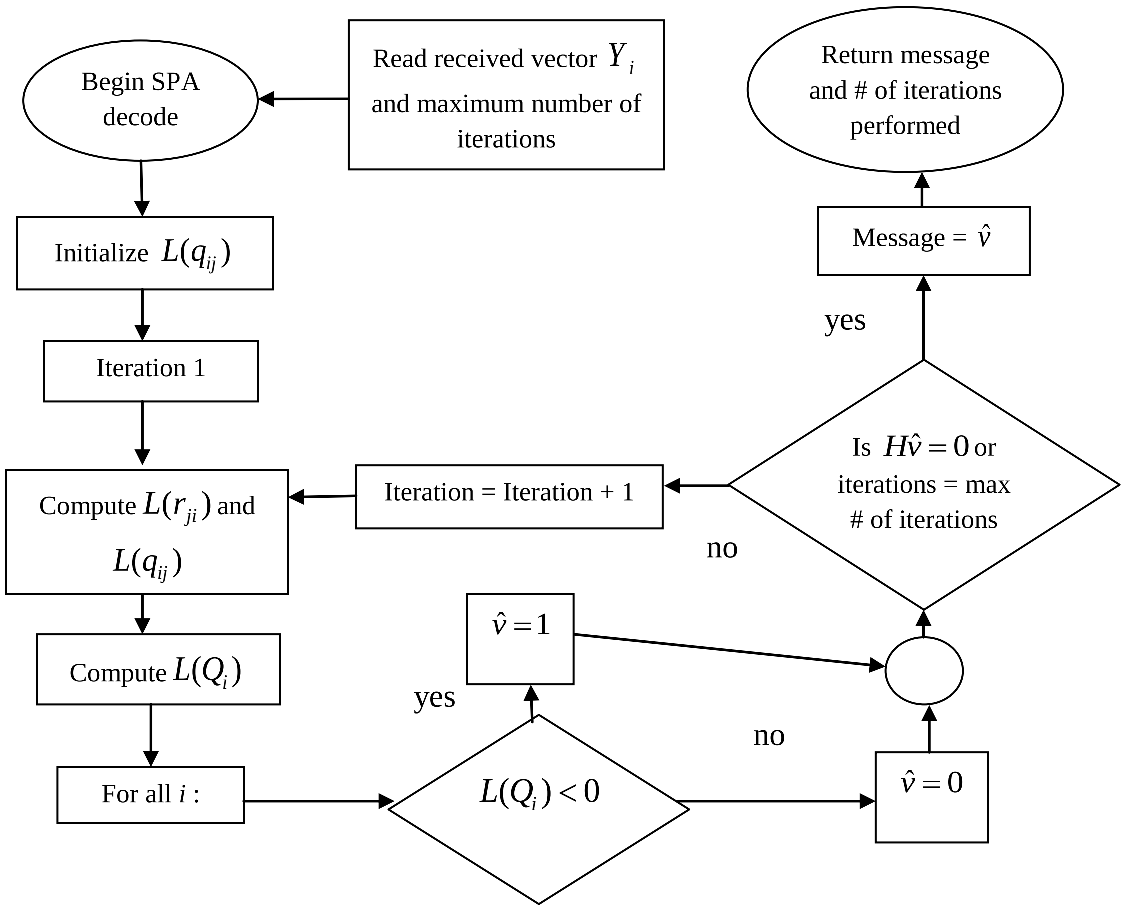


Fig. 3.7: Flowchart for SPA decoding

Example:

For the given tanner graph of H shown in Fig. 3.8, decoding for first iteration is done as following

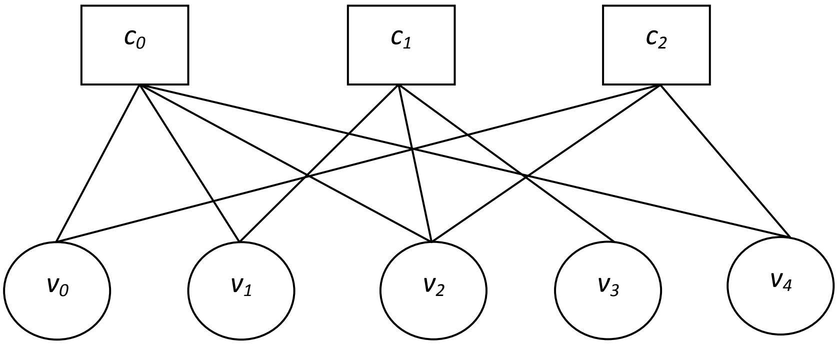


Fig. 3.8: Tanner graph of H matrix

Step 1: Initializing the value of $L(q_{ij}) = L(m_i)$,

$$L(q_{0j})_{0,2} = L(m_0) = 0.7$$

$$L(q_{1j})_{0,1} = L(m_1) = -1.5$$

$$L(q_{2j})_{1,2.3} = L(m_2) = 0.3$$

$$L(q_{3j})_{1,2} = L(m_3) = 1.1$$

$$L(q_{4j})_{0,2} = L(m_4) = -0.6$$

Step 2: Updating $L(r_{ji})$

$$L(r_{00}) = L(q_{10}) \boxplus L(q_{20}) \boxplus L(q_{40}) = -1.5 \boxplus 0.3 \boxplus -0.6 \approx 0.3$$

$$L(r_{01}) = L(q_{00}) \boxplus L(q_{20}) \boxplus L(q_{40}) = 0.7 \boxplus 0.3 \boxplus -0.6 \approx 0.3$$

$$L(r_{02}) = L(q_{00}) \boxplus L(q_{10}) \boxplus L(q_{40}) = 0.7 \boxplus -1.5 \boxplus -0.6 \approx 0.6$$

$$L(r_{04}) = L(q_{00}) \boxplus L(q_{10}) \boxplus L(q_{20}) = 0.7 \boxplus -1.5 \boxplus -0.6 \approx -0.3$$

$$L(r_{11}) = L(q_{21}) \boxplus L(q_{31}) = 0.3 \boxplus 1.1 \approx -0.3$$

$L(r_{12}) = L(q_{11}) \boxplus L(q_{31}) = -1.5 \boxplus 1.1 \approx -1.1$

$L(r_{13}) = L(q_{11}) \boxplus L(q_{21}) = -1.5 \boxplus 0.3 \approx -0.3$

$L(r_{20}) = L(q_{22}) \boxplus L(q_{32}) \boxplus L(q_{42}) = 0.3 \boxplus 1.1 \boxplus -0.6 \approx -0.3$

$L(r_{22}) = L(q_{02}) \boxplus L(q_{32}) \boxplus L(q_{42}) = 0.7 \boxplus 1.1 \boxplus -0.6 \approx -0.6$

$L(r_{23}) = L(q_{02}) \boxplus L(q_{22}) \boxplus L(q_{42}) = 0.7 \boxplus 0.3 \boxplus -0.6 \approx -0.3$

$L(r_{24}) = L(q_{02}) \boxplus L(q_{22}) \boxplus L(q_{32}) = 0.7 \boxplus 0.3 \boxplus 1.1 \approx 0.3$

Step 3: Updating $L(q_{ij})$

$L(q_{00}) = L(m_0) + L(r_{20}) = 0.7 - 0.3 = 0.4$

$L(q_{02}) = L(m_0) + L(r_{00}) = 0.7 + 0.3 = 1$

$L(q_{10}) = L(m_1) + L(r_{11}) = -1.5 + 0.3 = -1.2$

$L(q_{11}) = L(m_1) + L(r_{01}) = -1.5 - 0.3 = -1.8$

$L(q_{20}) = L(m_2) + L(r_{12}) + L(r_{22}) = 0.3 - 1.1 - 0.6 = -1.4$

$L(q_{21}) = L(m_2) + L(r_{02}) + L(r_{22}) = 0.3 + 0.6 - 0.6 = 0.3$

$L(q_{22}) = L(m_2) + L(r_{02}) + L(r_{12}) = 0.3 + 0.6 - 1.1 = -0.2$

$L(q_{31}) = L(m_3) + L(r_{23}) = 1.1 - 0.3 = 0.8$

$L(q_{32}) = L(m_3) + L(r_{13}) = 1.1 - 0.3 = 0.8$

$$L(q_{40}) = L(m_4) + L(r_{24}) = -0.6 + 0.3 = -0.3$$

$$L(q_{42}) = L(m_4) + L(r_{04}) = -0.6 - 0.3 = -0.9$$

Step 4: Update $L(Q_i)$

$$L(Q_0) = L(m_0) + L(r_{00}) + L(r_{20}) = 0.7 + 0.3 - 0.3 = 0.7$$

$$L(Q_1) = L(m_1) + L(r_{01}) + L(r_{11}) = -1.5 - 0.3 + 0.3 = -1.5$$

$$L(Q_2) = L(m_2) + L(r_{02}) + L(r_{12}) + L(r_{22}) = 0.3 + 0.6 - 1.1 - 0.6 = -0.8$$

$$L(Q_3) = L(m_3) + L(r_{13}) + L(r_{23}) = 1.1 - 0.3 - 0.3 = 0.5$$

$$L(Q_4) = L(m_4) + L(r_{04}) + L(r_{24}) = -0.6 - 0.3 + 0.3 = -0.6$$

Step 5: Find $\hat{v}$.

$$\hat{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

Step 6: Check if $\hat{v}H^T = 0$ or number of iterations equals to maximum number of iterations, if yes then stop otherwise go to step 2.

3.4 Designing and Optimizing LDPC Codes

For designing LDPC codes we should consider some of the parameters such as code rate, code size and code structure. Some of the parameters such as girth need to be optimized to achieve better BER performance. These parameters are affected by application needs.

3.4.1 Code size

Code size is given as (N, w_c, w_r). Codes with larger *N* have better performance than codes with smaller value of *N*, but require larger memory [29].

3.4.2 Code weight

Code weight consists of row (w_r) and column weight (w_c). Higher weights leads to increase in computations at each node, since nodes will be attached to more information bits in the decoding process but at the same time it leads to more consistent decoding, since more nodes will participate in estimating the probability of a bit thus leading to faster convergence.

3.4.3 Code rate

Code rate is given as $R = \frac{K}{N}$ as mentioned before. Higher rate means less redundancy which results in high throughput of information data and poor decoding performance and increases BER. Low rates means less throughput and better decoding performance.

3.4.4 Code structure

Random LDPC code that is code whose parity check matrix is constructed randomly (i.e., without any predefined pattern). These codes need to be stored in memory for encoding and decoding process. In case of long block codes, very large memory usage is needed to store the parity check matrix which deduces the computational efficiency of the code. To overcome this problem, structured LDPC codes are used whose parity check matrix consists of predefined patterns that need few inputs to generate a range of code words. These constructions have an advantage in reducing the cost, complexity, memory usage, and latency.

3.4.5 Decoder iterations

In a decoding process, the number of times the received bit is estimated before a hard decision is made by the decoding algorithm is called decoding iterations. With the increase in decoder iterations BER decreases and decoding convergence increases.

3.4.6 Minimum distance

Minimum distance of LDPC codes is better than linear codes, thus have better performance and can correct larger number of errors. Random LDPC codes have minimum distance proportional to the size of the code and the minimum distance of structured LDPC codes is restricted to an upper bound depending upon column weight and is not proportional to the code size [30].

3.4.7 Girth

Girth means the smallest cycle in the tanner graph. Girth should be greater than 4, since with small cycles a node gets a probability estimate depending mainly on its own probability contribution. With large girth, the probability estimation of bit decoding relies on the connected bits, which results in better estimation of the node [31].

3.4.8 Density evolution

The degree distribution of an irregular LDPC code determines the proportion of edges in the Tanner graph connected to check nodes and variable nodes of a particular degree. Density evolution (DE) [32] offers a reduced complexity method to determine the performance limit of a degree distribution. The analysis performed by DE determines what the `worst' channel for which the SPA decoder will decode with an arbitrarily small probability of error in the limit of long block lengths and using a large number of iterations of the SPA to decode. DE is based on the average performance of an ensemble of LDPC codes of a particular degree distribution. One can then state with certainty that there exists a code in this ensemble which outperforms the average. Using this analysis, one can optimize the number of iteration used in the decoding process and approximate the BER of the code [33].

Chapter 4

TYPES OF LDPC CODES ON THE BASIS OF CONSTRUCTION

LDPC codes is categorized mainly into two main categories on the basis of construction:

- Random LDPC codes
- Structured LDPC codes (QC-LDPC codes)

These are distinguished on the basis of connections between check nodes and variable nodes in Tanner graph. Each type of construction has its advantages over the other.

4.1 Random LDPC Codes

Random constructions connect rows and columns of a parity check matrix *H* without any structure or predefined connection pattern. The construction of random LDPC codes is done by adding random edges to a Tanner graph or '1' entries in the parity check matrix. Designing a code with a desired rate and girth can be achieved by post processing the connections of *H* matrix to maintain the desired rate and girth and to delete cycles of 4 which degrade the performance of the code. The advantage of random LDPC code is that they have better performance than structured codes in case of long codes. Disadvantage of this code occurs in implementing this long length code in practical system. Due to long lengths, they require large memory which affects the computational efficiency of the code which is more important than bit-error rate in real life [34].

Review of some random construction algorithms are discussed here :

4.1.1 Gallager's construction [17]

The original LDPC codes presented by Gallager are regular and defined by a banded structure in H. The rows of Gallager's parity-check matrices are divided into w_c sets with M/w_c rows in each set. The first set of rows contains w_c consecutive ones ordered from left to right across the columns. (i.e. for $i \le M/w_c$, the i-th row has non zero entries in the $((i-1)K+1)$-th to i-th columns). Every other set of rows is a randomly chosen column permutation of this first set. Consequently every column of H has a '1' entry once in every one of the w_c sets.

Example of such construction of (12, 3, 4) regular code is given below

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{4.1}$$

4.1.2 MacKay's construction [17]

- An all-zero matrix is taken and w_c bits in each column are randomly flipped.
- Matrix H is generated by randomly creating weight w_c columns.
- Matrix H is generated with weight w_c per column and uniform weight per row with a constrain that no two columns overlap in more than one position.
- Further constrain H such that its graph has large girth.
- Finally we get randomly constructed H of order $M \times N$.

Example of such construction of (12, 3, 4) regular code is given below

$$
H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad (4.2)
$$

4.2 Quasi Cyclic (QC) LDPC codes

For short and medium length codes, structured constructions can perform better than random one. Structured LDPC codes can be divided into two main categories. The first category is based on finite geometries [35], while the second category is based on algebraic and combinatorial methods [36]. In this thesis we will focus on second category. Example of such a code is QC-LDPC codes. QC-LDPC codes have been proposed to reduce the complexity of the LDPC codes. QC-LDPC codes solve memory problem since their parity check matrices consists of circulant permutation matrices which are usually derived from identity matrices [37]. Their main advantage against randomly constructed codes is that they involve easier implementation in terms of the encoding procedure, since they can be encoded using simple shift-registers [38], with a complexity linearly proportional to the code length. In case of random LDPC codes complexity is proportional to the square of the code length.

The parity check matrix *H* of such codes consists of square blocks with circulant permutation matrices [38]. Examples [39] of such sub-matrices are shown in (4.3), where I^{α} denotes an identity matrix I, whose columns are (α-1) times circularly shifted to the right (or rows are (α-1) times circularly shifted up). Notice that if the size of the sub-matrix is $P \times P$, then $I^{P+1} = I$ More generally, $I^{a} = I^{a \bmod P}$.

$$I = I^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} I^2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} I^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} I^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{4.3}$$

The parity check matrix of an algebraically structured regular LDPC code with size $Pw_c \times Pw_r$ is shown in Fig. 4.1 which is first proposed by Tanner in 2004. Similar codes are proposed by Myung in 2005 and Honary in 2005 as shown in Fig. 4.2 and Fig. 4.3 respectively.

$$H = \begin{bmatrix} I & I^a & \cdots & I^{a(w_r-1)} \\ I^b & I^{ab} & \cdots & I^{a(w_r-1)b} \\ \vdots & \vdots & \ddots & \vdots \\ I^{b(w_c-1)} & I^{ab(w_c-1)} & \vdots & I^{a(w_r-1)b(w_c-1)} \end{bmatrix}$$

Fig. 4.1: Structure of a regular QC parity check matrix of size $Pw_c \times Pw_r$ proposed by Tanner [39]

$$H = \begin{bmatrix} I & I & I & I & I \\ I & I^a & I^{a^2} & \cdots & I^{a(w_r-1)b} \\ I & I^{a^2} & I^{a^3} & \cdots & \cdots \\ I & \vdots & \vdots & \ddots & \vdots \\ I & I^{a^{(w_C-1)}} & I^{a(w_C-1)+1} & \cdots & I^{a^{(w_r-1)}a^{(w_C-1)}} \end{bmatrix}$$

Fig. 4.2: Structure of a regular QC parity check matrix of size $Pw_c \times Pw_r$ proposed by Myung [39]

$$H=\begin{bmatrix} I & I & I & I & I \\ I & I & I^2 & \cdots & I^{(w_r-1)} \\ I & I^2 & I^4 & \cdots & I^{2(w_r-1)} \\ I & \vdots & \vdots & \ddots & \vdots \\ I & I^{(w_C-1)} & I^{2(w_C-1)} & \cdots & I^{(w_r-1)(w_C-1)} \end{bmatrix}$$

Fig. 4.3: Structure of a regular QC parity check matrix of size $Pw_c \times Pw_r$ proposed by Honary [39]

For (N, w_r, w_c) regular QC-LDPC code, the $Pw_c \times Pw_r$ parity check matrix is constructed from sub-matrices $P \times P$, where $P = N/w_r$. To have short cycles P should be greater than the number of sub matrices, since repetition of sub-matrices within H is the main cause of 4-cycles. Example of such case is shown in (4.3).

$$\begin{bmatrix} I & I \\ I & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.4}$$

In (4.3), similar rows 1^{st} and 5^{th}, 2^{nd} and 6^{th} , 3^{rd} and 7^{th}, 4^{th} and 8^{th} and similar columns 1^{st} and 5^{th}, 2^{nd} and 6^{th} , 3^{rd} and 7^{th}, 4^{th} and 8^{th} create cycles of length 4.

Assume $a = b = 2$, $w_c = 3$ and $w_r = 6$ as shown in (4.4),

$$H=\begin{bmatrix} I & I^2 & I^4 & I^8 & I^{16} & I^{32} \\ I^2 & I^4 & I^8 & I^{16} & I^{32} & I^{64} \\ I^4 & I^8 & I^{16} & I^{32} & I^{64} & I^{128} \end{bmatrix} \tag{4.5}$$

If the size of the sub matrix P = 48, since $I^{64} = I^{\bmod 48(64)} = I^{16}$ and $I^{128} = I^{\bmod 48(128)} = I^{32}$ the resulting 144×288 parity check matrix will have repeated sub-matrices which will create cycles of length 4. If P = 47, then the parity check matrix will be of order 141×282 will have no repetition of sub-matrices and consequently, no length four cycles. Therefore, when we construct parity check matrix, the number of distinct sub matrices should be checked before determining the size of the parity check matrix. For example, if we choose P = 56 instead of 47 in (4.5) then 6 distinct sub-matrices will be generated instead of 18 sub matrices.

Cycles of length four can exist even if there are no repetitions of sub-matrices as shown in (4.5).

$$\begin{bmatrix} I & I^2 \\ I^3 & I^4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{4.6}$$

It happens because of constant shift differences between consecutive sub matrices. In (4.5), there is a constant shift difference between $I - I^2$ and $I^3 - I^4$ sub matrices. Thus column pair 1-6, 3-8, 4-5, creates cycles of length 4.

4.2.1 Construction of QC-LDPC code

The construction of QC-LDPC code [40] starts by choosing a prime number P. The element from 0 to P – 1 form a field under addition and multiplication (mod P), and thus the nonzero elements of this field represent a cyclic multiplicative group. Now choose two non-zero elements a and b, a is with order of w_r and b is with order of w_c. Take elements from $GF(P)$ and form a matrix $\overline{H}$ of the order $w_c \times w_r$ as the following:

$$\overline{H} = \begin{bmatrix} 1 & a & a^2 & \cdots & a^{w_r-1} \\ b & ab & a^2b & \cdots & a^{w_r-1}b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b^{w_c-1} & ab^{w_c-1} & a^2b^{w_c-1} & \cdots & a^{w_r-1}b^{w_c-1} \end{bmatrix} \tag{4.7}$$

Then a $Pw_c \times Pw_r$, H matrix is constructed by having an $P \times P$ identity matrices inserted with their rows circularly shifted to the left by (x – 1) positions according to the values of $\overline{H}$ matrix as the following [41],

$$H = \begin{bmatrix} I & I^{a} & I^{a^2} & \cdots & I^{a(w_r-1)} \\ I & I^{a} & I^{a^2 b} & \cdots & I^{a(w_r-1)b} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ I^{b(w_C-1)} & I^{ab(w_C-1)} & I^{a^2 b(w_C-1)} & \cdots & I^{a(w_r-1)b(w_C-1)} \end{bmatrix} \tag{4.8}$$

Design rate of this code is given as

$$R = 1 - \frac{w_c}{w_r} \tag{4.9}$$

This construction can be extended to use nonprime integers with some modifications to sustain the regularity of the H matrix.

Table 4.1 [40]: Examples of QC-LDPC codes constructed from (prime) circulant sizes

Codeword Length N	Design Parameters w_c	w_r	Design Rate R	Actual Rate R	Circulant Size P
21	2	3	1/3	0.3809	7
91	2	3	1/3	0.3441	31
129	2	3	1/3	0.3411	43
155	3	5	2/5	0.4129	31
186	5	6	1/6	0.1882	31
305	3	5	2/5	0.4065	61
755	3	5	2/5	0.4423	151
905	3	5	2/5	0.4022	181
1055	3	5	2/5	0.4018	211
1205	3	5	2/5	0.4016	241
1477	3	7	4/7	0.5727	211

1477	5	7	2/7	0.2884	211
1703	5	13	5/8	0.6177	131
1928	3	8	5/8	0.626	241
1928	5	8	3/8	0.3771	241
1967	5	7	2/7	0.2877	281
2041	3	13	10/13	0.7702	157
2248	5	8	3/8	0.3768	281
2947	3	7	4/7	0.5721	421
2947	4	7	3/7	0.4296	421
3641	3	11	8/11	0.7278	331
3641	5	11	6/11	0.5465	331
5219	3	17	14/17	0.8239	307
11555	3	5	2/5	0.4001	2311

4.3 Encoding

In this section, encoding and decoding of both the types of LDPC codes are discussed.

4.3.1 Encoding of random LDPC code

Random LDPC code can be encoded with the help of generator matrix.The generator matrix is derived from parity check matrix H which is rearranged by Gauss Jordan elimination or approximate lower triangulation method to form the generator matrix as discussed in section 3.2 of Chapter 3. The generator matrix is then multiplied with data to get codeword.

4.3.2 Encoding of QC-LDPC code [42]

The QC-LDPC codes consist of horizontally concatenated circulant sub-matrices. Each circulant sub-matrix is a square matrix for which every row is the cyclic shift of the previous row, and the first row is obtained by the cyclic shift of the last row. In this way, every column of each circulant sub-matrix is automatically the cyclic shift of the previous column, and the first column is obtained by the cyclic shift of the last column. The parity check matrix of QC-LDPC code is given as

$$H = \begin{bmatrix} H_1 & H_2 & H_3 & \cdots & H_L \end{bmatrix} \tag{4.10}$$

This H is of dimension $P \times L_P$ and H_i is the i^{th} sub-matrix of order $P \times P$. Using this parity check matrix, generator matrix is obtained. The generator matrix is constructed in such a way that $GH^T = 0$. The generator matrix is expressed as

$$G = \begin{bmatrix} p_2^T & I_P & 0 & 0 & \cdots & 0 \\ p_3^T & 0 & I_P & 0 & \cdots & 0 \\ p_4^T & 0 & 0 & I_P & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ p_L^T & 0 & 0 & 0 & \cdots & I_P \end{bmatrix} \tag{4.11}$$

The rate of this generator matrix is given as

$$R = \frac{L-1}{L} \tag{4.12}$$

Since $GH^T = 0$,

$$\begin{bmatrix} p_2^T & I_P & 0 & 0 & \cdots & 0 \\ p_3^T & 0 & I_P & 0 & \cdots & 0 \\ p_4^T & 0 & 0 & I_P & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ p_L^T & 0 & 0 & 0 & \cdots & I_P \end{bmatrix} \begin{bmatrix} H_1^T \\ H_2^T \\ H_3^T \\ \vdots \\ H_L^T \end{bmatrix} = 0 \tag{4.13}$$

From this relation we can calculate parity bits p as

$$p = H_1^{-1} H_i H \quad \text{where } i = 1,2,\ldots,L \tag{4.14}$$

The inverse of a circulant matrix is a circulant matrix and product of two circulant matrix is also a circulant matrix. Therefore generator matrix G will be a circulant matrix. Since the generator matrix is quasi-cyclic in nature, the first row of each circulant sub-matrix is stored, and successive rows can be generated by a shift register generator. This greatly simplifies the encoder design [43].

4.4 Decoding

Random LDPC codes and QC- LDPC codes can use the same decoding algorithm. In this thesis, sum product decoding algorithm is used for both of these codes. Sum product decoding algorithm is discussed in section 3.3 of Chapter 3.

4.5 Simulation Results

The performance of regular LDPC codes constructed with different manner as described in Section 4.1 and 4.2 with different code lengths are shown in Fig. 4.4 and Fig. 4.5 with SPA decoding. Design parameters for these simulation results are shown in Table 4.1.

In Fig. 4.4 and Fig. 4.5, regular randomly constructed LDPC codes are compared with regular QC-LDPC codes of similar rates and code word lengths for a BPSK-modulated AWGN channel. The maximum number of iterations in SPA is taken as 50 iterations. SPA decoder stops when either we get a valid codeword or number of iterations equals to maximum number of iterations. All codes have $w_c = 3$ and $w_r = 5$. The QC-LDPC is seen to outperform the random LDPC for short to moderate code word lengths as shown in Fig. 4.4, codeword length is taken as 155. However, at longer block lengths say at code word length 10000 as shown in Fig. 4.5, the QC-LDPC codes are not as good as the random LDPC codes. The QC-LDPC codes show an error floor, which may be due to their limited minimum distance. For $w_c = 3$, the minimum distance is given as $(w_c + 1)! = (3+1)! = 24$. On the other hand, the minimum distance of random LDPC codes can grow linearly with the code word length and they do not exhibit an error floor behaviour.

Table 4.2: Simulation parameters for LDPC codes

Parameters	Values
Modulation	BPSK
Channel Model Used	Rayleigh fading channel
Channel Fading	Rayleigh Independent and identically distributed
Channel Estimation	Ideal
Codeword length (N)	155 and 10000
Number of '1's Per Column	3
Number of '1's Per Row	5
Number of Iterations	50
LDPC Code Rate	½

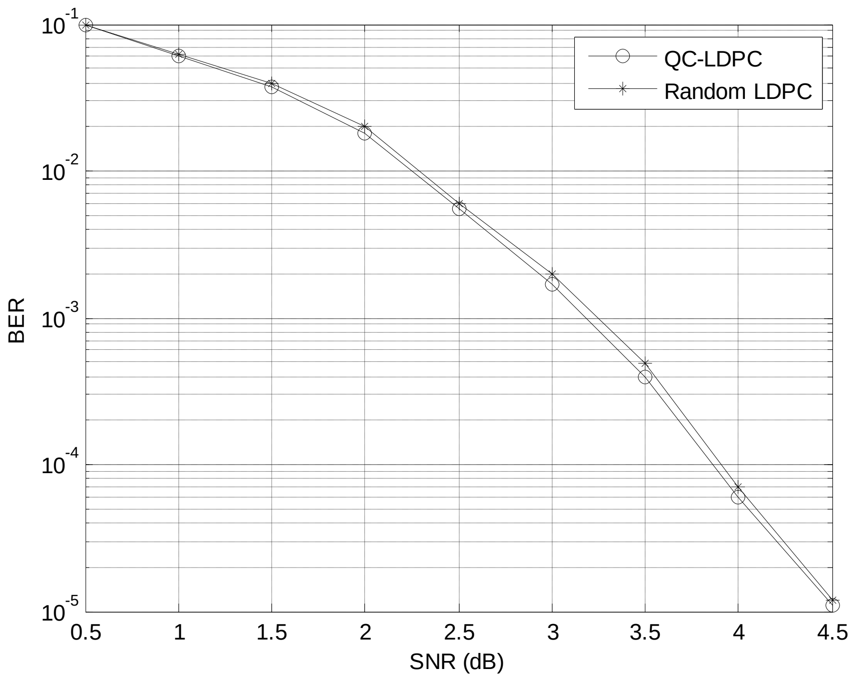


Fig. 4.4: Types of LDPC for N = 155

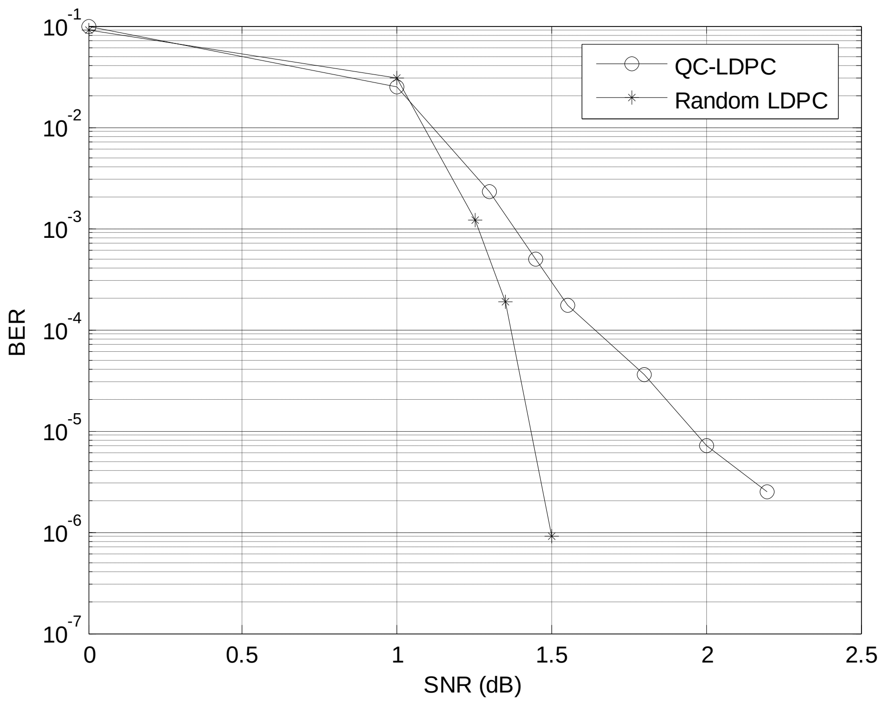


Fig. 4.5: Types on LDPC for N = 10000

Chapter 5

LDPC CODED MU-MIMO-OSTBC SYSTEM

MIMO systems are adopted in current broadband wireless standards to achieve high capacity, high data rate and improved reliability. To further improve reliability and performance of MIMO systems, coding gain benefit can be added by concatenating MIMO systems with error correcting codes such as LDPC codes which are capacity approaching codes that are especially well suited for implementation in integrated circuits due to their inherent parallelizability. The performance of LDPC codes depends on their code structure and decoding algorithms. On the basis of parity check matrix structure, we have random and structured LDPC codes.

5.1 LDPC Coded MIMO-OSTBC System model

The transceiver structure of Random and QC-LDPC coded MIMO system is same and is shown in Fig. 5.1.

Initially, the binary input signals are encoded using an LDPC encoder. These encoded bits are mapped into data symbols via modulation techniques such as BPSK and QPSK. After modulation, incoming serial stream is converted into multiple parallel sub-streams. These parallel sub-streams are then transmitted over the MIMO channel.

The channel output vector Y is given by

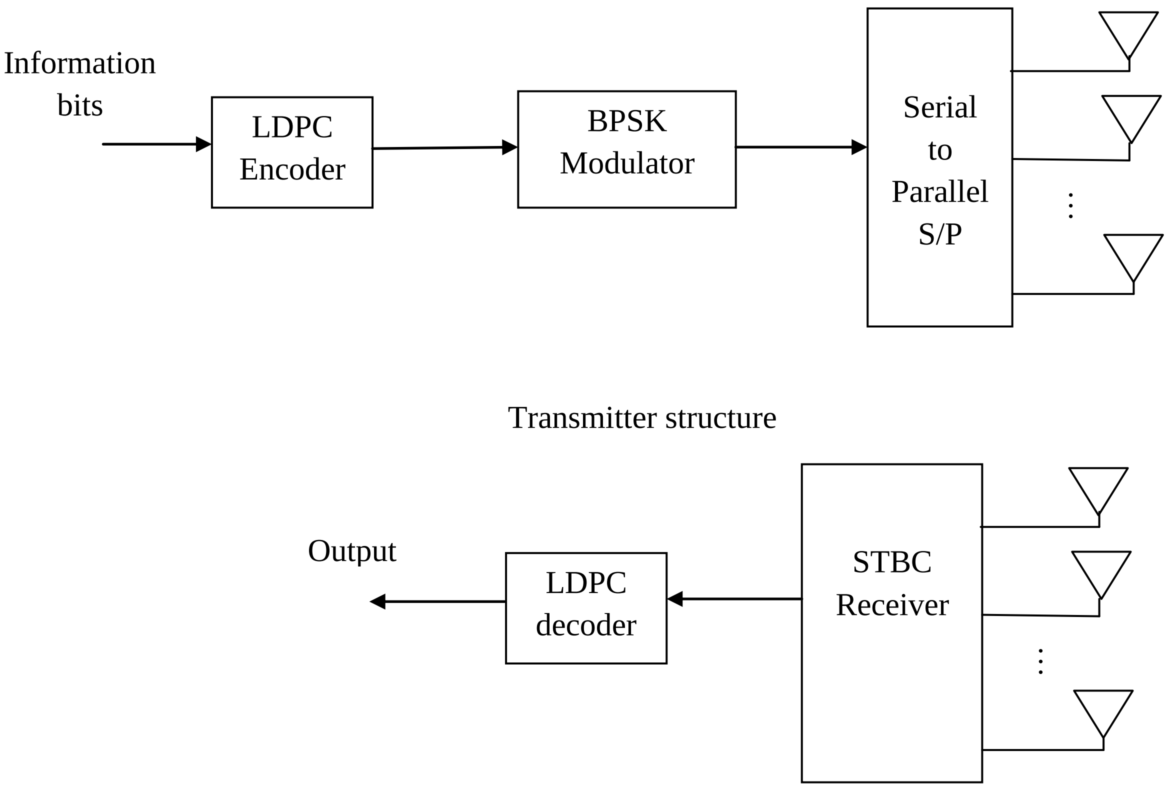
$$Y_i(1) = \frac{1}{\sqrt{M_T}}(\mathrm{H}_{1,i}X_1 + \mathrm{H}_{2,i}X_2) + N_0(1) \tag{5.1}$$

$$Y_i(2) = \frac{1}{\sqrt{M_T}}(-\mathrm{H}_{1,i}X_1 + \mathrm{H}_{2,i}X_2) + N_0(2) \tag{5.2}$$

where $X = [X_1, X_2, ...X_{M_T}]^T$ and $Y = [Y_1, Y_2, ...Y_{M_R}]^T$ are the transmitted and received vectors, $N_0 = [N_{01}, N_{02}, ...N_{0M_T}]^T$ is the independent and identically distributed (i.i.d) circularly symmetric, complex additive white Gaussian noise vector with zero-mean and covariance matrix $\sigma_n^2 I$, H denotes the $M_R \times M_T$ MIMO channel matrix which is given as

$$\mathrm{H} = \begin{bmatrix} h_{11} & \cdots & h_{1M_R} \\ \vdots & \ddots & \vdots \\ h_{M_R 1} & \cdots & h_{M_R M_T} \end{bmatrix} \tag{5.3}$$

where, h_{ij} is the fading coefficient or the complex path gain from transmit antenna i to receive antenna j.



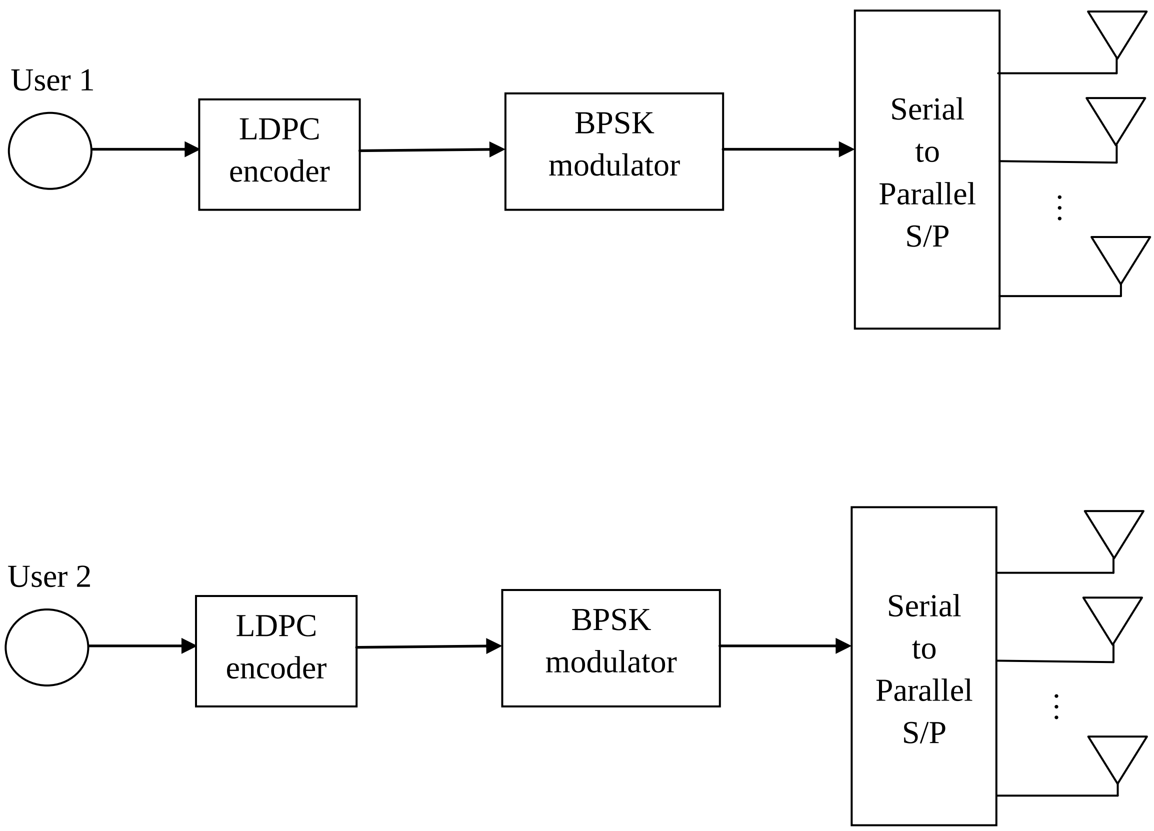
Receiver structure

Fig. 5.1: LDPC coded MIMO-OSTBC system

At the receiver, the received signal is processed by the MIMO detector which can be linear as well as non-linear. The processed signals will be passed on to the LDPC decoder which decode the signal using log domain sum product decoding algorithm to get the desired output. This output signal is compared with the input signal i.e., the information bits and number of bits in error is estimated. Thus we can analyse the BER performance of the system.

5.2 LDPC Coded Multi-user MIMO System Model

Consider a LDPC coded multiuser MIMO system model with 2 users, number of transmit antennas as M_T and number of receive antennas as M_R as shown in Fig. 5.2. Assume that the channel matrix H independently changes per channel-use, which is usually called the fast fading channel.



Transmitter structure

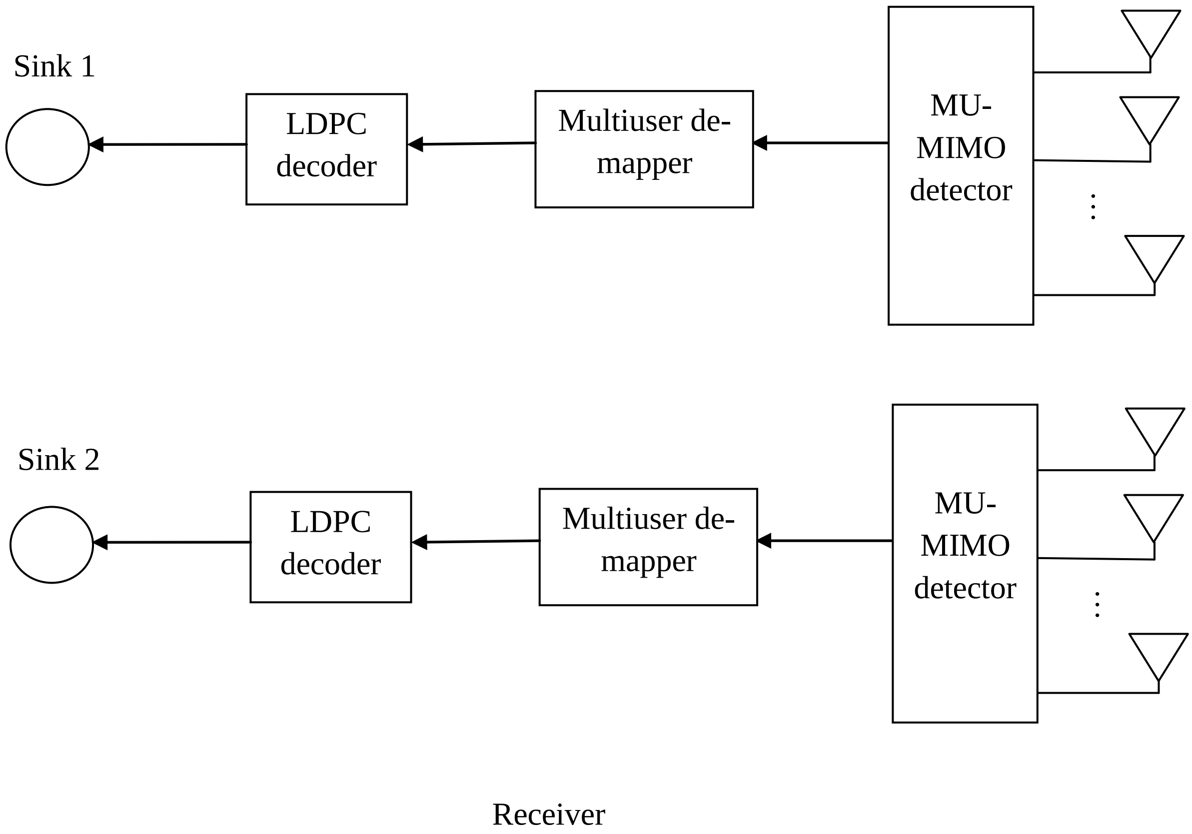


Fig. 5.2: LDPC coded MU-MIMO system

For each user, the binary input signals are encoded using an LDPC encoder. These encoded bits are mapped into data symbols via modulation techniques such as BPSK and QPSK. After modulation, incoming serial stream is converted into multiple parallel sub-streams. These parallel sub-streams are then transmitted over the MIMO channel. In this case, the received signal vector Y_t at t-th channel-use is given by

$$Y_t = \mathrm{H}^t X_t + N_{0t} \qquad \text{for } t = 1, \ldots, T \tag{5.4}$$

where,

$$Y_t := \begin{bmatrix} Y_{1t} \\ \vdots \\ Y_{M_R t} \end{bmatrix} \tag{5.5}$$

$$N_{0t} := \begin{bmatrix} N_{01t} \\ \vdots \\ N_{0M_R t} \end{bmatrix} \tag{5.6}$$

$$X_t := \begin{bmatrix} X_t^1 \\ X_t^2 \end{bmatrix}$$

$$X^u{}_t := \begin{bmatrix} X^u{}_{1t} \\ \vdots \\ X^u{}_{M_T t} \end{bmatrix} \tag{5.7}$$

$$\mathrm{H}^t = \begin{bmatrix} \mathrm{H}^{1t} & \mathrm{H}^{2t} \end{bmatrix} = \begin{bmatrix} h^{1t}{}_{11} & \cdots & h^{1t}{}_{1M_R} & h^{2t}{}_{11} & \cdots & h^{2t}{}_{1M_R} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ h^{1t}{}_{M_R 1} & \ldots & h^{1t}{}_{M_R M_T} & h^{2t}{}_{M_R 1} & \ldots & h^{2t}{}_{M_R M_T} \end{bmatrix} \tag{5.8}$$

and T is the total number of channel uses.

The matrix H^t is an matrix of order $M_R \times 2M_T$ with independent identically distributed (i.i.d.) complex-valued Gaussian random elements with zero mean and unit variance. The noise N_{0t} is circularly symmetric complex Gaussian with zero mean and variance σ_n^2.

At the receiver, the received signals are processed by the channel inversion method to remove interference due to number of users. The processed signals will be passed on to the de-mapper where it is iteratively de-mapped and thus decoded using log-domain sum product decoding algorithm to get the desired output.

5.3 BER Performance Analysis of LDPC Coded MIMO System

For BER analysis, initially it is assumed that all zero codeword is sent over a Rayleigh fading channel and densities of the variable and check node messages are Gaussian distributed. LLR's are considered to be independent random variables. Therefore, the summation of channel and incoming LLRs from check nodes is equivalent to convolution of their respective probability density functions (PDF's) [44]. The following steps are involved in derivation of BER expressions,

- Initially the PDF of channel LLR's derived.
- Convolve this channel PDF with the PDF of incoming LLR's from check node.
- Integrate the resultant PDF to get the BER expression [45].

To derive PDF of channel LLR's for LDPC coded 2×2 OSTBC MIMO system. The signal at receiver output can be expressed as [46]-[47]

$$Y = \frac{aX}{\sqrt{2}} + N_0 \tag{5.9}$$

where a is receiver combiner channel output gain. The conditional PDF of Y is given by

$$p(Y \mid A, X) = \frac{1}{\sqrt{2\Pi\sigma^2}} \exp\left(-\frac{(Y - aX/\sqrt{2})^2}{2\sigma^2} \right) \tag{5.10}$$

and LLR is given as

$$l_N = \ln\left[\frac{p(Y \mid a, X = +1)}{p(Y \mid a, X = -1)} \right] \tag{5.11}$$

$$= \ln\left[\frac{\frac{1}{\sqrt{2\Pi\sigma^2}} \exp\left(-\frac{(Y - a/\sqrt{2})^2}{2\sigma^2} \right)}{\frac{1}{\sqrt{2\Pi\sigma^2}} \exp\left(-\frac{(Y + a/\sqrt{2})^2}{2\sigma^2} \right)} \right]$$

$$= \ln\left[\exp\left(-\frac{(Y - a/\sqrt{2})^2}{2\sigma^2} + \frac{(Y + a/\sqrt{2})^2}{2\sigma^2} \right) \right]$$

$$l_N = \frac{2aY}{\sigma^2\sqrt{2}} \tag{5.12}$$

In the above equation substitute the value of Y from (5.9) and keeping X=1, we get the mean value as

$$\text{mean} = \frac{a^2}{\sigma^2} \tag{5.13}$$

and variance is twice of mean so

$$\text{variance} = \frac{2a^2}{\sigma^2} \tag{5.14}$$

using (5.12), conditional PDF of l_N is expressed as

$$p(l_N \mid a, X = +1) = \frac{\sigma}{2a\sqrt{\Pi}} \exp\left[-\frac{\left(l_N - 2a^2/\left(2\sigma^2\right)\right)^2}{8a^2/(2\sigma^2)} \right] \tag{5.15}$$

In STBC coded system, PDF of the receiver combiner output is given by

$$p(a) = a^3 \exp\left(-a^2\right) \qquad a \ge 0 \tag{5.16}$$

The PDF of channel LLR is evaluated as [45, eq. (3.325)]

$$p_c(l_N) = \int_0^\infty p(l_N \mid a, X = +1) p(a) da$$

$$= \frac{(-1)}{12} \frac{\partial^3}{\partial q^3} \frac{\sigma}{\sqrt{q}} \exp\left(\frac{l_c}{2} - \sqrt{q}\sigma |l_N| \right) \tag{5.17}$$

where $q = \dfrac{1 + 4\sigma^2}{4\sigma^2}$

The PDF of incoming LLR from check nodes is given by

$$p_i(l_N) = \frac{1}{\sqrt{4\Pi m_i}} \exp\left(-\frac{(l_N - m_i)^2}{4m_i} \right) \tag{5.18}$$

Convolution of channel PDF with PDF of incoming LLR from check node can be evaluated as [45, eq. (3.322(2))]

$$p_T(l_N) = p_c(l_N) * p_i(l_N) \tag{5.20}$$

$$= \frac{(-1)}{12} \frac{\partial^3}{\partial q^3} \frac{\sigma}{\sqrt{q}} \exp\left(\frac{\left(4q\sigma^2 - 1\right)m_i}{4} \right) \left[\exp\left(\frac{1 - \sqrt{4q}}{2} l_N \right) erfc\left(\frac{-l_N + \sqrt{4q}\sigma m_i}{2\sqrt{m_i}} \right) + \exp\left(\frac{1 + \sqrt{4q}}{2} l_N \right) erfc\left(\frac{l_N + \sqrt{4q}\sigma m_i}{2\sqrt{m_i}} \right) \right] \tag{5.21}$$

By integrating this resultant PDF $p_T(l_N)$, we can evaluate BER expression as

$$\text{BER} = \int_{-\infty}^{0} p_T(l_N) dl_N \tag{5.22}$$

$$= \frac{-1\sqrt{2}}{6}\frac{\partial^3}{\partial q^3}\left[\frac{\sigma}{\sqrt{2q}-4\sqrt{2}q^{3/2}\sigma^2}\exp\left(\frac{(4q\sigma^2-1)m_i}{4}\right)erfc\left(\sqrt{qm_i}\,\sigma\right)-\frac{\sqrt{2}\sigma^2}{1-4q\sigma^2}erfc\left(\frac{\sqrt{m_i}}{2}\right)\right]$$

(5.23)

Equation (5.23) can be further generalized for LDPC coded 4×4 OSTBC MIMO system as shown below

BER

$$= \frac{(-1)\sqrt{2}}{15!}\frac{\partial^{15}}{\partial q^{15}}\left[\frac{\sigma}{\sqrt{2q}-8\sqrt{2}q^{3/2}\sigma^2}\exp\left(\frac{(8q\sigma^2-1)m_i}{4}\right)erfc\left(\sqrt{2qm_i}\,\sigma\right)-\frac{\sqrt{4}\sigma^2}{1-8q\sigma^2}erfc\left(\frac{\sqrt{m_i}}{2}\right)\right]$$

(5.24)

5.4 Simulation Results

In this section, BER performance of different LDPC codes when they combined with OSTBC coded MIMO systems is analyzed. Design parameters for these simulations are shown in Table 5.1.

Table 5.1: Simulation parameters for LDPC coded MIMO

Parameters	Values
M_T	2 and 4
M_R	2 and 4
Modulation	BPSK
Channel Model Used	Rayleigh fading channel
Channel Fading	Rayleigh Independent and identically distributed
Channel Estimation	Ideal
Codeword length (*N*)	155 and 10000
Number of '1's Per Column	3

Number of '1's Per Row	5
Number of Iterations	50
LDPC Code Rate	½
Number of Users	4

Fig. 5.3 shows BER performances for 2×2 MIMO-STBC systems and 2×2 LDPC coded MIMO-STBC systems. Fig. 5.3 shows that we obtain BER of 10^{-3} around for 2×2 MIMO-STBC and 2×2 LDPC coded MIMO-STBC systems respectively. Thus, it shows that we get better BER performance when STBC is combined with LDPC codes.

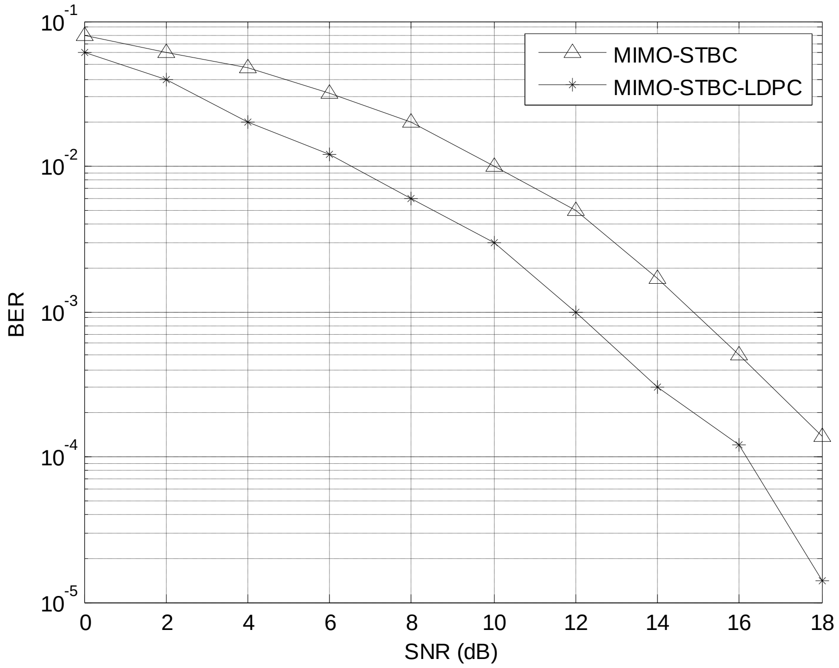


Fig. 5.3: BER performance of 2×2 STBC-MIMO system with 2×2 LDPC coded STBC-MIMO system

Fig. 5.4 shows BER of 10^{-3} around using random LDPC, QC-LDPC. Results show that at code length of 155 QC-LDPC coded MIMO-STBC systems gives better performance than Random-LDPC coded MIMO-STBC systems at similar design parameters.

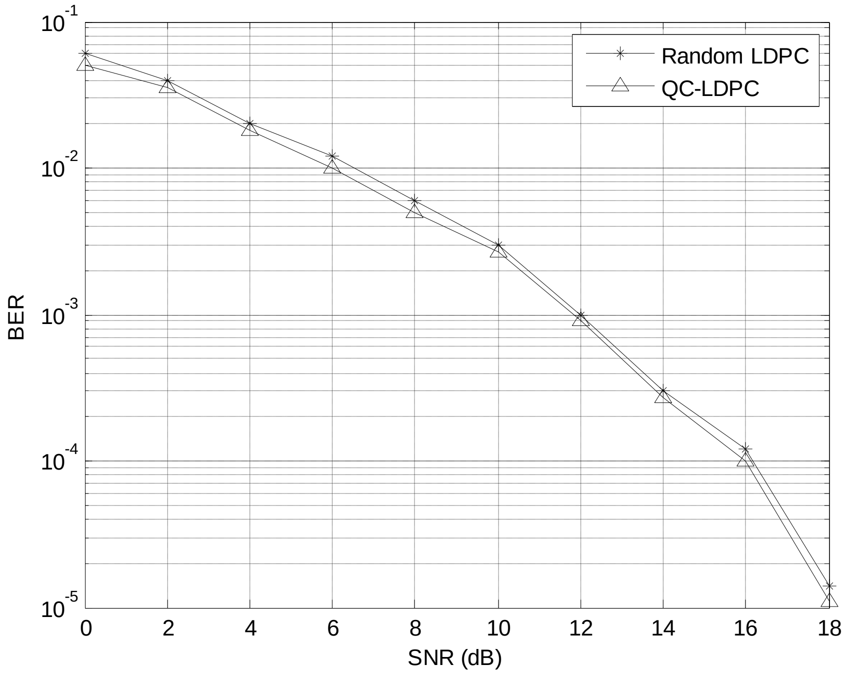


Fig. 5.4: BER performance of 2×2 MIMO-STBC system with different LDPC codes (N = 155)

Fig. 5.5 shows effect of increasing code length on BER performance of 2×2 MIMO system. Results show that BER performance is better with random LDPC code as compared to QC-LDPC at code length of 10000. Earlier it is better with QC-LDPC as it shown in Fig. 5.4 but at code length of 155.

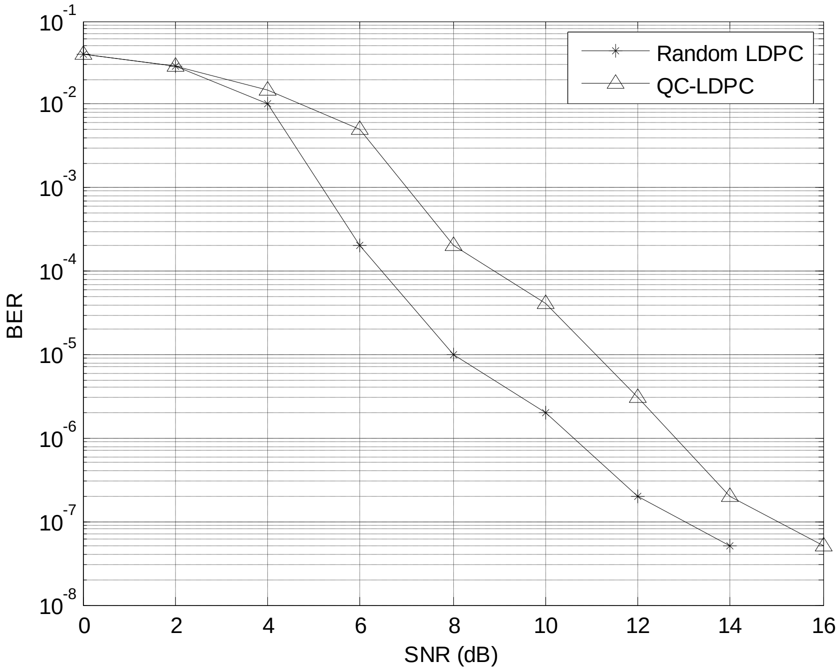


Fig. 5.5: BER performance of 2×2 MIMO-STBC system with different LDPC codes (N = 10000)

On comparing results of Fig. 5.4 and Fig. 5.6, it can be concluded that we get better BER performance with 4×4 configuration because of its higher diversity order.

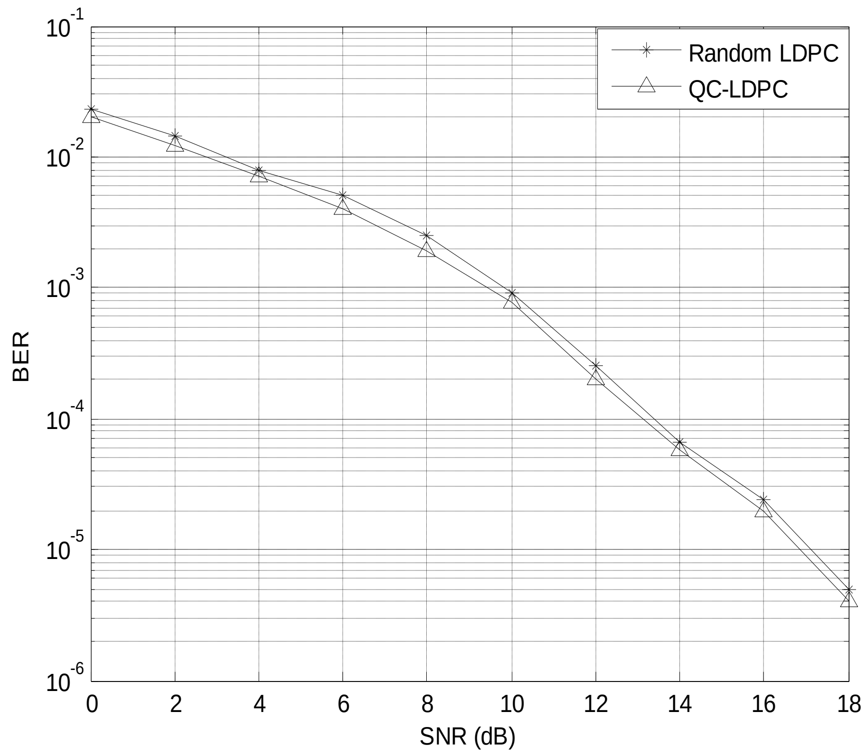


Fig. 5.6: BER performance of 4×4 MIMO-STBC system with different LDPC codes (N = 155)

In Fig. 5.7, BER performance of 2×2 MU-MIMO-STBC system with different LDPC codes are shown with code length of 155 and number of users equals to 4. On comparing this graph with Fig. 5.4, it can be seen that performance of multiuser system is bad as with the increase of number of user interference increases.

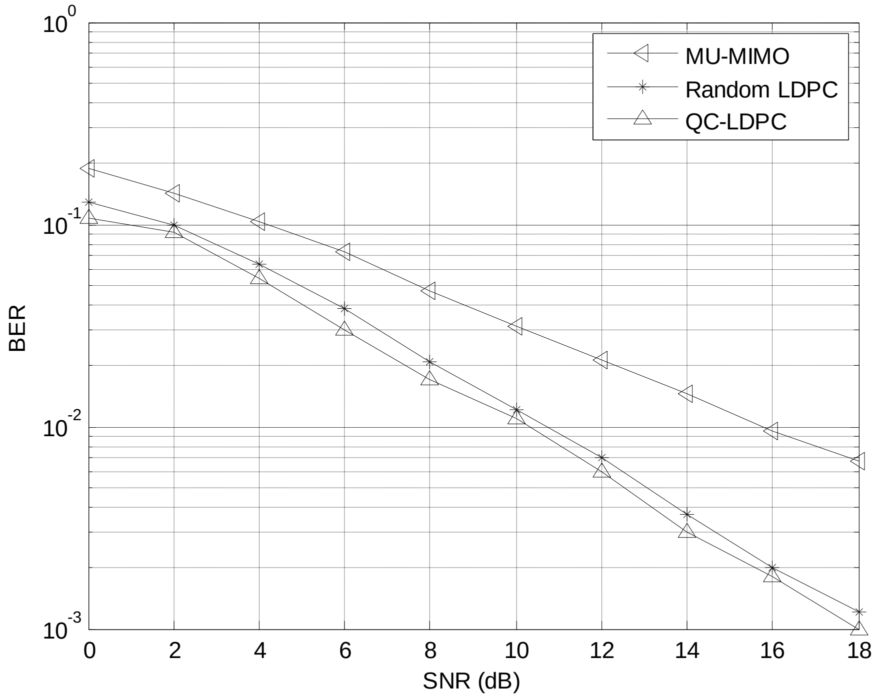


Fig. 5.7: BER performance of 2×2 MU-MIMO-STBC system with different LDPC codes (N = 155) and number of users equals to 4

Chapter 6

CONCLUSION AND FUTURE SCOPE

6.1 Conclusion

The goal of this thesis was to study LLR based BER performance of different LDPC codes when they are combined with OSTBC coded MIMO systems. In this thesis, we studied various construction methods of LDPC codes which are concatenated with MIMO-OSTBC systems with single user as well as multiple users. The main issue between random LDPC coded MIMO-OSTBC system and QC-LDPC coded MIMO-OSTBC system due to different construction methods is the trade-off between the need of high memory storage for randomly constructed codes and the easy implementation of QC codes. We concluded that BER performance of QC-LDPC is better than random LDPC with a lower code length and vice-versa with higher code length. The system performance is analyzed with LLR based low complexity decoding i.e. log-SPA. We also analyzed BER performance of multiuser MIMO-OSTBC system which is coded with both types of LDPC codes. We concluded that performance of multiuser system is bad when compared with single user case as interference exists due to multiuser. Further, approximate BER expressions are derived for both 2×2 and 4×4 MIMO configurations.

6.2 Future Scope

In this thesis, approximate BER expressions of LDPC coded MIMO systems is derived. Further these expressions can be extended for LDPC coded MU-MIMO systems.

LIST OF PUBLICATION

P. Gupta, B. Gupta, “ LLR Based Analysis of LDPC codes Concatenated with Orthogonal STBC Coded MIMO Systems”, Accepted in IEEE International Conference on Signal Propagation and Computer Technology (ICSPCT), 2014.

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RESUME

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Objective

A challenging career in the field of Engineering in a dynamic environment that allows me to utilize my inherent strengths and acquired skills and offers scope for learning and growth.

Education

M.Tech. in **Electronics and Communication** from **Jaypee University of Information Technology**

Academic Profile

M.Tech.

Year	University	Percentage/CGPA
2012-2014	Jaypee University of Information Technology	77% (till 3rd Sem.)

B.Tech.

Year	University	Percentage/CGPA
2008-2012	Uttar Pradesh Technical University	72.18%

10+2

Year	Board	Percentage
2008	CBSE	76.4%

10th

Year	Board	Percentage
2006	CBSE	83.4%

Trainings and Workshops

- Workshop on 'Amateur Radio Communications, software and Computer Networks in Education", in **Jaypee University of Information Technology.**
- Seminar by IEEE i.e. "2013 IEEE International Conference on Signal Processing, Computing and Control", in **Jaypee University of Information Technology.**
- Duration- 2 weeks, Training in **CETPA INFOTECH Private Limited**, "VHDL**"**.
- Duration- 4 weeks, Training in **CETPA INFOTECH Private Limited**, "Embedded System**"**.

Projects

M.Tech Project

Project Title : LLR Based Analysis of Quasi-Cyclic and Random LDPC Codes Combined with Multi-User MIMO Systems.
Duration : 1 Year
Team size : Individual Project
Organization : Jaypee University of Information Technology

B.Tech Project

Project Title	: Gesture Control Vehicle
Duration	: 1 year
Team size	: Two
Organization	: Hindustan Institute of Technology

Skill Set

Languages	**: C, C++, VHDL, VERILOG**
Software	**: MATLAB, PSPICE, LabVIEW, Embedded System**
Packages	**: Ms-Office**
Operating Systems	**: Windows XP/Vista/7**

Personal details

Name	**: Pallavi Gupta**
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Father's Name	**: Mahesh Gupta**
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