PERFORMANCE ANALYSIS OF MULTI-USER MIMO SYSTEMS WITH SPHERE DECODER

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DECLARATION

I, **PALLVI CHAWLA**, hereby declare that thesis entitled "PERFORMANCE ANALYSIS OF MULTI-USER MIMO SYSTEMS WITH SPHERE DECODER", has been carried out by me under the supervision of **Dr. BHASKER GUPTA**, Department of Electronics and Communication Engineering, Jaypee University of Information Technology, Waknaghat, Solan-173234, Himachal Pradesh, and has not been submitted for any degree or diploma to any other university. All assistance and help receive during the course of the investigation has been duly acknowledge.

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This is to certify that thesis entitled "PERFORMANCE ANALYSIS OF MULTI-USER MIMO SYSTEMS WITH SPHERE DECODER", submitted by PALLVI CHAWLA in partial fulfilment for the award of degree of Master of Technology in Electronics and Communication Engineering to Jaypee University of Information Technology, Waknaghat, Solan has been carried out under my supervision.

This work has not been submitted partially or fully to any other University or Institute for the award of this or any other degree or diploma.

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List of Abbreviations

ADC	Analog digital converter
AP	Access point
BC	Broadcast channel
BD	Block Diagonalization
BER	Bit error rate
BPSK	Binary Phase Shift Keying
BS	Base station
CQI	Channel quality indicator
CSI	Channel state information
DL	Downlink
DPC	Dirty-paper coding/Dirty-paper code
eNB	Evolved Node B
FDD	Frequency division duplex
FDMA	Frequency division multiple access
I.I.D.	Independent and Identically Distributed Channel
IQ	In-phase / quadrature
LNA	Low-noise amplifier
LOS	Line of sight
MAC	Multiple access channel
MGF	Moment Generating Function
MIMO	Multiple-input multiple-output
ML	Maximum Likelihood
MMSE	Minimum mean square error
MSE	Mean square error
MU	Multi-user
MUI	Multi-user interference
NLOS	Non-line-of sight
OFDM	Orthogonal frequency division multiplex
OFDMA	Orthogonal frequency division multiple access
OSTBC	Orthogonal space-time block code
PEP	Pair-wise Error Probability
QAM	Quadrature amplitude modulation
QoS	Quality of service
RF	Radio frequency
RMS	Root-mean-squared
QOSTBC	Quasi-orthogonal space time block coding
QPSK	Quadrature Phase Shift Keying
SD	Sphere Decoder
SDMA	Spatial division multiple access
SIC	Successive interference cancellation
SINR	Signal to interference plus noise ratio
SISO	Single-input single-output

SNR	Signal to noise ratio
SMUX	Spatial Multiplexing
STC	Space Time Coding
SU	Single user
SVD	Singular value decomposition
TDD	Time division duplex
TDMA	Time division multiple access
UL	Uplink
UT	User Terminal
V-BLAST	Vertical Bell Labs Layered Space-Time
ZF	Zero forcing

ABSTARCT

Multiple-input, multiple-output (MIMO) systems are the most promising technique in wireless communication systems, because of their improvement in terms of performance and bandwidth efficiency. The major advantages of MIMO systems are to increase system reliability through diversity and to achieve higher data rates through spatial multiplexing. MIMO techniques were first investigated for single-user scenario and now it is extended to multi-user MIMO (MU-MIMO) systems. Now days, the important research topic is the study of multi-user MIMO (MU-MIMO) systems. MU-MIMO systems are a key technology for future wireless communication systems because these systems satisfy most of the requirements of the next generations. Such systems have the potential to combine the high throughput achievable with MIMO processing with the benefits of space division multiple access (SDMA). Joint processing of MIMO channels yields maximum diversity regardless of the level of multi-user interference.

In this thesis, dirty paper coding (DPC) and various decoding schemes are discussed where there is no limit of transmitting antennas, receiving antennas and users. I have introduced linear and non-linear MU-MIMO processing (decoding) techniques. Linear equalizers are zero forcing (ZF) and minimum mean square error (MMSE) while maximum likelihood (ML), sphere decoder (SD) and fixed sphere decoder (FSD) are non-linear equalizers. The MU-MIMO decoding techniques that are proposed in this thesis are ZF, MMSE, ML and SD. The major focus will be on SD.

As it was previously reported in the literature, ZF technique nullifies the interference or in other words, it inverts the effect of channel. MMSE technique is optimum for single-antenna UTs. However, MMSE suffers from a performance loss when users are equipped with more than one antenna. ML calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. SD intends to find the transmitted signal vector with minimum ML metric (i.e. to find the ML solution vector) inside the sphere. However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors. Thus, complexity and processing time reduce in case of sphere decoder.

Then, BER performance is analyzed using SINR, MGF and PEP based approach in MU-MIMO systems with different equalizing techniques (ZF, MMSE, ML and SD). First, assuming the perfect channel knowledge at the transmitter and receiver i.e. the channel state information (CSI) is available both side. Thus, transmitted signal is defined such that the channel fading effect is greatly mitigated. This will improve the bit error rate (BER) performance of the MU-MIMO system.

For proposed schemes, it is observed that BER performance improves as I go toward non-linear equalizes. After simulation, it is also observed that the BER performance of MU-MIMO system with SD is better than the ML, MMSE and ZF.

Chapter 1
Introduction

The next generation of wireless mobile communication systems requires the reliable transmission of high-rate data under various types of channels and scenarios. Current wireless mobile, data, and fixed access communication systems are converging into a data oriented wireless networks with high spectral efficiency. Future wireless communication systems should be flexible and adaptive to various scenarios and Quality-of-Service (QoS) requirements. The system should be robust to the influence of fading, interference, and hardware imperfections.

The very high data rate that is required for future wireless systems in reasonably large areas do not appear to be feasible with the conventional techniques and architectures. Frequency bands that are envisioned for future wireless communication systems are well above 2 GHz. The radio propagation in these bands is significantly more vulnerable to non-line-of sight (NLOS) conditions, which is typical in modern urban communications.

The efficient design of wireless systems will require the use of multiple antennas, advanced adaptive modulation and coding schemes, relaying nodes, cooperative networks and users, and cross-layer design. The goal of reaching high data rates is particularly challenging for systems that are power, bandwidth, and complexity limited. However, another domain can be exploited to significantly increase channel capacity: the use of multiple transmit and receive antennas.

Pioneering work done in [1], [2], and [3], ignited much interest in this area by predicting remarkable spectral efficiencies for wireless systems with multiple antennas when the channel exhibits rich scattering and the channel state information (CSI) can be accurately tracked. This initial promise of exceptional spectral efficiency resulted in an explosion of research activities to characterize the theoretical and practical issues associated with MIMO channels and to extend these concepts to multi-user systems.

The main question from both a theoretical and practical standpoint is whether the enormous initially predicted capacity gains can be obtained in a more realistic operating scenarios and what specific gains result from adding more antennas and computational power to obtain CSI at the transmitter and receiver.

For single-user (SU) systems, a transmission and reception strategy that exploits this structure achieves capacity on approximately min (M_T, M_R) separate channels, where M_T is the number of transmit antennas and M_R is the number of receive antennas. Thus, capacity scales linearly with min (M_T, M_R) relative to a system with one transmit and one receive antenna. The capacity increase requires a scattering environment such that the matrix of channel gains between each transmit and receive antenna pair has full rank and independent entries and that perfect estimates of these gains are available at the transmitter and receiver.

Space-time coding (STC) [4], [5], and spatial multiplexing (SMUX) [3], [6], provide full diversity and achieve high data rates over MIMO channels, respectively. Spatial multiplexing involves transmitting independent streams of data across multiple antennas to maximize throughput, whereas space-time coding maps input symbol streams across space and time for diversity and coding gain at a given data rate. Neither scheme requires CSI at the transmitter. However, to achieve the maximum information rate and/or the diversity and array gain afforded by increased computational complexity, appropriate precoding and modulation techniques are necessary.

Generalized designs of a jointly optimum linear precoder and decoder for a SU MIMO system, using a mean-squared error (MSE) criterion are presented in [7] and [8]. The framework presented in this thesis is general and addresses several optimization criteria like minimum MSE (MMSE), minimum bit error rate (BER) and maximum information rate. It is assumed that the channel is known at the receiver as well as at the transmitter.

An important research topic is the study of multi-user MIMO (MU-MIMO) systems. Such systems have the potential to combine the high capacity achievable with MIMO processing with the benefits of space division multiple access (SDMA). In the MU-MIMO scenario, a base station (BS) or an access point (AP) is equipped with multiple antennas and it is simultaneously communicating with a group of users. Each of these users is also equipped with multiple antennas. We focus on systems where the complex signal processing is performed at the BS/AP. The BS/AP will use the CSI available at the transmitter to allow these users to share the same channel and mitigate or completely eliminate multi-user interference (MUI) in an ideal case.

In an MU scenario, capacity becomes a K-dimensional region defining the set of all rate vectors (R_1, . . . , R_K) simultaneously achievable by all K users. Two MU-MIMO scenarios can be distinguished. In the first scenario, multiple non-cooperative terminals are transmitting to a single receiver. This scenario is often referred to as the MU-MIMO uplink (UL) channel. In the information theory, it is known as the MIMO multiple access channel (MAC). The scenario, in which a single terminal is transmitting to multiple non-cooperative receivers, is referred to as MU-MIMO downlink channel or broadcast channel (BC).

The capacity region of a general MIMO MAC was obtained in [2], [9]-[10]. It has been shown that a linear detection with successive interference cancellation (SIC) provides the maximum sum rate capacity of a MU MAC system. However, the capacity of a MIMO BC is an open problem due to the lack of a general theory on non-degraded broadcast channels. Pioneering work done in [11], a set of achievable rates for the MIMO BC was obtained by applying Costa's "dirty-paper" coding (DPC) technique at the transmitter [12]. In [12], Costa proved that DPC is a technique that allows non-causally known interference to be "pre-subtracted" at the transmitter. It was also shown in [11] that the sum rate MIMO BC capacity equals the maximum sum rate DPC achievable region by demonstrating that the achievable rate meets the Sato upper bound [13]. In [14], [15] it was shown that the achievable region of the MIMO BC obtained using DPC is equal to the capacity region of the MIMO MAC using uplink-downlink duality. DPC can achieve the maximum sum rate of the system and provide the maximum diversity order [16], [17]. However, these techniques require the use of a complex sphere decoder or an approximate closest-point solution, which makes them hard to implement in practice, especially when the number of users is large [17]. So, to decode the data, various decoding algorithms are used. These algorithms can be linear or non-linear. Linear decoding techniques are zero-forcing, minimum mean square error while non-linear decoding techniques are maximum likelihood, sphere decoder and fixed complexity sphere decoder etc.

Linear MU-MIMO processing techniques [18]-[24] are less computationally demanding than non-linear, and they can use either instantaneous channel knowledge or long-term statistics of the channel to perform precoding or decoding. In this thesis it will be empirically shown that linear processing techniques reach the sum-rate capacity of the BC channel also when the total number of antennas at the user terminals is equal to or greater than the number of antennas at the base station. Non-linear MU-MIMO processing techniques [24]-[27] require the instantaneous knowledge of the channel transfer function at the BS. On the other hand, linear MU-MIMO processing techniques can be used with various degrees of channel state information. Thus, linear techniques are more flexible and more favourable for practical implementation than non-linear techniques.

1.1 Objective of Thesis and Thesis Organization

In this thesis, a general framework is introduced for the design of the MU-MIMO decoding matrices [28]-[40]. Main goal is to define MU-MIMO algorithm that will be able to address several optimization criteria like minimum MSE (MMSE), minimum bit error rate (BER), and maximum information rate [41],[42]. It has been shown in the literature that the simulation results are based on MU-MIMO precoding and decoding algorithms that are DPC [27],[41],[43]-[55], ZF [56]-[61], MMSE[60]-[79], ML[65],[80]-[95], SD [95]-[102] and FSD; considering all users are equipped with one antenna.

The link between the user terminals and the base station in a wireless multi-user MIMO scenario is the wireless propagation channel.

In Chapter 2 we describe the MIMO channel models that will be used in the simulations. An overview of the SU MIMO processing techniques is given in Chapter 2. First, we will review techniques that do not need any CSI at the transmitter to extract diversity gain or spatial multiplexing gain. These techniques do not require CSI at the transmitter to encode the user's data. A short overview of various MIMO gains like diversity gain, spatial multiplexing gain, array gain etc., is given in this Chapter.

In Chapter 3 provides an overview of fundamentals of multi-user MIMO channel. A brief introduction about the capacity of the MAC and the BC channels is provided. Some benefits and limitations of MU-MIMO systems are also discussed in this Chapter.

In Chapter 4, A short overview of the dirty paper coding has given. This is the most relevant multi-user precoding techniques that we have used as a starting point in our investigations.

In Chapter 5, a short overview has been given, of the most relevant multi-user decoding techniques that we have used as a starting point in our investigations. Each of these techniques has certain drawbacks that have significant impact on the performance and design of the multi-user MIMO systems. The zero forcing (ZF) decoder tries to nullify the interference but has noise enhancement. Minimum mean-square-error (MMSE) decoder balances the multi-user interference mitigation with noise enhancement and minimizes the total error. The drawback of this technique is that it is limited to single antenna user terminals. In a MU-MIMO system employing MMSE decoding, if the user terminal is equipped with more than one antenna, the signal transmitted over each antenna needs to be decoded independently. Block Diagonalization (BD) is more appropriate to be used with user terminals with multiple numbers of antennas [25]. However, it has a limitation that the total number of antennas at the user terminals has to be less than or equal to the number of antennas at the base station.

Maximum likelihood calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. SD [95-102] intends to find the transmitted signal vector with minimum ML metric (i.e. to find the ML solution vector) inside the sphere. However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors. Thus, complexity and processing time reduce in case of sphere decoder.

A novel approach that is sphere decoder for MU-MIMO systems has been implemented to reduce the computational complexity and to achieve better BER performance. At the end of Chapter 7, a short overview has been given, of the computational complexity of ML and SD.

Chapter 6 portrays SINR, MGF and PEP based BER performance analysis of MU-MIMO system with linear (ZF and MMSE) and non-linear (ML and SD) equalizers. BER analysis based on this unique approach is also done with i.i.d (independent and identically distributed) and spatially correlated channels.

The results of the system level investigations of MU-MIMO decoding techniques are given in this Chapter. The results show that MU-MIMO systems with sphere decoder can provide improved BER performance than ZF, MMSE and ML. The channel estimation errors are investigated on the performance of the MU-MIMO decoding techniques in this Chapter.

First, assuming the perfect channel knowledge at the transmitter and receiver i.e. the CSI is available both side. Thus, transmitted signal is defined such that the channel fading effect is greatly mitigated. This will improve the BER performance of the MU-MIMO system. For proposed schemes, it is observed that BER performance improves as I go towards non-linear equalizer. After simulation, it is also observed that the BER performance of MU-MIMO system with SD is better than the ML, MMSE and ZF.

Finally, in chapter 7, conclusions are drawn. Chapter 7 also portrays the future scope of the work done.

Chapter 2
MIMO SYSTEM MODEL

The goal of future wireless communication systems is to provide a wide variety of high quality high rate services with minimum requirements on spectrum, power consumption and hardware complexity. Towards this end, proper system structures as well as robust system designs are required to meet the challenges in wireless transmissions, such as multipath fading, limited spectrum resource and interference. Recent research results have unveiled the multiple input multiple output (MIMO) system as a potential candidate to play a key role in future wireless. MIMO is a multiple antenna technology which uses number of antennas at both transmitter and receiver side. Early work on multi antenna systems involves the use of antenna arrays at the receiver to provide spatial diversity against the random destructive effect of fading. MIMO technique is advancement over various previous technologies like SISO, SIMO and MISO etc. The improvement in reliability of a MIMO system compared to that of a traditional SISO system is typically quantified by the diversity gain and coding gain.

A profound understanding of MIMO system model is crucial in selecting proper signalling strategies in MIMO wireless systems. Simple models have been used to get the insight into the impact of propagation conditions on MIMO channel. In this chapter we review the construction of wireless MIMO channels which we will use in simulations, its sampled model, as well as the input-output signal model.

2.1 MIMO Communication System

The basic building blocks those comprise a MIMO communication system is shown in Fig. 2.1. The information bits to be transmitted are encoded and interleaved. The interleaved codeword is mapped to data symbols (QAM) by the symbol mapper. These data symbols are input to a space time encoder that outputs one or more spatial data streams. The spatial data streams are mapped to the transmit antennas by the space-time precoding block. The signals launched from the transmit antennas propagate through the channel and arrive at the receiver antenna array.

The receiver collects the signal at the output of each receive antenna element and reverses the transmitter operation in order to decode the data: receive space-time processing, followed by space –time decoding, symbol de-mapping, de-interleaving and decoding. Each of the building blocks offers the opportunity for significant design challenges and complexity performance trade-offs.

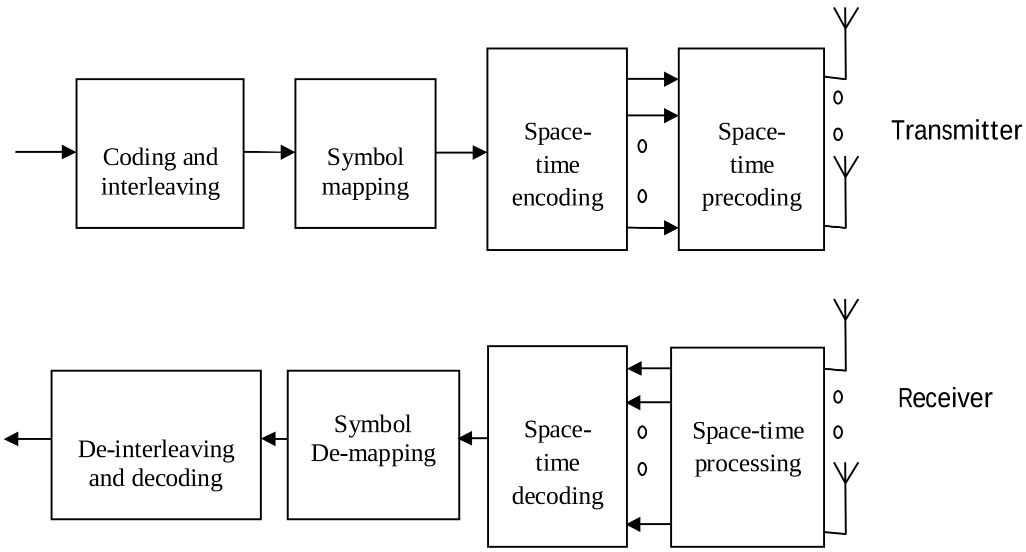

Figure 2.1: Block Diagram of MIMO Communication System

2.2(i) MIMO System Model

If we want to design the efficient communication algorithms for MIMO systems and to get the knowledge of the performance limits, it is important to understand the nature of the MIMO system and channel. Figure 2.2 shows the generalized MIMO system model.

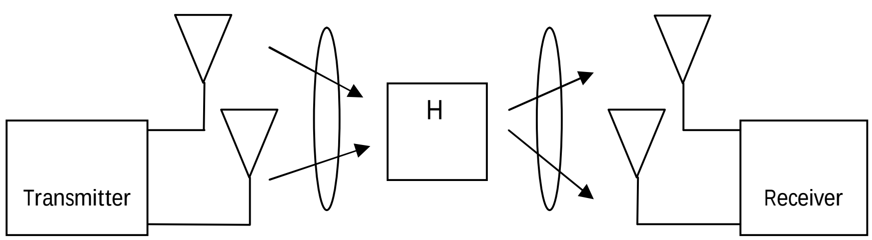

Figure 2.2: Block Diagram of MIMO System Model

A MIMO channel with M_T transmit antennas and M_R receive antennas is typically represented as a matrix H of dimension M_R x M_T, where $[H]_{i,j}$ represents the complex Gaussian random variable that models fading gain between the j^{th} transmitter and the i^{th} receiver. The MIMO channel model is presented in (2.1)-

$$Y = H x + w \tag{2.1}$$

where w is a vector of additive Gaussian noise with zero mean and unit variance and having dimension of M_R 1, Y is the vector of received data with dimension M_R 1, X is the vector of transmitted data with dimension M_T 1 and H is channel matrix with dimension M_R M_T.

2.2 (ii) Wireless Channel/MIMO Channel Model

The link between the user terminals and the base station in a wireless multi-user MIMO scenario is the wireless propagation channel. One of the distinguishing characteristics of wireless channel is the fact that there are various paths between transmitter and receiver. These various components experience different path loss and phases. If there is a direct path between the transmitter and the receiver, it is called the line of sight (LOS) path. LOS path does not exist when large objects obstruct the line between Tx and Rx. If LOS exists, the corresponding signal received through LOS is the strongest and the dominant signal. The signal from the LOS is more deterministic while its strength and phase may change due to mobility.

There is various non-line of sight (NLOS) path along with LOS. An electromagnetic wave experiences different propagation mechanisms. These propagation mechanisms which are used to characterize the channel model are path loss exponent, reflection, diffraction, scattering, refraction, absorption, attenuation, fading, shadowing etc.

The effect of these propagation mechanisms result in many properties of the received signal that are unique to wireless channel. These effects may reduce the power of the signal in different ways. There are two general aspects of such a power reduction that require separate treatments. One facet is the large scale effect which corresponds to the characterization of the signal power over large distances or the time-average behaviours of the signal. This is called attenuation or path loss and sometimes large-scale fading. The other aspect is the rapid change in the amplitude and power of the signal and this is called small-scale fading or simply fading.

- **Attenuation-** Attenuation or large scale fading is caused by many factors including propagation losses, antenna losses and filter losses. The average received signal or the large scale fading factor, decreases logarithmically with distance. The logarithm factor or the path loss exponent depends on the propagation medium and the environment between Tx and Rx.

- **Fading** – Fading is basically a random variation in signal strength. Fading or small scale fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at different times. These signals, called multipath waves, combine at the receiver antenna and provide an effective combined signal. This resultant signal can vary widely in amplitude and phase.

2.2.1 SU-MIMO System Model

A MIMO channel with M_T transmit antennas and M_R receive antennas comprises of $M_T M_R$ SISO channels.

In case of single-user MIMO (SU-MIMO) system as shown in Fig. 2.3), all the received data is available for processing while in the case of MU-MIMO, received data is distributed among different users. If each user has only one receive antenna then user is restricted to access only one element of the received data Y.

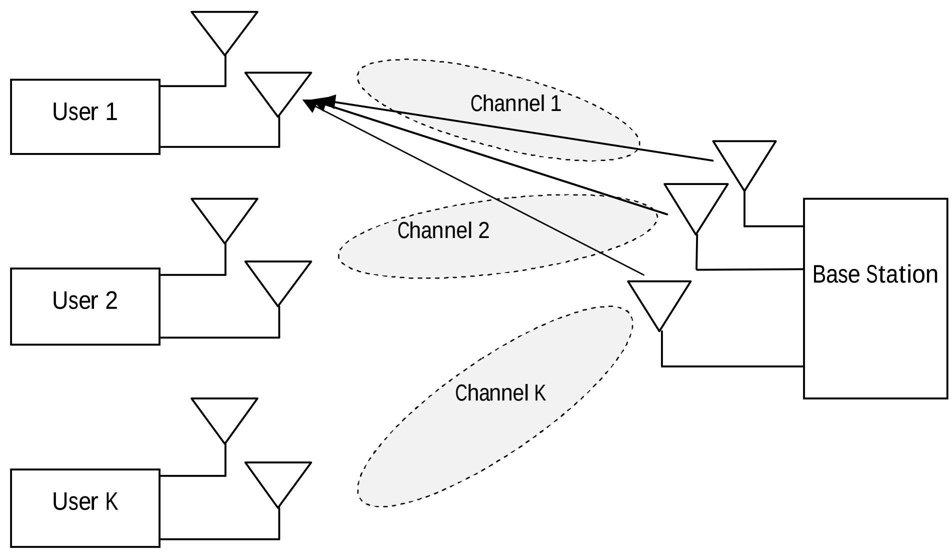

Figure 2.3: Single-User MIMO System Model

The channel matrix is denoted by H. The ij-th component of the matrix H, denoted by h_{ij} , represents the channel gain between the j^{th} transmit and i^{th} receive antenna pair.

$$H = \begin{bmatrix} h_{11} & h_{12} & . & . & h_{1M_T} \\ h_{21} & h_{22} & . & . & h_{2M_T} \\ . & . & . & . & . \\ . & . & . & . & . \\ h_{M_R 1} & h_{M_R 2} & . & . & h_{M_R M_T} \end{bmatrix} \quad (2.2)$$

The i^{th} column of H is often referred to as the spatial signature of the i^{th} transmit antennas across the receive antenna array. The relative geometry of the M_T spatial signatures determines the distinguishability of the signals launched from the transmit antennas at a receiver. This is particularly important when independent data streams are launched from the transmit antennas, as is done in the case of spatial multiplexing. The MIMO system model as shown in Fig. 1 uses multiple antennas at both transmitter and receiver side to improve the communication performance.

2.3 Gain/ Benefits of MIMO Systems

By using multiple antennas various gains can be achieved-

- Diversity gain
- Capacity gain
- Array gain
- Interference mitigation

2.3.1 Spatial Diversity Gain

Spatial diversity gain is realized by providing the receiver with multiple copies of the transmitted signals in space, time or frequency. With an increasing number of independent copies (the number of copies is often referred as the diversity order), the probability that at least one of the copies is not experiencing a deep fade increases, thereby improving the quality and reliability of reception. A MIMO channel with M_T transmit antennas and M_R receive antennas potentially offers M_RM_T independently fading links, and hence a spatial diversity order of M_RM_T.

Space-time diversity methods assume that the receiver has perfect channel knowledge and the transmitter has no channel knowledge. They are designed to perform well over averaged channel statistics, to provide diversity gain. Diversity gain reduces fading effect, BER and improves the quality of signal. Thus, diversity gain provides reliable communication.

2.3.2 Spatial Multiplexing Gain

MIMO systems offer a linear increase in data rate through spatial multiplexing [12,17, 40], i.e., transmitting multiple, independent data streams within the bandwidth of operation.

Each data stream experiences at least the same channel quality that would be experienced by a SISO system, effectively enhancing the capacity by a multiplicative factor equal to the number of streams. In general, the number of data streams that can be reliably supported by a MIMO channel equals the minimum of the number of transmit antennas and the number of receive antennas, i.e., min. $\{M_T, M_R\}$. The spatial multiplexing gain increases the capacity of a wireless network.

By transmitting and receiving parallel independent symbol streams in the same frequency bandwidth, spatial multiplexing obtains a linear increase in data rates, with increase in the number of antennas. We note that spatial channel multiplexing does not require channel knowledge at the transmitter.

2.3.3 Array Gain

Array gain is the increase in receive SNR that results from a coherent combining effect of the wireless signals at the receive antennas array and/or spatial pre-processing at the transmit antenna array. Array gain improves resistance to noise, thereby improving the coverage and the range of a wireless network. Array gain is proportional to the number of receive antennas used.

2.3.4 Interference Mitigation

Co-channel interference adds to the overall noise of the system and deteriorates performance. Interference reduction allows use of aggressive reuse factors and improves the system capacity. Interference in wireless networks results from multiple users sharing time and frequency resources. Interference may be mitigated in MIMO systems by exploiting the spatial dimension to increase the separation between users. For instance, in the presence of interference, array gain increases the tolerance to noise as well as the interference power, hence improving the SINR. Additionally, the spatial dimension may be leveraged for the purposes of interference avoidance, i.e., directing signal energy towards the intended user and minimizing interference to other users. Interference reduction and avoidance improve the coverage and range of a wireless network.

In general, it may not be possible to exploit simultaneously all the benefits described above due to conflicting demands on the spatial degrees of freedom. However, using some combination of the benefits across a wireless network will result in improved capacity, coverage and reliability.

Interference reduction can also be implemented at the transmitted side, where the goal is to enhance the signal power at the intended receiver and minimize the interference energy sent towards co-channel users.

MIMO systems add the diversity so the robustness of the system improves. MIMO systems also provide high data rate and high spectral efficiency. To achieve high data rate with low BER, there is a trade-off between data rate and BER.

Drawbacks of MIMO systems are that these systems are very costly and complex because it requires large number of antenna array and powerful DSP unit. Thus, signal processing also becomes very complex.

Chapter 3
Multi-User MIMO channels

3.1 Multi-User MIMO (MU-MIMO) System Model

MU-MIMO is a MIMO system in which multiple users can take participation in data transmission simultaneously. In the uplink of cellular network, users transmit signals to the base station over the same channel but it is difficult for the base station to separate these signals. If transmitter provides channel feedback information back to the users then coordination among users may be possible. For this coordination each user must know channels experienced by other users as well as its own channel. In uplink, base station receives the data from multiple users. It is also known as uplink- MAC (multiple access channel). It is a multipoint to point communication.

In the downlink, base station transmits information simultaneously to a group of users. But there is some inter-user interference because signal received by one user will act as interference signal for other remaining users. It is also known as downlink-BC (broadcast). It is a point to multipoint communication.

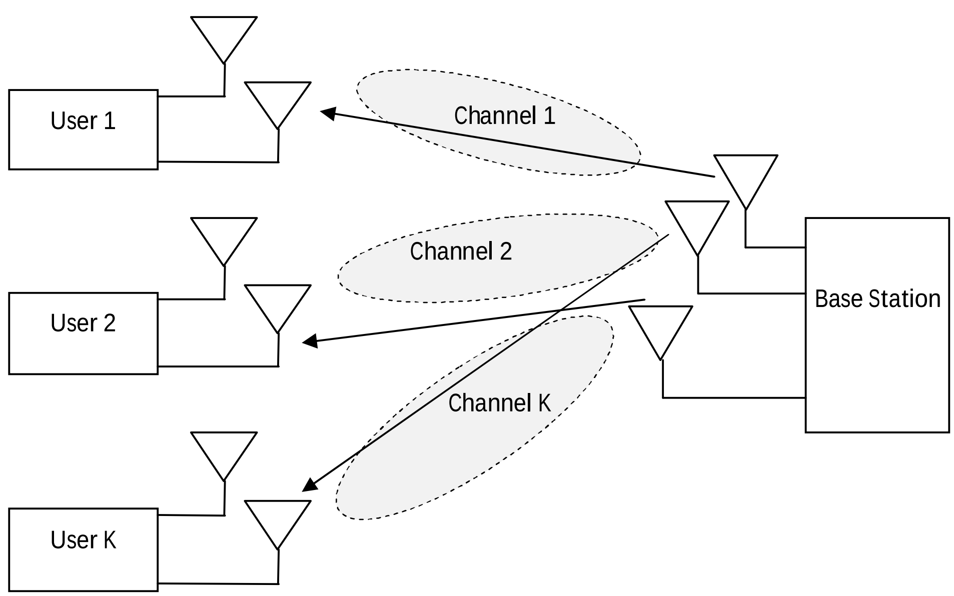

Figure 3.1: Multi-User MIMO system model

In SU-MIMO channel, MIMO system takes the advantage of coordination among all the transmitters and receivers but there is no coordination among the users in case of MU-MIMO channel.

Consider an MU-MIMO system model for downlink data transmission for a cellular system [1]-[3]. Fig. 3.1 shows the MU-MIMO system model. This system employs a single base station (BS) which has M_{Tx} transmit antennas and K users where each user has M_{Rx} receive antenna. Let s_k denotes the transmitted data intended for user k. For each user symbol s_k is multiplied by a beam former vector c_k, thus the signal vector $\mathbf{x}$ $\hat{C}^{M_{T_x}\times 1}$ can be written as

$$\mathbf{x} = \sum_{k=1}^{K} c_k s_k = C\, s \tag{3.1}$$

where $C = [c_{1,}c_{2,\ldots\ldots\ldots}c_k]$ $\hat{C}^{M_{T_x}\times K}$ is a beam forming matrix, and $s = \left[s_{1,}s_{2,\ldots\ldots\ldots}s_k\right]^{M_{T_x}}$ $\hat{C}^{K\times 1}$ is the signal vector. Hence, the received signal vector Y_i of the i^{th} user can be expressed as

$$Y_i = H_i\, \mathbf{x} + w_i \tag{3.2}$$

$$Y_i = H_i\, C\, s + w_i$$

$$Y_i = H_i\, c_i\, s_i + \sum_{k=1, k\neq i}^{K} H_i c_k s_k + w_i \tag{3.3}$$

where $Y = \left[y_1,\ y_{2,\ldots\ldots\ldots}y_K\right]^{M_{R_x}}$; $\quad s = \left[s_{1,}s_{2,\ldots\ldots\ldots}s_k\right]^{M_{T_x}}$

$$H = [\,(h_1)^{M_{T_x}}, (h_2)^{M_{T_x}}, \ldots\ldots, (h_K)^{M_{T_x}}\,];$$

$$C = [c_1,\ c_2,\ \ldots\ldots\ldots, c_K]\ ; \qquad w = \left[w_1, w_{2,\ldots\ldots\ldots}w_K\right]^{M_{R_x}}$$

The noise w_i $\hat{C}^{(M_{R_x})_i\times 1}$ is independent complex Gaussian distributed noise with zero mean and unit variance. The MIMO channel H for the i^{th} user is H_i $\hat{C}^{(M_{R_x})_i\times M_{T_x}}$. MIMO channel is basically a realization of standard i.i.d Rayleigh fading channel. The received signal vector Y_i of the i^{th} user at the t^{th} symbol interval can also be written as

$$Y_i[t] = H_i[t]\, C[t]\, s[t] + w_i[t] \tag{3.4}$$

$$Y_i[t] = H_i[t]\, c_i[t]\, s_i[t] + \sum_{k=1, k\neq i}^{K} H_i[t] c_k[t] s_k[t] + w_i[t] \tag{3.5}$$

where s_k satisfies $E[s_k\, s_k^{*}] = E_s$ and E_s is the symbol energy.

3.2 Capacity Region of Multi-User MIMO Channels

In this section, single-user and multi-user MIMO channel capacities in the Shannon theoretic sense are given. The Shannon capacity of a time-invariant channel is defined as the maximum mutual information between the channel input and output. This is the maximum data rate that can be transmitted over the channel with arbitrarily small error probability. When the CSI is perfectly known at both the transmitter and the receiver, the transmitter can adapt its transmission strategy relative to the instantaneous channel state. If the channel is time variant, the ergodic capacity is the maximum mutual information averaged over all channel states. The ergodic capacity is typically achieved using an adaptive transmission policy where the power and data rate vary relative to the channel state variations.

In a single-user MIMO system the link is point-to-point with a defined capacity. In a multi-user MIMO system, the link is a multiple access channel on the uplink and broadcast channel on the downlink. SU-MIMO suffers only a small penalty in information rate without CSI at the transmitter. MU-MIMO has a much larger penalty on the downlink. In a SU-MIMO system, precoding at the transmitter and decoding at the receiver can be done with full cooperation between the collocated antennas. In a MU-MIMO system, the antennas can cooperate at the base station for precoding on the downlink and for decoding on the uplink. However, the users cannot cooperate in decoding on the downlink or during the precoding on the uplink. In a MU-MIMO system, cooperation between the users may be possible in terms of power rates assigned to the users. In a SU-MIMO system, the information rate is identical on the uplink and downlink for same transmitting power if the channel is known at the transmitter and the receiver.

3.2.1 Single-user MIMO capacity

When the channel is constant and known perfectly at the transmitter and the receiver, the capacity of the channel can also be determined. If H is random, the channel capacity is a random variable too, and can vary from zero to infinity. If the exact CSI is not known at the transmitter, the information rate maximization can now be performed only in terms of the outage or ergodic capacity.

Capacity can be classified as-

- Outage Capacity
- Ergodic Capacity
- Asymptotic Capacity

Asymptotic capacity is used to characterize the distribution of capacity. Capacity can be written as-

$$C = \log_2 \left| I_{M_T} + \frac{1}{\sigma_w^2} H R_{M_T} H^H \right|$$

where $R_{M_T} = R_x = E[xx^H]$ = covariance matrix of transmitted vector.

For SISO channel, capacity can be presented as-

$$C = M_T \log_2 \left(1 + \frac{P_T}{\sigma_w^2} \right)$$

Similarly, For MIMO channel, capacity can be written as-

$$C = \min.(M_R, M_T) \log_2 \left(1 + \frac{P_T}{\sigma_w^2} \right)$$

3.2.2 Capacity region of MAC and BC channel

The union of achievable rates under all transmission strategies is called the capacity region of the multi-user system. It defines the limit of error-free communications given certain channel characteristics and it is used as the ultimate measure of channel capacity.

The MU-MIMO downlink channel in general belongs to the class of non-degraded Gaussian channels. The sum-rate capacity of a Gaussian broadcast channel, for multiple-users each having multiple antennas, has been shown to satisfy [13]. The Sato bound is not tight in general, but by introducing noise correlation at the different receivers, we can get a much stronger bound [39], [40]. The downlink problem at the BS is to broadcast the user signals with appropriate processing and spatial weighting, such that each user receives a maximum or desired signal-to-interference and noise ratio (SINR), information rate or BER. Antennas at the base station can cooperate during the encoding phase. Cooperation between the users might either entail cooperative management of the rates or SINR at each user.

The capacity region of the general non-degraded broadcast channels is unknown. However, in [11] it was shown that Costa's "dirty-paper" coding is optimal in achieving the sum-rate capacity, by demonstrating that the achievable rate meets the Sato upper bound.

The basic premise of DPCs is that if the transmitter has perfect, non-causal knowledge of additive Gaussian interference in the channel, then the capacity of the channel is the same as if there was no additive interference. DPC allow non-causally interference to be "pre-subtracted" at the transmitter, but in such a way that the transmit power is not increased.

The channel capacity is different for the uplink and the downlink due to the fundamental differences between these channels. However, the fact that the downlink and the uplink channels look like mirror images of each other implies that there is a duality between these channels that allows the capacity region of either channel to be obtained from the capacity region of the other. In a point-to-point communication, the capacity is unchanged when the role of transmitters and receivers is interchanged. In case of downlink linear processing followed by SU receivers at the user terminals (UTs), the choice of transmit and receive matrices is closely related to a virtual uplink problem. Finally, the capacity region of degraded Gaussian channels is the same as the capacity region of the corresponding MAC with the transmit power constraint of the BC translated to the sum of powers in the MAC [41], [15]. The difference between the uplink and the downlink channel is that on the downlink there is an additive noise term associated with each user terminal, while on the uplink there is only one. Another important difference is that on the downlink there is a single power constraint associated with the transmitter, whereas on the uplink there is a different power constraint associated with each user. Finally, on the downlink both the signal and interference associated with each user travel through the same channel, whereas on the uplink these signals travel through the different channels.

3.3 Advantages and Limitations

LOS communication has highly correlated channel. Thus, the channel rank degrades in highly correlated channel. Whereas in MU-MIMO system, correlation among channel coefficients is less, so channel rank loss does not happen. Simultaneously, MU-MIMO system provides the capacity/ spatial multiplexing gain without the need of multiple antennas at the UE.

MU-MIMO has various benefits over SU-MIMO such as better spectral efficiency, high diversity gain, high capacity and interference suppression etc.

There are also some limitations of MU-MIMO systems.

- Each user has different channel characteristics so each user has to deal with different channel conditions.

- MU-MIMO systems consume more power because of multi-user and multiple antennas. Thus, additional units are required which makes the system complex and costly.

Chapter 4
Dirty Paper Coding Technique

To overcome the multipath effect and achieve high throughput transmission, channel equalization or precoding techniques can be used. The original principle of precoding is that if transmit side knows channel information, we can design the transmit signal so that the ISI at the receive side is mitigated. In MIMO or MU-MIMO systems, if channel information is known to the transmitter, precoding can be used to further improve the system performance based on various design criteria.

4.1 Introduction to precoding techniques

Precoding is a technique that exploits the channel information available at the transmitter. Precoding schemes may be divided into three categories according to the accessibility of channel information.

- Tx has full channel information
- Tx has partial channel information
- Tx does not have any channel information

The precoding scheme with full channel information can achieve better performance than other two schemes. Interference among multiple base stations that co-exists in the same location, limits the capacity of wireless networks. The spectral efficiency in existing cellular mobile and WLANs networks is also limited by interference. In cellular mobile networks, the dominant interference comes from adjacent cells, while in co-working WLANs, the interference from other networks, operating in the same area, is a major limiting factor. In the cooperative transmission scheme, multiple base stations (BSs) share information about the transmitted messages to their respective users and wireless channels via a backbone network. Individual BSs are equipped with multiple transmit antennas. Each BS transmitter uses the information of the transmitted signals from other BSs and wireless channel condition to precode its own signal. The precoded signal for each BS is broadcast through all BS transmit antennas in the same frequency band at a given time slot. The precoding operation and transmit-receive antenna coefficients are chosen in such a way as to minimize the interference coming from other BS transmissions. The calculated receive antenna coefficients are then sent from the transmitter to the receiver through the wireless channel prior to the data transmission.

As MIMO is a widely accepted technology for all future wireless standards, due to its high spectral efficiency, we consider multiple antennas at both the transmitter and receivers.

A multi-user MIMO systems with multiple transmit-receive antennas has been considered by several researchers. In precoding techniques, a ZF method can be used to exploit the availability of multiple receive antennas. Here transmit-receive antenna weights are first jointly optimized by a ZF diagonalization technique and then a water-filling power allocation method is applied to allocate power to each user. Nonlinear methods, utilizing a combination of a ZF method with DPC and a combination of a ZF method with THP for a multi-user MIMO system can be used to improve the BER performance by reducing interference. In this thesis, I have implemented the combination of ZF with DPC.

The ZF method is used to eliminate part of the inter-link interference. DPC or THP are then used to cancel the remaining interference. These schemes, however, are not practical for cooperative MIMO systems, since their symbol-error-rate (SER) performance varies from user to user. In particular, this SER variation is not desirable since the MIMO systems can be deployed by different operators and they expect the systems to have similar performance.

In this chapter, a cooperative transmission scheme employing precoding and beamforming is brought in for the downlink of multi-user MIMO systems. In this algorithm, the DPC cancels part of the interference while transmit-receive antennas weights cancel the remaining interference. The DPC precoding technique offers a significant improvement over a various precoding algorithms.

- The first improvement is the enhancement of the SER performance.

- The second improvement is the relaxation of the zero forcing constraints. Assuming transmit signals intended for different users to interfere with each-other. This interference is cancelled at the receiver where the signal is multiplied by the receive antenna weights.

- The third improvement comes from the complexity reduction. DPC scheme has a much lower computational complexity than other methods.

- The fourth improvement comes from the elimination of the dependency of the number of receive antennas to the number of transmit antennas. In this precoding technique, it is not necessary for the number of receive antennas to be at least equal to the number of transmit antennas.

4.2 Dirty Paper Coding

In this Chapter, we have illustrated an idea of dirty paper coding (DPC), showing that an interference-free transmission can be realized by subtracting the potential interferences before transmission.

In theory, DPC would be implemented when channel gains are completely known on the transmitter side. Dirty paper coding (DPC) is a method of precoding the data such that the effect of the interference can be cancelled subject to some interference that is known to the transmitter. More specifically, the interferences due to the first up to (k-1)th user signals are cancelled in the course of precoding the kth user signal. To simplify the exposition, we just consider the case of M_T=3, K=3, and $M_{R,i}$=1, i=1,2,3. If the ith user signal is given by $\tilde{x}_i \in$, then the received signal is given as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ H_3^{DL} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \tag{4.1}$$

where $H_i^{DL} \in {}^{1\times 3}$ is the channel gain between base station and the i-th user. The channel matrix H^{DL} can be LQ-decomposed as

$$H^{DL} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{4.2}$$

where $L = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ and $Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$

where $\{q_i\}_{i=1}^{3} \in {}^{1\times 3}$ are ortho-normal row vectors. Let $x=[x_1\ x_2\ x_3]^T$ denote a precoded signal for $\tilde{x} = [\tilde{x}_1\ \tilde{x}_2\ \tilde{x}_3]^T$. By transmitting $Q^H x$, the effect of Q in equation (9) is eliminated through the channel. Leaving the lower-triangular matrix after transmission, the received signal is given as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ H_3^{DL} \end{bmatrix} Q^H x + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \tag{4.3}$$

From (4.3), the received signal of the first user is given as

$$Y_1 = l_{11}x_1 + w_1 \tag{4.4}$$

From the first-user perspective, therefore, the following condition needs to be met for the interference-free data transmission

$$x_1 = \tilde{x}_1 \tag{4.5}$$

From (4.5), it can be seen that the following precoding cancels the interference component, $l_{21}x_1$ or $l_{21}\tilde{x}_1$, on the transmitter side

$$x_2 = \tilde{x}_2 - \frac{l_{21}}{l_{22}} x_1 = \tilde{x}_2 - \frac{l_{21}}{l_{22}} \tilde{x}_1 \tag{4.6}$$

From (4.6), it can be seen that the precoded signal x_2 is now composed of the user signals, $\tilde{x}_1$ and $\tilde{x}_2$. Finally, the received signal of the third user is given as

$$Y_3 = l_{31}x_1 + l_{32}x_2 + l_{33}x_3 + w_3 \tag{4.7}$$

where the precoded signals, x_1 and x_2, are composed of the known user signals, $\tilde{x}_1$ and $\tilde{x}_2$, given in (4.5) and (4.6). From the perspective of the third user, the precoded signals, x_1 and x_2, are interference components, which can be cancelled by the following precoding on the transmitter side:

$$x_3 = \tilde{x}_3 - \frac{l_{31}}{l_{33}} x_1 - \frac{l_{32}}{l_{33}} x_2 \tag{4.8}$$

The precoded signals in (4.6), (4.7), and (4.8) can be expressed in a matrix as

$$\begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} \tag{4.9}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}, \tag{4.10}$$

and

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{l_{31}}{l_{33}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \tilde{x}_3 \end{bmatrix}, \tag{4.11}$$

Combining the above three precoding matrices, we can express the DPC in the following matrix form:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -\frac{l_{31}}{l_{33}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} \tag{4.12}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32}}{l_{33}} \frac{l_{21}}{l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}$$

Using the above precoding matrix, (4.3) can be re-written as

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32}}{l_{33}} \frac{l_{21}}{l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} \tag{4.13}$$

From (4.13), it is obvious that the interference-free detection can be made for each user. We can see that the precoding matrix in DPC is a scaled inverse matrix of the lower triangular matrix which is obtained from the channel gain matrix, that is,

$$\begin{bmatrix} 1 & 0 & 0 \\ -\frac{l_{21}}{l_{22}} & 1 & 0 \\ -\frac{l_{31}}{l_{33}} + \frac{l_{32}}{l_{33}}\frac{l_{21}}{l_{22}} & -\frac{l_{32}}{l_{33}} & 1 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}^{-1} \begin{bmatrix} l_{11} & 0 & 0 \\ 0 & l_{22} & 0 \\ 0 & 0 & l_{33} \end{bmatrix} \tag{4.14}$$

This technique, “writing on dirty paper,” was with interference cancellation in mind; it was shown that the capacity of a channel where the transmitter knows the interfering signal is the same as if there were no interference. The dirty paper analogy comes from comparing the interference in a communications channel to dirt that is present on a piece of paper. The signal is the ink, which is chosen based on the interference (dirt) that is present.

We now turn to a nonlinear technique based on the concept of “writing on dirty paper” introduced by Costa. In the thesis, the traditional additive Gaussian noise channel is modified to include an additive interference term that is known at the transmitter:

$$\text{Received signal} = \text{transmitted signal} + \text{interference} + \text{noise}.$$

The simplest thing to do in such a scenario would be to set the transmitted signal equal to the desired data minus the interference, but such an approach requires increased power. Costa proved the surprising result that the capacity of this channel is the same as if the interference was not present; no more power is needed to cancel the interference than is used in a nominal additive Gaussian noise channel. To use Costa’s analogy, writing on dirty paper is information theoretically equivalent to writing on clean paper when one knows in advance where the dirt is. Costa’s approach is theoretical, however, and does not provide a practical technique for approaching capacity.

The application of this principle to downlink transmission in multi-user MIMO channels was proposed. Because the transmitter has CSI, it knows what interference user 1’s signal will produce at user 2, and hence can design a signal for user 2 that avoids the known interference.

The most well-known dirty paper technique for the MIMO downlink uses a QR decomposition of the channel, which we write here as the product of a lower triangular matrix $\mathbf{L}$ with a unitary matrix $\mathbf{H} = \mathbf{LQ}$. The signal to be transmitted is precoded with the Hermitian transpose of $\mathbf{Q}$, resulting in the effective channel $\mathbf{L}$. The first user of this system sees no interference from other users; its signal may be chosen without regard for the other users. The second user sees interference only from the first user; this interference is known and thus may be overcome using dirty paper coding. Subsequent users are dealt with in a similar manner.

An important difference between the multi-user MIMO channel and the interference channels for which dirty paper techniques are designed is that the interference depends on the signal being designed. This problem is solved using QR-type decomposition, so the interference for any particular user depends only on the interference generated by previous users. Dirty paper coding is then applied to cancel this interference. Techniques based on dirty paper coding perform much better and approach the theoretical limits of the channel, but require complicated coding schemes.

Chapter 5
Decoding Techniques

Decoders are used at the receiver side to detect or recover back the signal. Decoders can be linear and non-linear. Linear signal detection method treats all transmitted signals as interference except for the desired stream from the target transmit antenna. Therefore, interference signals from other transmit antennas are minimized or nullified to detect the desired signal from the target transmit antenna. To facilitate the detection of desired signals from each antenna, the effect of the channel is inverted by a weight matrix. The detection of each symbol is given by a linear combination of the received signals. The standard linear detection methods include ZF technique and MMSE technique.

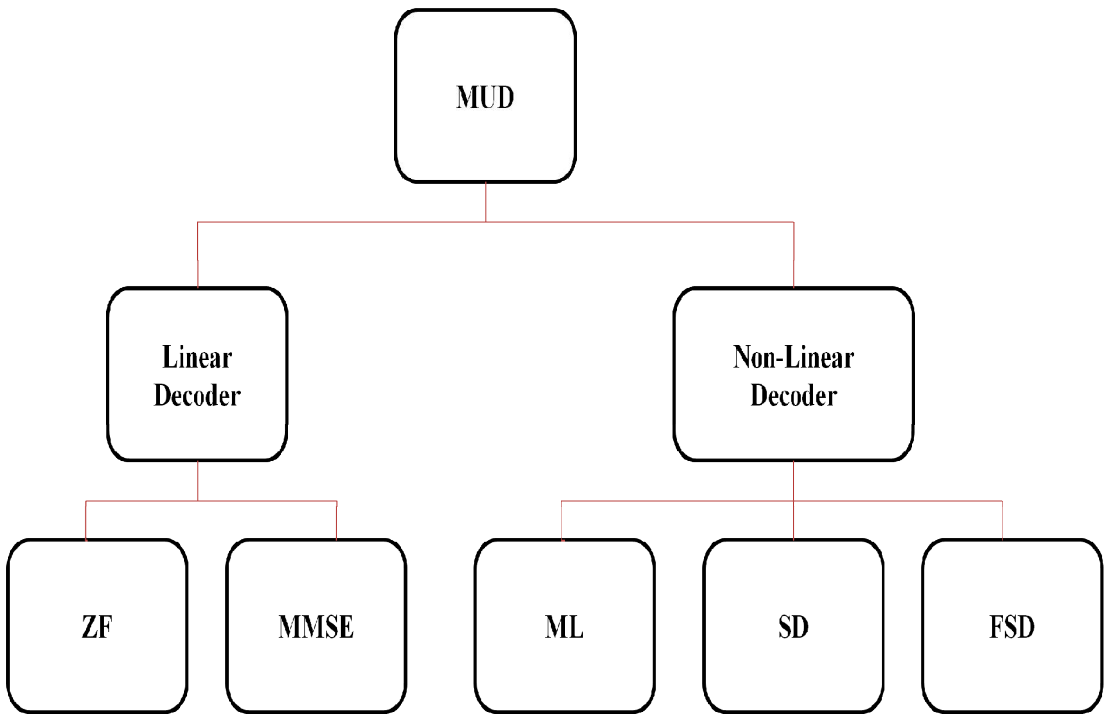

Figure 5.1: Classification of various decoding techniques

Whereas, non-linear equalizers are adaptive in nature. Non-linear equalizers consider channel effect optimization as well as inter symbol interference (ISI). Non-linear decoding techniques are maximum likelihood (ML), sphere decoder (SD) and fixed complexity sphere decoder (FSD).

Any receiver technique such as zero-forcing (ZF), minimum mean square error estimation and optimal maximum-likelihood detection (MLD) can be applied directly. MLD gives the best performance with a high computational complexity. ZF and MMSE are less expensive techniques. In case of ZF, complexity reduction comes, however, at the expense of noise enhancement which in general results in a significant performance degradation (compared to the ML decoder). The diversity order achieved by each of the individual data streams equals $M_T - M_R + 1$. The MMSE receiver balances inter-stream interference mitigation with noise enhancement and minimizes the total error. At low SNRs the MMSE receiver approximates a matched filter and is near optimal. It outperforms the ZF receiver that continues to enhance noise. At high SNRs, the MMSE receiver approaches ZF and therefore realizes the same diversity order as ZF for each data stream [9].

By analyzing the various decoding techniques, it is said that, ML gives better performance than ZF and MMSE but it has large computational complexity. To eradicate this drawback, sphere decoder came into picture. SD gives slightly better performance than ML with very less computations.

When channel state information is available at transmitter and receiver sides, channel dependent decoding of data streams improves the system performance. The linear decoder operates in the complex domain and removes any redundancy that has been introduced by the precoder. The framework is general and addresses several optimization criteria like minimum MSE (MMSE), minimum bit error rate (BER), and maximum information rate.

In this Chapter, a new approach will be introduced which will be used to derive a general framework for MU-MIMO decoding design so we can target any optimization criteria with one universal algorithm like in the case of single-user MIMO processing.

5.1 Linear MU-MIMO decoding Techniques

Linear MU-MIMO decoding techniques are ZF and MMSE. These techniques consider only channel effect.

5.1.1 Zero forcing decoding

Since the base station has no influence on the noise at the user terminals, the most intuitive approach for decoding is a zero forcing equalizer (ZF) which eliminates all interference at the user terminals. ZF decoding for single antenna receivers was investigated extensively in the literature [56], [9].

The zero forcing technique nullifies the interference by the following matrix, when $M_T=M_{R;}$

$$B_{ZF}= H^{-1} \tag{5.1}$$

When $M_T \neq M_R$, then, the solution to the optimization problem is a pseudo-inverse of the combined channel matrix H:

$$B_{ZF}= (H^H H)^{-1} H^H \tag{5.2}$$

Where $(.)^H$ denotes the Hermitian transpose operation. In other words, it inverts the effect of channel as

$$\begin{aligned} \hat{x}_{ZF} &= B_{ZF} Y \\ &= x + (H^H H)^{-1} H^H w \\ &= x + \hat{w}_{ZF} \end{aligned} \tag{5.3}$$

Where $\hat{w}_{ZF} = B_{ZF} * w = (H^H H)^{-1} H^H w$. Note that the error performance is directly connected to the power of $\hat{w}_{ZF}$. We assume that the complex data symbols are i.i.d. uniformly distributed random variables and that the samples of the additive noise at the input of receive antennas are i.i.d. complex Gaussian white random variables with mean and variance, respectively.

The ZF decoding suffers from the noise enhancement problem and requires increased transmit power. It is sub-optimal and results in significant performance degradation. The diversity order and array gain of each stream is proportional to $M_T - M_R + 1$ [9].

5.1.2 Minimum mean-square-error decoding

The ZF decoder completely eliminates multi-user interference at the expense of noise enhancement. The minimum mean-square-error (MMSE) decoder balances the multiuser interference mitigation with noise enhancement and minimizes the total error. Unlike the ZF decoder, the MMSE decoder cannot be designed in such a straightforward way.

MMSE decoding is optimum when all users in the system are equipped with only one antenna. MMSE balances the MUI in order to reduce the performance loss and improves the diversity. However, MMSE suffers a performance loss when it attempts to mitigate the interference between two closely spaced antennas as in the case when the user terminal is equipped with more than one receive antenna.

In order to maximize the post-detection SINR, The MMSE decoder is defined as

$$B_{MMSE} = (H^H H + \sigma_w^2 I)^{-1} H^H \tag{5.4}$$

Note that the MMSE receiver requires the statistical information of noise σ_w^2. Using the MMSE weight matrix-

$$\begin{aligned} \tilde{x}_{MMSE} &= B_{MMSE} Y \\ &= x + (H^H H + \sigma_w^2 I)^{-1} H^H w \\ &= x + \tilde{w}_{MMSE} \end{aligned}$$

where $\tilde{w}_{MMSE} = B_{MMSE}\, w = (H^H H + \sigma_w^2 I)^{-1} H^H w$.

The MMSE decoder approximates a matched filter at low SNRs and is near optimal. At high SNRs, the MMSE decoder converges to a ZF decoder and we can expect it to extract $M_T - M_R + 1$ order diversity.

5.2 Non-Linear MU-MIMO decoding Techniques

Non-Linear MU-MIMO decoding techniques are ML,SD and FSD. These techniques consider IST as well as channel effect.

5.2.1 Maximum Likelihood

Maximum likelihood (ML) detection calculates the Euclidean distance the received signal vector and the product of all possible transmitted signal vectors with the given channel H, and finds the one with the minimum distance. Let C denotes a set of signal constellation symbol points. Then, ML detection determines the estimate of the transmitted signal vector x as

$$\hat{x}_{ML} = \underset{x \in C^{M_T}}{\arg\min} \| Y - Hx \|^2 \tag{5.5}$$

Where $\| Y - Hx \|^2$ corresponds to the ML metric. The ML method achieves the optimal performance as the maximum a posterior detection when all the transmitted vectors are equally likely. However, its complexity increases exponentially as modulation order and/or the number of transmit antennas increases. The required number of ML metric calculation is $|C|^{M_T}$, that is, the complexity of metric calculation exponentially increases with the number of antennas. Even if this particular method suffers from computational complexity, its performance serves as a reference to other detection methods since it corresponds to the best possible performance.

5.2.2 Sphere Decoding

Sphere decoding (SD) method intends to find the transmitted signal vector with minimum ML metric, that is, to find the ML solution vector. However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors.

SD adjusts the sphere radius until there exists a single vector (ML solution vector) within a sphere. It increases the radius when there exists no vector within a sphere, and decreases the radius when there exist multiple vectors within the sphere. The SD method can be exploited as

$$\arg\min_{x} \| Y - Hx \|^2 = \arg\min_{x} (x - \hat{x})^T H^T H (x - \hat{x}) \tag{5.6}$$

where $\hat{x} = (H^H H)^{-1} H^H Y$, which is the unconstrained solution of the real system. Consider the following radius R_{SD} of sphere-

$$(x - \hat{x})^T H^T H (x - \hat{x}) \le R_{SD}^2 \tag{5.7}$$

The SD method considers only the vector inside a sphere. The complexity of SD in terms of the sum of multiplication, division, and square-root operations, varies, as SNR varies. As the SNR increases, the ZF solution $\hat{x}$ becomes more likely to coincide with the ML solution vector. The main drawback of SD is that its complexity depends on SNR. Furthermore, the worst-case complexity is the same as that of ML detection, although the average complexity is significantly reduced.

MIMO decoding should be done in such a way that the complexity of decoder gets reduced while the performance of the system should be improved.

Sphere decoding (SD) method intends to unearth the transmitted signal vector with bare minimum ML metric, that is, to find the ML solution vector. However, it considers only a small set of vectors within a given sphere rather than all possible transmitted signal vectors. SD adjusts the sphere radius until there exists a single vector (ML solution vector) within a sphere. It increases the radius when there exists no vector within a sphere, and decreases the radius when there exist multiple vectors within the sphere.

In the sequel, we sketch the idea of SD through an example. Consider a square QAM in a 2 2 complex MIMO channel. The underlying complex system can be converted into an equivalent real system. Let Y_{jR} and Y_{jI} denote the real and imaginary parts of the received signal at the j^{th} receive antenna, that is, $Y_{jR} = \mathrm{Re}\,\{Y_j\}$ and $Y_{jI} = \mathrm{Im}\{Y_j\}$.

Similarly, the input signal x_i from the ith antenna can be represented by $x_{iR} = R\{x_i\}$ and $x_{iI} = \mathrm{Im}\{x_i\}$. For the 2 2 MIMO channel, the received signal can be expressed in terms of its real and imaginary parts as follows:

$$\begin{bmatrix} Y_{1R}+jY_{1I} \\ Y_{2R}+jY_{2I} \end{bmatrix} = \begin{bmatrix} h_{11R}+jh_{11I} & h_{12R}+jh_{12I} \\ h_{21R}+jh_{21I} & h_{22R}+jh_{22I} \end{bmatrix} \begin{bmatrix} x_{1R}+jx_{1I} \\ x_{2R}+jx_{2I} \end{bmatrix} + \begin{bmatrix} w_{1R}+jw_{1I} \\ w_{2R}+jw_{2I} \end{bmatrix} \tag{5.8}$$

where hij = Re{hij}, hij = Im{hij}, wi = Re{wi}, and wi = Im{wi}. The real and imaginary parts of (5.8) can be respectively expressed as

$$\begin{bmatrix} Y_{1R} \\ Y_{2R} \end{bmatrix} = \begin{bmatrix} h_{11R} & h_{12R} \\ h_{21R} & h_{22R} \end{bmatrix} \begin{bmatrix} x_{1R} \\ x_{2R} \end{bmatrix} - \begin{bmatrix} h_{11I} & h_{12I} \\ h_{21I} & h_{22I} \end{bmatrix} \begin{bmatrix} x_{1I} \\ x_{2I} \end{bmatrix} + \begin{bmatrix} w_{1R} \\ w_{2R} \end{bmatrix}$$

$$= \begin{bmatrix} h_{11R} & h_{12R} & -h_{11I} & -h_{12I} \\ h_{21R} & h_{22R} & -h_{21I} & -h_{22I} \end{bmatrix} \begin{bmatrix} x_{1R} \\ x_{2R} \\ x_{1I} \\ x_{2I} \end{bmatrix} + \begin{bmatrix} w_{1R} \\ w_{2R} \end{bmatrix} \tag{5.9}$$

$$\begin{bmatrix} Y_{1I} \\ Y_{2I} \end{bmatrix} = \begin{bmatrix} h_{11I} & h_{12I} & h_{11R} & h_{12R} \\ h_{21I} & h_{22I} & h_{21R} & h_{22R} \end{bmatrix} \begin{bmatrix} x_{1R} \\ x_{2R} \\ x_{1I} \\ x_{2I} \end{bmatrix} + \begin{bmatrix} w_{1I} \\ w_{2I} \end{bmatrix} \tag{5.10}$$

The following expression can be yield by combining (5.9) and (5.10)

$$\begin{bmatrix} Y_{1R} \\ Y_{2R} \\ Y_{1I} \\ Y_{2I} \end{bmatrix} = \begin{bmatrix} h_{11R} & h_{12R} & -h_{11I} & -h_{12I} \\ h_{21R} & h_{22R} & -h_{21I} & -h_{22I} \\ h_{11I} & h_{12I} & h_{11R} & h_{12R} \\ h_{21I} & h_{22I} & h_{21R} & h_{22R} \end{bmatrix} \begin{bmatrix} x_{1R} \\ x_{2R} \\ x_{1I} \\ x_{2I} \end{bmatrix} + \begin{bmatrix} w_{1R} \\ w_{2R} \\ w_{1I} \\ w_{2I} \end{bmatrix} \tag{5.11}$$

here $\bar{Y} = \begin{bmatrix} Y_{1R} \\ Y_{2R} \\ Y_{1I} \\ Y_{2I} \end{bmatrix}$, $\bar{H} = \begin{bmatrix} h_{11R} & h_{12R} & -h_{11I} & -h_{12I} \\ h_{21R} & h_{22R} & -h_{21I} & -h_{22I} \\ h_{11I} & h_{12I} & h_{11R} & h_{12R} \\ h_{21I} & h_{22I} & h_{21R} & h_{22R} \end{bmatrix}$, $\bar{x} = \begin{bmatrix} x_{1R} \\ x_{2R} \\ x_{1I} \\ x_{2I} \end{bmatrix}$ and $\bar{w} = \begin{bmatrix} w_{1R} \\ w_{2R} \\ w_{1I} \\ w_{2I} \end{bmatrix}$.

The SD method can be exploited as

$$\underset{\bar{x}}{\arg\min} \| \bar{Y} - \bar{H}\bar{x} \|^2 = \underset{\bar{x}}{\arg\min} (\bar{x} - \hat{\bar{x}})^T \bar{H}^T \bar{H} (\bar{x} - \hat{\bar{x}}) \tag{5.12}$$

where $\hat{\bar{x}} = (\bar{H}^H \bar{H})^{-1} \bar{H}^H \bar{Y}$, which is the unconstrained solution of the real system. It shows that the ML solution can be determined by the different metric $(\bar{x} - \hat{\bar{x}})^T \bar{H}^T \bar{H} (\bar{x} - \hat{\bar{x}})$. Consider the following radius R_{SD} of sphere-

$$(\bar{x} - \hat{\bar{x}})^T \bar{H}^T \bar{H} (\bar{x} - \hat{\bar{x}}) \leq R_{SD}^2 \tag{5.13}$$

The SD method considers only the vector inside a sphere. Fig. 5.2 (a) illustrates a sphere with the centre of $\hat{\bar{x}} = (\bar{H}^H \bar{H})^{-1} \bar{H}^H \bar{y}$ and radius of R_{SD}. In this example, this sphere includes four candidate vectors, one of which is the ML solution vector. No vector outside the sphere can be the ML solution vector because their ML metric values are bigger than the ones inside the sphere. The radius of the sphere can be reduced so that there remains a single vector inside the sphere.

In other words, the ML solution vector is now contained in this sphere with a reduced radius as illustrated in Fig. 5.2 (a), (b). Note that the new metric in (5.12) is also expressed as

$$(\bar{x} - \hat{\bar{x}})^T \bar{H}^T \bar{H} (\bar{x} - \hat{\bar{x}}) = (\bar{x} - \hat{\bar{x}})^T R^T R (\bar{x} - \hat{\bar{x}}) = \| R(\bar{x} - \hat{\bar{x}}) \|^2 \tag{5.14}$$

where R is obtained from QR decomposition of the real channel matrix $\bar{H}$= QR. When $N_T = N_R = 2$, the metric in (5.14) is given as

$$\| R(\bar{x} - \hat{\bar{x}}) \|^2 = \left\| \begin{bmatrix} r_{11} & r_{12} & r_{13} & r_{14} \\ 0 & r_{22} & r_{23} & r_{24} \\ 0 & 0 & r_{33} & r_{34} \\ 0 & 0 & 0 & r_{44} \end{bmatrix} \begin{bmatrix} \bar{x}_1 - \hat{\bar{x}}_1 \\ \bar{x}_2 - \hat{\bar{x}}_2 \\ \bar{x}_3 - \hat{\bar{x}}_3 \\ \bar{x}_4 - \hat{\bar{x}}_4 \end{bmatrix} \right\|^2 \tag{5.15}$$

$$\begin{aligned} \| R(\bar{x} - \hat{\bar{x}}) \|^2 = \ & | r_{44}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 + | r_{33}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{34}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 \\ & + | r_{22}(\bar{x}_2 - \hat{\bar{x}}_2) + r_{23}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{24}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 \\ & + | r_{11}(\bar{x}_1 - \hat{\bar{x}}_1) + r_{12}(\bar{x}_2 - \hat{\bar{x}}_2) + r_{13}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{14}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 \end{aligned}$$

From (5.14) and (5.15), the sphere in (5.13) can be expressed as

$$\begin{aligned} & | r_{44}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 + | r_{33}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{34}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 + | r_{22}(\bar{x}_2 - \hat{\bar{x}}_2) + r_{23}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{24}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 \\ & + | r_{11}(\bar{x}_1 - \hat{\bar{x}}_1) + r_{12}(\bar{x}_2 - \hat{\bar{x}}_2) + r_{13}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{14}(\bar{x}_4 - \hat{\bar{x}}_4) |^2 \leq R_{SD}^2 \end{aligned} \tag{5.16}$$

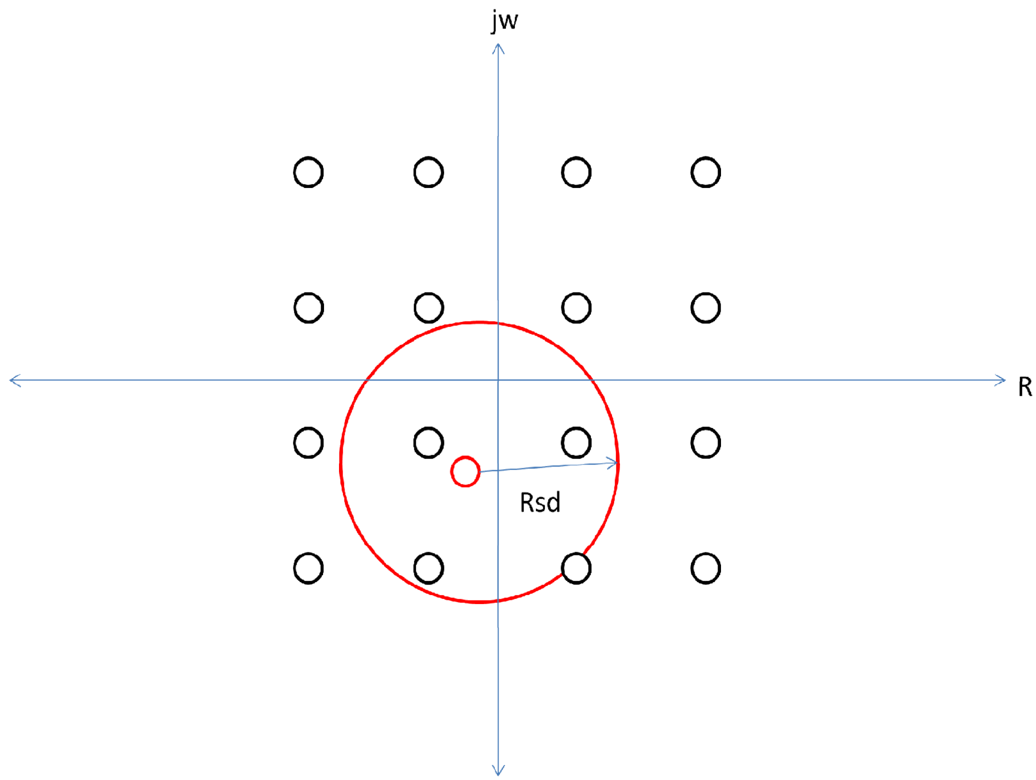

Figure 5.2 (a): Original Sphere in Working of Sphere Decoding

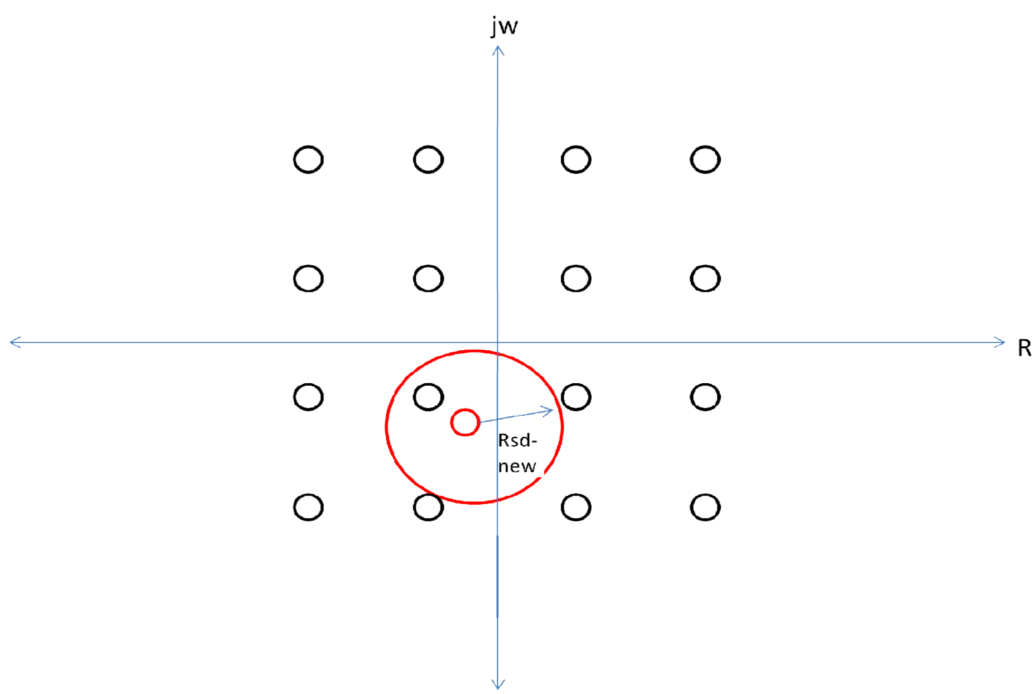

Figure 5.2 (b): New Sphere with Reduced Radius in Sphere Decoding

Using the sphere in (5.16), the details of SD method are now described with the following four steps:

Step 1: Referring to (5.16), we first consider a candidate value for $\bar{x}_4$ in its own single dimension, that is, which is arbitrarily chosen from the points in the sphere $|r_{44}(\bar{x}_4 - \hat{\bar{x}}_4)|^2 \le R_{SD}^2$. In other words, this point must be chosen in the following range:

$$\hat{\bar{x}}_4 - \frac{R_{SD}}{r_{44}} \le \bar{x}_4 \le \hat{\bar{x}}_4 + \frac{R_{SD}}{r_{44}} \tag{5.17}$$

Let $\hat{\hat{\bar{x}}}_4$ denote the point chosen in step 1. If there exists no candidate point satisfying the inequalities, the radius needs to be increased. We assume that a candidate value was successfully chosen. Then we proceed to step 2.

Step 2: Referring to (5.16) again, a candidate value for $\bar{x}_3$ is chosen from the points in the following sphere:

$$|r_{44}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2 + |r_{33}(\bar{x}_3 - \hat{\bar{x}}_3) + r_{34}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2 \le R_{SD}^2 \tag{5.18}$$

which is equivalent to

$$\hat{\bar{x}}_3 - \frac{\sqrt{R_{SD}^2 - |r_{44}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2} - r_{34}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)}{r_{33}} \le \bar{x}_3 \le \hat{\bar{x}}_3 + \frac{\sqrt{R_{SD}^2 - |r_{44}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2} - r_{34}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)}{r_{33}} \tag{5.19}$$

Note that $\hat{\hat{\bar{x}}}_4$ in (5.19) is the one already chosen in step 1. If a candidate value for $\bar{x}_3$ does not exist, we go back to step 1 and choose other candidate value $\hat{\hat{\bar{x}}}_4$. Then search for $\bar{x}_3$ that meets the inequalities in (5.19) for the given $\hat{\hat{\bar{x}}}_4$. In case that no candidate value $\bar{x}_3$ exists with all possible values of $\hat{\hat{\bar{x}}}_4$, we increase the radius of sphere, RSD, and repeat the step 1. Let $\hat{\hat{\bar{x}}}_4$ and $\hat{\hat{\bar{x}}}_3$ denote the final points chosen from step 1 and step 2, respectively.

Step 3: Given $\hat{\hat{\bar{x}}}_4$ and $\hat{\hat{\bar{x}}}_3$, a candidate value for $\bar{x}_2$ is chosen from the points in the following sphere:

$$|r_{44}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2 + |r_{33}(\hat{\hat{\bar{x}}}_3 - \hat{\bar{x}}_3) + r_{34}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2 + |r_{22}(\bar{x}_2 - \hat{\bar{x}}_2) + r_{23}(\hat{\hat{\bar{x}}}_3 - \hat{\bar{x}}_3) + r_{24}(\hat{\hat{\bar{x}}}_4 - \hat{\bar{x}}_4)|^2 \le R_{SD}^2 \tag{5.20}$$

Arbitrary value is chosen for $\bar{x}_2$ inside the sphere of (5.20). In choosing a point, the inequality in (5.20) is used as in the previous steps. If no candidate value for $\bar{x}_2$ exists, we go back to step 2 and choose another candidate value $\tilde{x}_3$. In case that no candidate value for $\bar{x}_2$ exists after trying all possible candidate values for $\tilde{x}_3$, we go back to step 1 and choose another candidate value for $\tilde{x}_4$. The final points chosen from step 1 through step 3 are denoted as $\tilde{x}_4$, $\tilde{x}_3$, and $\tilde{x}_2$, respectively.

Step 4: Now, a candidate value for $\bar{x}_1$ is chosen from the points in the following sphere:

$$|r_{44}(\tilde{x}_4-\hat{x}_4)|^2+|r_{33}(\tilde{x}_3-\hat{x}_3)+r_{34}(\tilde{x}_4-\hat{x}_4)|^2+|r_{22}(\tilde{x}_2-\hat{x}_2)+r_{23}(\tilde{x}_3-\hat{x}_3)+r_{24}(\tilde{x}_4-\hat{x}_4)|^2+|r_{11}(\bar{x}_1-\hat{x}_1)$$
$$+r_{12}(\tilde{x}_2-\hat{x}_2)+r_{13}(\tilde{x}_3-\hat{x}_3)+r_{14}(\tilde{x}_4-\hat{x}_4)|^2\leq R_{SD}^2 \tag{5.21}$$

An arbitrary value satisfying (5.21) is chosen for $\bar{x}_1$. If no candidate value for $\bar{x}_1$ exists, we go back to step 3 to choose other candidate value for $\tilde{x}_2$. In case that no candidate value for $\bar{x}_1$ exists after trying with all possible candidate values for $\tilde{x}_2$, we go back to step 2 to choose another value for $\bar{x}_3$. Let $\tilde{x}_1$ denote the candidate value for $\bar{x}_1$. Once we find all candidate values, $\tilde{x}_4$, $\tilde{x}_3$, $\tilde{x}_2$, and $\tilde{x}_1$, then the corresponding radius is calculated by using (5.21). Using the new reduced radius, step 1 is repeated. If $[\tilde{x}_1\ \tilde{x}_2\ \tilde{x}_3\ \tilde{x}_4]$ turns out to be a single point inside a sphere with that radius, it is declared as the ML solution vector and our searching procedure stops.

ML detection over MIMO channels can achieve the lowest BER for a given scenario, but at the expense of prohibitive complexity [1]. Thus, there is a continuous search for computationally efficient detectors, such as the sphere decoding (SD) algorithms, which are a set of tree search detectors with reduced complexity compared to the ML exhaustive search detector due to setting a radius constraint [2].

These algorithms perform a closest lattice point search for each component of the received vector, which is feasible due to the fact that the constellation set to which the transmitted symbols belong is known in advance. The existing SD algorithms can be implemented to operate within a finite set of real numbers, thus called real sphere decoders (RSD) [3], or to perform the search directly within a finite set of complex numbers, commonly known as complex sphere decoders (CSD) [4]. Since these detectors provide the ML solution to the detection problem, their evaluation focuses only on their complexity.

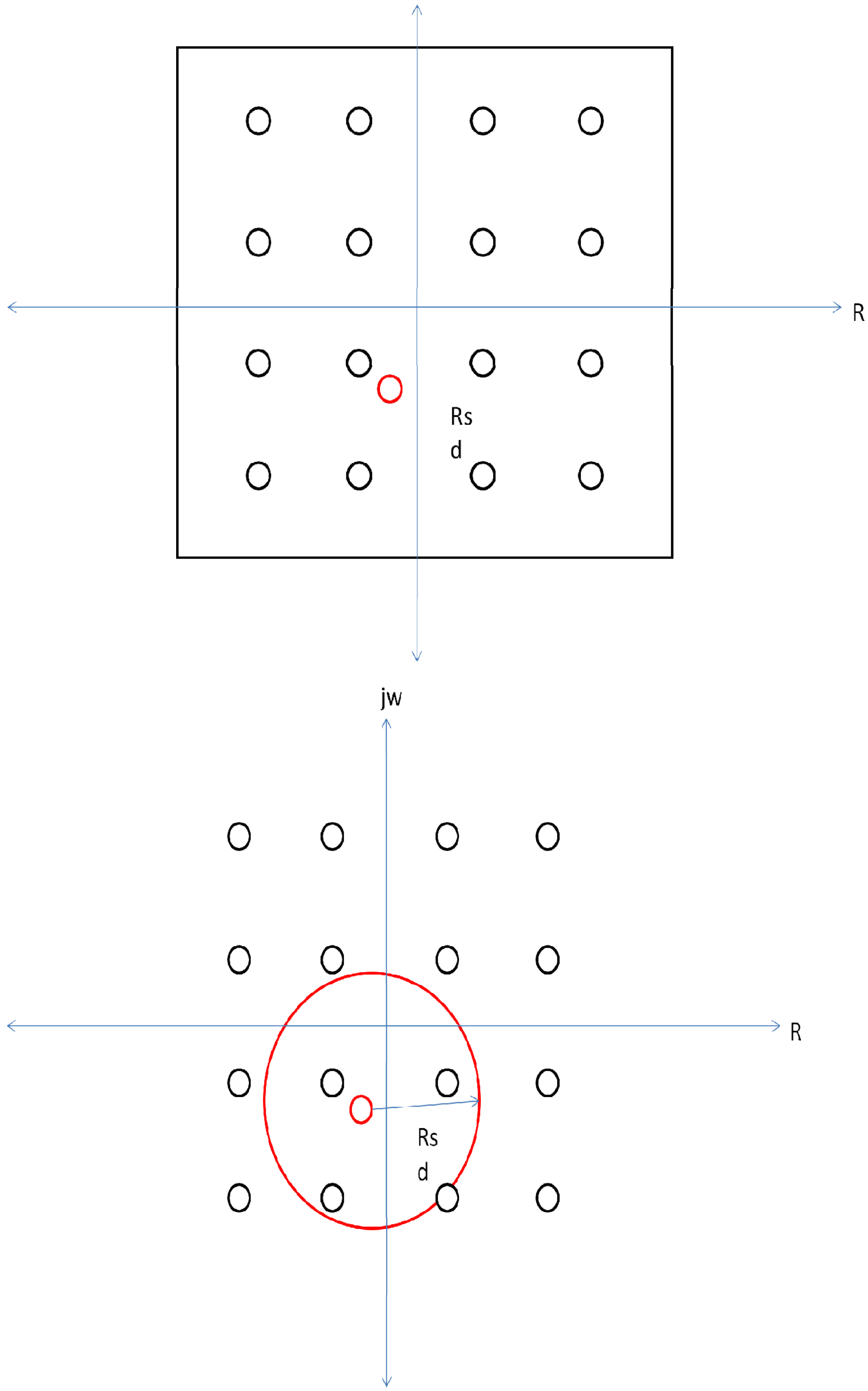

Figure 5.3: Comparison of ML Decoder and Sphere Decoder

The complexity of SD in terms of the sum of multiplication, division, and square-root operations, varies, as SNR varies. As the SNR increases, the ZF solution $\hat{\hat{x}}$ becomes more likely to coincide with the ML solution vector. The main drawback of SD is that its complexity depends on SNR. Furthermore, the worst-case complexity is the same as that of ML detection, although the average complexity is significantly reduced.

5.3 Channel estimation errors

An important issue in a multi-user MIMO transmission is the impact of imperfect channel state information on the general system performance.

In low SNR regions, MSE is dominated by the error due to the noise. Hence, the MSE linearly decreases with the SNR. The influence of the channel estimation errors is much higher at the user terminals than at the base station due to the processing and power limits. In order to reduce the influence of the channel estimation errors on the performance of the decoding techniques we can invest in more processing power to implement more efficient channel estimation techniques.

One more straightforward way of reducing the channel estimation errors is to increase the transmit power or antenna gain. Values of the transmit power and the antenna gain are set by the government regulations so there we do not have too many options. Therefore, a logical conclusion is that if we cannot improve the estimator performance by increasing the processing power, then the other option is to either increase the number of antennas or to reduce the number of spatial data streams.

In the next chapter, system level investigations and simulation results show that the last approach (SD) gives very good results.

Chapter 6
MGF and PEP based BER Performance Analysis of Multi-User MIMO System with Linear and Non-Linear Equalizers

MIMO is a 4G Wireless Communication Technique and is supposedly has the most promising reign. MU-MIMO has an edge over 4G wireless systems due to their better spectral efficiency, diversity gain and interference suppression. This paper incorporates the moment generating function (MGF) based closed form upper bounds of pair wise error probability (PEP), which are derived for MU-MIMO systems. These upper bounds are then extended for different equalizers like, ZF , MMSE , ML and SD. BER expressions are also derived for MU-MIMO system under independent and spatially correlated quasi-static Rayleigh fading channels. Simulation results for upper bounds are drawn and akin performance with the analytical results have been subsume in this paper.

In this Chapter, unique approach of coalescing SINR, MGF and PEP [94] is being employed to derive bounds on BER for above mentioned MU-MIMO system. At the outset, MGF of SNR is calculated for independent and identically distributed (i.i.d) and spatially correlated channels [95]. Later on, it is used to estimate PEP for the same system.

PEP [96]-[97] will act as basic building block for the derivation of union bounds to the error probability. The Chernoff bound [98] is then applied to get closed form PEP expression. These expressions are then extended for ZF, MMSE, ML and SD equalizers in multi-user scenarios [97]-[98]. It is scrutinized that the computational complexity of SD gets reduced than the ML. Also, we achieved diversity order of $M_R \times M_T$ with ML-decoder and $M_R + M_T - K+1$ with ZF and MMSE equalizers, where K is the number of users.

The reported work incorporated the effect of imperfect CSI which caused due to both CSI delay and channel estimation error [8]. This upshot is analyzed by calculating post processing SINR and its relation with BER [99]-[102].

In this chapter, our intention is to analyze the BER performance for multi-user case. A distinctive approach based on SINR, MGF and PEP is used to derive and evaluate generalized closed-form BER expressions for MU-MIMO system. BER performance is analyzed for the MU-MIMO system under different equalizers and different no. of users. The analysis of BER performance is also done for different modulation schemes with independent and spatially correlated channels.

MU-MIMO channel is basically a realization of standard i.i.d Rayleigh fading channel. The received signal vector Y_i of the i^{th} user at the t^{th} symbol interval can also be written as

$$Y_i[t] = H_i\,[t]\; C[t]\; s[t] + w_i[t] \tag{6.1}$$

$$Y_i[t] = H_i\,[t]\; c_i[t]\; s_i[t] + \sum_{k=1,k\neq i}^{K} H_i[t] c_k[t] s_k[t] + w_i[t] \tag{6.2}$$

where s_k satisfies $E[s_k\, s_k^*] = E_s$ and E_s is the symbol energy. Assuming that the delay is equal to t_d symbol intervals, then the beam forming matrix C[t] can be generated with the help of CSI ($\hat{H}[t-t_d]$). Here, delay t_d is different for the different downlink symbol time slot. For MU-MIMO downlink system with ZF equalizer, C[t] is normalized and inferred as

$$C[t] = \frac{(\hat{H}[t-t_d])^{\dagger}}{\|(\hat{H}[t-t_d])^{\dagger}\|_F} \tag{6.3}$$

where $\hat{H}[t-t_d]$ is the channel estimation result at the $(t-t_d)^{th}$ symbol interval. For the given $\hat{H}\,[t-t_d]$, post-processing SINR is calculated. Substitute (6.3) into (6.1), the received signal vector can be expressed as

$$Y_i[t] = H_i\,[t]\; \frac{(\hat{H}[t-t_d])^{\dagger}}{\|(\hat{H}[t-t_d])^{\dagger}\|_F}\, s[t] + w_i[t] \tag{6.4}$$

$$\begin{bmatrix} h_1[t-t_d] \\ h_2[t-t_d] \\ . \\ . \\ h_K[t-t_d] \end{bmatrix} = \begin{bmatrix} \rho_{e,1}\hat{h}_1[t-t_d] \\ \rho_{e,2}\hat{h}_2[t-t_d] \\ . \\ . \\ \rho_{e,K}\hat{h}_K[t-t_d] \end{bmatrix} + \begin{bmatrix} \sqrt{1-|\rho_{e,1}|^2}\,\zeta_1[t-t_d] \\ \sqrt{1-|\rho_{e,2}|^2}\,\zeta_2[t-t_d] \\ . \\ . \\ \sqrt{1-|\rho_{e,K}|^2}\,\zeta_K[t-t_d] \end{bmatrix} \tag{6.5}$$

The elements of random matrix $\hat{H}\,[t-t_d]$ are i.i.d zero mean complex Gaussian random variables with unit variance having dimension $K \times M_T$, the elements of the random vector $\zeta_K[t-t_d]$ are also i.i.d zero mean complex Gaussian random variables with unit variance but with dimension $1\times M_T$, $\rho_{e,k}$ is the correlation coefficient between the actual channel gain and its estimation for user k. It can be expressed as

$$\rho_{e,k} \quad E\{h_{k,1j}[t-t_d]\hat{h}^*_{k,1j}[t-t_d]\} \tag{6.6}$$

where k=1, 2,….K; j=1,2,….. M_T ; $0 \le \rho_{e,k} \le 1$. Both H[t] and H $[t-t_d]$ follow jointly complex Gaussian distribution which can be presented as

$$\begin{bmatrix} h_1[t] \\ h_2[t] \\ . \\ . \\ h_K[t] \end{bmatrix} = \begin{bmatrix} \rho_{d,1}h_1[t-t_d] \\ \rho_{d,2}h_2[t-t_d] \\ . \\ . \\ \rho_{d,K}h_K[t-t_d] \end{bmatrix} + \begin{bmatrix} \sqrt{1-\left|\rho_{d,1}\right|^2}\varepsilon_1[t] \\ \sqrt{1-\left|\rho_{d,2}\right|^2}\varepsilon_2[t] \\ . \\ . \\ \sqrt{1-\left|\rho_{d,K}\right|^2}\varepsilon_K[t] \end{bmatrix} \tag{6.7}$$

where the elements of the random vector $\varepsilon_K[t]$ are i.i.d zero mean complex Gaussian random variables with unit variance having dimension 1 × M_T, $\rho_{d,k}$ is the correlation coefficient between the current channel gain and the delayed one for user k and is defined as

$$\rho_{d,k} \quad E\{h_{k,1j}[t]\hat{h}^*_{k,1j}[t-t_d]\} \tag{6.8}$$

where k=1, 2,….K; j=1,2,….. M_{Tx}; $0 \le \rho_{d,k} \le 1$. Because each user can have a different mobile velocity, $\rho_{d,k}$ of each user can be different. Defining $\rho_k = \rho_{d,k}\ \rho_{e,k}$ and substituting (6.5) and (6.7) into (6.6), then, the received signal vector can be expressed as

$$Y[t] = s_{eq}[t] + w_{eq}[t] \tag{6.9}$$

where $s_{eq}[t]=\left[s_{eq,1}[t],s_{eq,2}[t],..,s_{eq,K}[t]\right]^{M_{T_x}}$. $s_{eq}[t]$ is directed to effective post processing signal which is given by

$$s_{eq}[t]=\frac{1}{\|(\hat{H}[t-t_d])^\dagger\|_F}\begin{bmatrix} \rho_1[t]s_1[t] \\ \rho_2[t]s_2[t] \\ . \\ . \\ \rho_K[t]s_K[t] \end{bmatrix} \tag{6.10}$$

where $w_{eq}[t]=\left[w_{eq,1}[t],w_{eq,2}[t],..,w_{eq,K}[t]\right]^{M_{T_x}}$. $w_{eq}[t]$ is referred to effective post processing noise which is given by

$$w_{eq}[t]=\begin{bmatrix} \rho_{d,1}\sqrt{1-|\rho_{e,1}|^2}\zeta_1+\sqrt{1-|\rho_{d,1}|^2}\varepsilon_1 \\ \rho_{d,2}\sqrt{1-|\rho_{e,2}|^2}\zeta_2+\sqrt{1-|\rho_{d,2}|^2}\varepsilon_2 \\ . \\ . \\ \rho_{d,K}\sqrt{1-|\rho_{e,K}|^2}\zeta_K+\sqrt{1-|\rho_{d,K}|^2}\varepsilon_K \end{bmatrix}\frac{(\hat{H}[t-t_d])^\dagger}{\|(\hat{H}[t-t_d])^\dagger\|_F}s[t]+w[t] \tag{6.11}$$

The covariance of $w_{eq,k}[t]$ can be computed as

$$E\,[w_{eq,k}[t]\; w^{*}_{eq,k}[t]] = E_s(1-|\rho_k|^2) + \sigma^2_{w,k} \tag{6.12}$$

Based on (6.10) and (6.12), the post-processing SINR per symbol ($\gamma_k[t]$) on the data stream of user k can be derived as

$$\gamma_k[t] = \frac{\gamma_{s,k}\,|\rho_k|^2}{[\gamma_{s,k}(1-|\rho_k|^2)+1]\,\|(\hat{H}[t-t_d])^{\dagger}\|_F^2} \tag{6.13}$$

where $\gamma_{s,k} = E_s/\,\sigma^2_{w,k}$. $\gamma_{s,k}$ is the SINR of downlink data symbol for MU-MIMO ZF system. The expression for the average BER [3] of MU-MIMO ZF system with M-QAM modulated signals can be obtained from (6.13). If the SINR is $\gamma_{p,k}$ and MMSE is chosen for channel estimation for user k, then

$$|\rho_{e,k}| = \sqrt{\gamma_{p,k}/(1+\gamma_{p,k})} \tag{6.14}$$

The power spectrum for a time-varying Rayleigh fading channel follows the Jakes model, which can be expressed as

$$PS_{d,k} = J_o(2\pi t_d T_s.F_{d,k}) \tag{6.15}$$

where J_o is a zero order Bessel function of the first kind, $F_{d,k}$ is the maximal Doppler frequency shift of user k, and T_s is the symbol duration. So correlation coefficient can be written as

$$\rho_k = J_o\left(2\pi t_d T_s.F_{d,k}\right)\sqrt{\gamma_{p,k}/(1+\gamma_{p,k})} \tag{6.16}$$

The expression of SNR γ is further used in the calculation of MGF, PEP and BER.

6.1 Performance Analysis of MU-MIMO Systems

MU-MIMO system is analyzed using MGF, PEP and BER and their expression are derived in subsequent sections.

6.1.1 Moment Generating Function (MGF) Analysis

For a given random variable ψ and probability distribution function p (ψ), MGF can be written as

$$M_{\psi}(s) \quad E[\exp(-s\psi)] \tag{6.17}$$

The evaluation of the probability is p $\quad \mathbb{P}$ (ξ>ω), whose MGF can be expressed as

$$M_{\xi}(s) \quad E[\exp(-s\xi)] \tag{6.18}$$

$$M_{\omega}(s) \quad E[\exp(-s\omega)] \tag{6.19}$$

where ξ and ω are random variables and consider $\Delta = \xi - \omega$. Now probability is p=$\mathbb{P}$ (Δ<0). Assume that the MGF of Δ is as follows

$$M_{\Delta}(s) \quad E[\exp(-s\Delta)] \tag{6.20}$$

If random variables are independent, then MGF can be written as

$$M_{\Delta}(s) = M_{\omega}(s)\, M_{\xi}(-s) \tag{6.21}$$

$$\begin{aligned} M_{\Delta}(s) &\quad E[\exp(-s\Delta)] \\ &= M_{\xi}(s)\, M_{\omega}(-s) \\ &= M_{\xi}(s)\,(1-2s)^{-1/2} \end{aligned} \tag{6.22}$$

For MU-MIMO system, the receiver combines the signals through M_{Rx} receive antennas coming from M_{Tx} transmit antennas optimally for k users. Thus, the MU-MIMO channel can be represented with fading coefficients as

$$H = \frac{1}{M_R M_T k} \sum_{i=1}^{M_R} \sum_{j=1}^{M_T} \sum_{k=1}^{K} h_k(j,i) \tag{6.23}$$

The equivalent SNR can be expressed as

$$\gamma_k = \bar{\gamma}_k \, \|H\|_F^2 \tag{6.24}$$

$$\bar{\gamma}_k = \frac{1}{M_R M_T k} \frac{RE_s}{N_o} \tag{6.25}$$

where R is code rate, E_s is symbol energy and N_o is block length [4]. Now, MGF is calculated on the basis of correlation among fading coefficients [5].

6.1.1.1 Uncorrelated channels

When the coefficients of the MU-MIMO channel H are independent, the resulting SNR will be the sum of M_R, M_T and k independent exponential variables and follows chi-square distribution. When the fading coefficients of channel matrix H are independent i.e. there is no correlation among them. Then, the MGF of probability distribution function can be presented as

$$M_\gamma(s) = \prod_{k=1}^{K} (1\text{-}s\,\bar{\gamma}_k)^{-D} \tag{6.26}$$

where D= $M_R.M_T.k$, is known as diversity factor.

6.1.1.2 Spatial-correlation channels

If the fading coefficients are spatially correlated and assuming that the correlation among the fading coefficients is time-invariant. Then, the MGF of SNR is given by

$$M_\gamma(s) = \prod_{i=1}^{M_R}\prod_{j=1}^{M_T}\prod_{k=1}^{K}(1-s\lambda_{i,k}\lambda_{j,k}\bar{\gamma}_k)^{-1} \tag{6.27}$$

where $\lambda_{i,k}$ and $\lambda_{j,k}$ are Eigen values of correlation matrices of MU-MIMO system at the receiver and transmit side respectively. From (6.23) and (6.24), MU-MIMO channel can be presented as

$$H = \frac{1}{M_R M_T k}\sum_{i=1}^{M_R}\sum_{j=1}^{M_T}\sum_{k=1}^{K} h_k(j,i)$$

$$\gamma_k = \bar{\gamma}_k \,\| H \|_F^2$$

(6.24) can be rewritten as

$$\gamma_k = \prod_{i=1}^{M_R}\prod_{j=1}^{M_T}\prod_{k=1}^{K}\lambda_{i,k}\lambda_{j,k}\bar{\gamma}_k\,|\bar{h}_k(j,i)|^2 \tag{6.28}$$

Then by using (6.28) into (6.27), the expression of MGF can be derived as

$$M_{\gamma}(s) = E\left[\exp\left(-s\,\bar{\gamma}_k \,\|H\|_F^2\right)\right] \tag{6.29}$$

$$M_{\gamma}(s) = \prod_{i=1}^{M_R}\prod_{j=1}^{M_T}\prod_{k=1}^{K} E\left\{\exp\left(-s\lambda_{i,k}\lambda_{j,k}\bar{\gamma}_k \,|\bar{h}_k(j,i)|^2\right)\right\} \tag{6.30}$$

PEP can be expressed in terms of MGF as follows

$$P = E\left[Q(\xi)\right] \tag{6.31}$$

where ξ is random variable which can be represented for IF (Independent fading) and BF (block fading) channel as shown below

$$\xi = \begin{cases} \left\{\sqrt{\dfrac{1}{2N_o}\displaystyle\sum_{i=1}^{M_{R_x}}\sum_{j=1}^{M_{T_x}}\sum_{k=1}^{K}\left\{\| H_k(j,i)x_k(i,1) - \hat{x}_k(i,1)\|^2\right\}}\right. & \text{(IF channel)} \\ \left\{\sqrt{\dfrac{1}{2N_o}\displaystyle\sum_{i=1}^{M_{R_x}}\sum_{j=1}^{M_{T_x}}\left\{\| H(j,i)x(i,1) - \hat{x}(i,1)\|^2\right\}}\right. & \text{(BF channel)} \end{cases} \tag{6.32}$$

6.1.2 Pair wise Error Probability (PEP) Analysis

PEP serve as basis for calculation of union bounds on the error probability. An MGF-based approach is presented here for the exact calculation of the PEP which guarantees arbitrarily high accuracy [6], [7]. This approach is described here for the calculation of E $[Q(\sqrt{\xi})]$ for a non-negative random variable ξ

$$M_{\xi}(s) \quad E[\exp(-s\xi)] \tag{6.33}$$

The PEP calculation is done for different equalizers which is described below

6.1.2.1 PEP calculation with ZF Equalizer

ZF equalizer is also known as linear equalizer which has lower complexity than ML equalizer. In MU-MIMO system model ZF equalizer can be implemented as

$$B \quad (H^{\dagger}H)^{-1}H^{\dagger} \tag{6.34}$$

This matrix is known as ZF equalizer. By multiplying (6.35), on the both side of (3.2)

$$BY_i = BH_i\,\mathbf{x} + Bw_i \tag{6.35}$$

where $\mathbf{x} = \sum_{k=1}^{K} c_k s_k = C\,s$

Equation (6.35) can be further decomposed, then the covariance of noise is calculated as $(H^{\dagger}H)^{-1}_{k,k}$. PEP given on H with ZF equalizer can be calculated as

$$P\,(x_{k,1} \rightarrow x_{k,2}|H) = Q(d_2/\sqrt{2}) \tag{6.36}$$

where $\Delta x_k = x_{k,1} - x_{k,2}$, $d_2 = \sqrt{\dfrac{\gamma_k \,\|\,\Delta x_k\,\|^2}{(H^{\dagger}H)^{-1}_{k,k}}}$ and $\|\Delta x_k\|^2 = \|x_{k1} - x_{k2}\|^2$

The density function of random variable $\dfrac{1}{(H^{\dagger}H)^{-1}_{k,k}}$ -

$$= \frac{1}{(M_R + M_T - K)!} a^{M_R + M_T - K} \exp(-a) \tag{6.37}$$

Therefore, PEP with ZF equalizer can be calculated using (6.37) and can be expressed as

$$P\,(x_{k,1} \rightarrow x_{k,2}|H) = \int_0^{\infty} Q\left(\sqrt{\frac{\gamma_k \,\|\,\Delta x_k\,\|^2\, a}{2}}\right) \frac{a^{M_R + M_T - K} \exp(-a)}{(M_R + M_T - K)!} da \tag{6.38}$$

$$\begin{aligned} &\approx \frac{1}{12}\int_0^{\infty} \exp\left(-\frac{\gamma_k \,\|\,\Delta x_k\,\|^2\, a}{4}\right) \frac{a^{M_R + M_T - K} \exp(-a)}{(M_R + M_T - K)!} da \\ &+ \frac{1}{6}\int_0^{\infty} \exp\left(-\frac{\gamma_k \,\|\,\Delta x_k\,\|^2\, a}{3}\right) \frac{a^{M_R + M_T - K} \exp(-a)}{(M_R + M_T - K)!} da \end{aligned} \tag{6.39}$$

$$\approx \frac{1}{12}\left(1 + \frac{\gamma_k \,\|\,\Delta x_k\,\|^2}{4}\right)^{-(M_R + M_T - K + 1)} + \frac{1}{6}\left(1 + \frac{\gamma_k \,\|\,\Delta x_k\,\|^2}{3}\right)^{-(M_R + M_T - K + 1)} \tag{6.40}$$

By comparing the upper bound, it can be observed that the diversity order of ZF equalizer is $M_{Rx}+ M_{Tx}$ -K+1. The diversity order of ZF is less than the ML equalizer. Hence decoding complexity decreases in ZF equalizer. Actually, the complexity of ZF increases linearly with the user number K, while in ML-decoder it increases exponentially [8]-[10].

6.1.2.2 PEP calculation with MMSE Equalizer

MMSE equalizer is also a linear equalizer which has lower complexity than ZF equalizer. MMSE equalizer can be implemented as

$$F_{MMSE} \quad (\frac{1}{\gamma_k} I_k + H^{\dagger} H)^{-1} H^{\dagger} \tag{6.41}$$

By multiplying F_{MMSE}, on the both side of (3.2) we get

$$F_{MMSE}\, Y_i = F_{MMSE}\, H_i\, \mathbf{x} + F_{MMSE}\, w_i \tag{6.42}$$

where $\mathbf{x} = \sum_{k=1}^{K} c_k s_k = C\, s$. Equation (6.42) can be further decomposed. The SINR (γ) can be calculated as

$$\gamma_{MMSE,k} = \frac{\overline{\gamma}_k}{\left(\frac{1}{\overline{\gamma}_k} I_k + H^{\dagger} H\right)^{-1}_{k,k}} - 1 \tag{6.43}$$

Then, the PEP calculation on channel matrix H, can be done with the help of MGF as shown below

$$M_{\gamma}(x) = Q(\frac{\sqrt{\gamma_{MMSE,k}}\, |\Delta x_k|}{\sqrt{2}}) \tag{6.44}$$

The PEP averaging on channel matrix H can be calculated as

$$P_{MMSE,k}(x_1 \rightarrow \hat{x}_1) = E(\, Q(\frac{\sqrt{\gamma_{MMSE,k}}\, |\Delta x_k|}{\sqrt{2}})) \tag{6.45}$$

$$\approx \frac{1}{12} E\left(\exp\left(-\frac{\gamma_{MMSE,k} |\Delta x_k|^2}{4}\right)\right) + \frac{1}{6} E\left(\exp\left(-\frac{\gamma_{MMSE,k} |\Delta x_k|^2}{3}\right)\right) \tag{6.46}$$

$$= \frac{1}{12^{M_{R_x} \times M_{T_x} \times k}} \int_H \exp(-\frac{\gamma_{MMSE,k} \,|\, \Delta x_k \,|^2}{4}) e^{-tr(H^\dagger H)} dH$$
$$+ \frac{1}{6^{M_{R_x} \times M_{T_x} \times k}} \int_H \exp(-\frac{\gamma_{MMSE,k} \,|\, \Delta x_k \,|^2}{3}) e^{-tr(H^\dagger H)} dH \quad (6.47)$$

Further it can be concluded that the diversity order of MMSE equalizer is M_{Rx}+ M_{Tx} - K+1 same as ZF equalizer. But the PEP of ZF equalizer is greater than the MMSE equalizer. Hence, the complexity of MMSE equalizer is less than the ZF equalizer. PEP error performance can be calculated and analyzed in two cases of fading distribution-

6.1.2.2 [A] IF Channel

In this fading channel, the transmitted symbols are affected by independent fading realization [10]. For this fading channel, PEP is given as

$$P(x_k \to \hat{x}_k) = \mathbb{P}\left(\sum_{i=1}^{M_{R_x}} \sum_{j=1}^{M_{T_x}} \sum_{k=1}^{K} \left\{ \| y_k(j,1) - H_k(j,i)\hat{x}_k(i,1) \|^2 - \{ \| y_k(j,1) - H_k(j,i)\hat{x}_k(i,1) \|^2 \} \right\} < 0 \right) \quad (6.48)$$

$$P(x_k \to \hat{x}_k) = E\left[Q(\sqrt{\frac{1}{2N_o} \sum_{i=1}^{M_{R_x}} \sum_{j=1}^{M_{T_x}} \sum_{k=1}^{K} \left\{ \| H_k(j,i) x_k(i,1) - \hat{x}_k(i,1) \|^2 \right\}}) \right] \quad (6.49)$$

where N_o is block length.

6.1.2.2 [B] BF Channel

In this fading channel, transmitted symbols are affected by the same fading realization. For this fading channel, PEP is given as

$$P(x \to \hat{x}) = E\left[Q(\sqrt{\frac{1}{2N_o} \sum_{i=1}^{M_{R_x}} \sum_{j=1}^{M_{T_x}} \left\{ \| H(j,i) x(i,1) - \hat{x}(i,1) \|^2 \right\}}) \right] \quad (6.50)$$

Here, consider d= $x \to \hat{x}$, where d is known as Hamming weight. Now assuming all-zero codeword is transmitted, the PEP of a codeword with weight d given the MU-MIMO channel H with SNR γ of the fading block is

$$P(d|\, H, \gamma) = Q(\sqrt{2 \sum_{i=1}^{M_{R_x}} \sum_{j=1}^{M_{T_x}} \sum_{k=1}^{K} \gamma_{i,j}(k)}) \quad (6.51)$$

Q-function can also be written as

$$Q(x) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \exp(-\frac{x^2}{2\sin^2\theta})d\theta \tag{6.52}$$

Now, by rewriting the conditional PEP and taking average over SNR γ to get

$$P\,(d|\,H) = E_\gamma[P\,(d|\,H, \gamma)]$$

$$= \frac{1}{\pi}\int_{\theta=0}^{\frac{\pi}{2}} \prod_{i=1}^{M_{R_x}} \prod_{j=1}^{M_{T_x}} \prod_{k=1}^{K} \int_{\gamma=0}^{\infty} \exp(-\frac{\gamma_{i,j}(k)}{2\sin^2\theta})d\gamma_{i,j}(k)d\theta \tag{6.53}$$

Here the inner integral is the MGF of SNR γ, $M(s) = E[e^{s\gamma}]$. It is evaluated at $s=1/\sin^2\theta$ and can be represented as

$$P(d|H) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{k=1}^{K}\left[M(-\frac{1}{2\sin^2\theta})\right]^{h_k} d\theta \tag{6.54}$$

For IF channel, expression of PEP using Chernoff bound can be expressed as

$$P\,(d|\,H) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{k=1}^{K}\left[(-\frac{d\bar{\gamma}_k}{\sin^2\theta})\right]^{-D} d\theta \le \frac{1}{2}(1+d\bar{\gamma}_k)^{-D} \tag{6.55}$$

For BF channel, expression of PEP using Chernoff bound (at high SNR) can be expressed as

$$P\,(d|\,H) \le \frac{1}{2}\prod_{k=1}^{K}(d\bar{\gamma}_k)^{-h_k D} \tag{6.56}$$

where $D= M_{R.}M_{T.}k$, D is diversity factor [10].

6.1.2.3 PEP Calculation with ML Equalizer

The density function of received signals is required to analyze the performance of ML-decoder. To get this function, let us assume that a signal vector x is transmitted from k users simultaneously. Here channel H is known to base station.

The density function denoted as f(Y), can be represented as

$$f(Y|H,x)= \sum_{i=1}^{M_{R_x}} \sum_{j=1}^{M_{T_x}} \sum_{k=1}^{K} \frac{1}{\pi^{M_{R_x}\times M_{T_x}\times k}} \exp(-\| Y_k(j,1)-H_k(j,i)x_k(i,1)\|^2) \tag{6.57}$$

From (6.57), ML- decoder for decoding signals x_k is designed as follows

$$(\hat{x}_1,\hat{x}_2,..,\hat{x}_K)=\arg\min_{(\hat{x}_1,\hat{x}_2,...,\hat{x}_K)\in\hat{x}_1\times\hat{x}_2\times..\times\hat{x}_K} \| Y-Hx \|^2 \tag{6.58}$$

To decode x, K comparisons of $|x_1|\times|x_2|\times...\times|x_K|$ are required. Where |x| means cardinality of a set x. Now, calculate the PEP of decoding signals x_k. Assuming the signal vector x_1 is transmitted and $\hat{x}_1$ is decoded.

Probability of event $x_1 \neq \hat{x}_1$ can be calculated as Q ($d_1/\sqrt{2}$), where Q is the error function, $\Delta x = x_1-\hat{x}_1$ and $d_1=\sqrt{\gamma \mathrm{tr}(H\Delta x(\Delta x)^\dagger H^\dagger)}$. Approximate of Q-function is as follows

$$Q(a)\approx \frac{1}{12}\exp(-a^2/2)+\frac{1}{6}\exp(-2a^2/3) \tag{6.59}$$

By using this approximation, the PEP can be approached as

$$P\{\ x_1\neq\hat{x}_1|H\}\approx \frac{1}{12}\exp(-d_1^2/4)+\frac{1}{6}\exp(-d_1^2/3) \tag{6.60}$$

$$=\frac{1}{12}\exp(-\frac{\gamma}{4}\mathrm{tr}(\Delta x(\Delta x)^\dagger H^\dagger H))+\frac{1}{6}\exp(-\frac{\gamma}{3}\mathrm{tr}(\Delta x(\Delta x)^\dagger H^\dagger H)) \tag{6.61}$$

From assumptions on the channel matrix H, it is known that the density function of H is $\frac{1}{\pi^{M_R\times M_T\times K}}\exp(-\mathrm{tr}(H^\dagger H))$. Therefore, the PEP averaging on H is presented as

$$P\{\ x_1\neq\hat{x}_1\}=\int_H P\{x_1\neq\hat{x}_1\,|\,H\}\frac{1}{\pi^{M_R\times M_T\times K}}\exp(-\mathrm{tr}(H^\dagger H))dH \tag{6.62}$$

$$\frac{1}{12\pi^{M_R\times M_T\times K}}\int_H \exp(-\mathrm{tr}((HG_1)^\dagger(HG_1)))dH+\frac{1}{6\pi^{M_R\times M_T\times K}}\int_H \exp(-\mathrm{tr}((HG_2)^\dagger(HG_2)))dH \tag{6.63}$$

where $G_1=\sqrt{I_K+\frac{\gamma}{4}\Delta x(\Delta x)^\dagger}$, $G_2=\sqrt{I_K+\frac{\gamma}{3}\Delta x(\Delta x)^\dagger}$.

To calculate the integrals, use integral transforms $A_1=HG_1$ and $A_2=HG_2$. Thus, $dH_1=|\det(G_1)|^{-2(M_R\times M_T\times k)}\, dA_1$ and $dH_2=|\det(G_2)|^{-2(M_R\times M_T\times k)}\, dA_2$ respectively. Hence, PEP is written as

$$P(x_1 \neq \hat{x}_1) \approx \frac{1}{12}\left(\frac{1}{\det(G_1)}\right)^{2(M_R\times M_T\times K)} + \frac{1}{6}\left(\frac{1}{\det(G_2)}\right)^{2(M_R\times M_T\times K)} \tag{6.64}$$

By putting the value of G_1 and G_2 into (6.64)

$$= \frac{1}{12}\left(\frac{1}{\det\left(I_k + \frac{\gamma}{4}\Delta x(\Delta x)^{\dagger}\right)}\right)^{M_R\times M_T\times K} + \frac{1}{6}\left(\frac{1}{\det\left(I_k + \frac{\gamma}{3}\Delta x(\Delta x)^{\dagger}\right)}\right)^{M_R\times M_T\times K} \tag{6.65}$$

$$= \frac{1}{12}\left(\frac{1}{\det\left(I_k + \frac{\gamma}{4}\|\Delta x\|^2\right)}\right)^{M_R\times M_T\times K} + \frac{1}{6}\left(\frac{1}{\det\left(I_k + \frac{\gamma}{3}\|\Delta x\|^2\right)}\right)^{M_R\times M_T\times K} \tag{6.66}$$

Above approximation is very similar to Chernoff bound of PEP for a MIMO system with M_T transmit antenna and M_R receive antennas. Since each user or transmitter is independent of transmitting signals, so there is no cooperation among the transmitters. Moreover, the major concern is the PEP for each user in the system. Hence, this approximation is required to be changed into PEP for each user.

To get the PEP for each user, say the i^{th} user, $x_{i1}= x_{i2}$ for $k \neq i$, $1\leq k \leq K$, and $x_{k1}\neq x_{k2}$. Thus, $\|\Delta x\|^2$ in (70) is equal to $\|\Delta x_k\|^2=\|x_{k1}- x_{k2}\|^2$, and

$$P(x_{k1} \neq \hat{x}_{k2}) \approx \frac{1}{12}\left(1+\frac{\gamma_k}{4}\|\Delta x_k\|^2\right)^{-(M_R\times M_T\times K)} + \frac{1}{6}\left(1+\frac{\gamma_k}{3}\|\Delta x_k\|^2\right)^{(M_R\times M_T\times K)} \tag{6.67}$$

From this upper bound it can be concluded that diversity order of the performance is M_T x M_R, which is consistent with the case of MIMO system having M_{Tx} transmit antenna and M_R receive antennas.

Although the diversity order is high, but simultaneously the decoding complexity is also very high. It increases exponentially with the user number [8]-[10].

6.1.2.4 PEP calculation with SD Equalizer

Sphere Decoder is non-linear equalizer which has lower computational complexity than ML equalizer. Sphere Decoder adjusts the radius of sphere until there exists a single ML solution vector within the sphere. The density function f(Y) of SD, can be represented as

$$f(Y|H, X) = \sum_{i=1}^{M_R}\sum_{j=1}^{M_T}\sum_{k=1}^{K}\frac{1}{M_R M_T k}\exp\left\{-(x-\hat{x})^T H^T H(x-\hat{x})\right\} \tag{6.68}$$

From (6.68), SD-decoder for decoding signals x_k is designed as follows

$$(\hat{x}_1,\hat{x}_2,..,\hat{x}_K) = \arg\min_{(\hat{x}_1,\hat{x}_2,...,\hat{x}_K)\in\hat{x}_1\times\hat{x}_2\times.\times\hat{x}_K}(x-\hat{x})^T H^T H(x-\hat{x}) \tag{6.69}$$

where $\hat{x} = (H^H H)^{-1} H^H Y$. ML solution vector for SD can be determined by the metric $(x-\hat{x})^T H^T H(x-\hat{x})$. Considering the following sphere with radius R_{SD}-

$$(x-\hat{x})^T H^T H(x-\hat{x}) \le R_{SD}^2 \tag{6.70}$$

Probability of event $x_1 \neq \hat{x}_1$ can be calculated as Q ($R_{SD}/\sqrt{2}$), where Q is the error function and $\Delta x = x_1 - \hat{x}_1$. Approximate of Q-function is as follows

$$Q(R_{SD}) \approx \frac{1}{12}\exp\left(-\frac{R_{SD}^2}{2}\right)+\frac{1}{6}\exp\left(-\frac{2R_{SD}^2}{3}\right) \tag{6.71}$$

By using this approximation, the PEP can be approached as

$$P\{x_1 \neq \hat{x}_1|H\} \approx \frac{1}{12}\exp\left(-\frac{R_{SD}^2}{4}\right)+\frac{1}{6}\exp\left(-\frac{R_{SD}^2}{3}\right) \tag{6.72}$$

$$= \frac{1}{12}\exp(-\frac{\gamma}{4}tr(\Delta x(\Delta x)^\dagger H^\dagger H)) + \frac{1}{6}\exp(-\frac{\gamma}{3}tr(\Delta x(\Delta x)^\dagger H^\dagger H)) \tag{6.73}$$

From assumptions on the channel matrix H

$$(x-\hat{x})^T H^T H(x-\hat{x}) = (x-\hat{x})^T R_{SD}^T R(x-\hat{x}) = \| R_{SD}(x-\hat{x})\|^2 \tag{6.74}$$

It is known that the density function of H is $\exp(-tr(R_{SD}^\dagger R_{SD}))$. Therefore, the PEP averaging on H is presented as

$$P\{\,x_1 \neq \hat{x}_1\} = \int_H P\{x_1 \neq \hat{x}_1 \mid R_{SD}\} \frac{1}{\pi^{M_{R_x} \times M_{T_x} \times K}} \exp(-\mathrm{tr}(R_{SD}^{\dagger} R_{SD}))\, dR_{SD} \tag{6.75}$$

Hence, SD reduces the computational complexity than the ML solution vector. The main drawback of SD is that its complexity depends on SNR [8]-[10]. The worst case complexity of SD is the same as that of ML equalizer.

6.2 BER Analysis for Multi-User MIMO System

In this section, expression for average BER can be presented using two different approaches: direct SNR based approach and MGF based approach.

7.2.1 BER analysis using SNR

If the modulation scheme used for transmitting signals is uncoded M-QAM and the constellation size is M= 2^q , BER in AWGN is

$$\rho_b \approx 0.2 \exp\left\{-\frac{1.6\gamma_k}{2^q - 1}\right\} \tag{6.76}$$

where γ is post processing SNR [3]. Substituting (6.8) into (6.76), then the BER at the t^{th} symbol interval, denoted by $\rho_{b,k}[t]$, is as follows for the data stream of user k .

$$\rho_{b,k}[t] \approx 0.2 \exp\left\{-\frac{1.6\gamma_k(t)}{2^q - 1}\right\} \tag{6.77}$$

Because $\rho_{b,k}$ in (6.77) depends on channel estimate $\hat{H}$ [t- t_d], then the expectation of $\rho_{b,k}$ on $\hat{H}$ [t- t_d] is given as

$$\rho_{b,k}[t] \approx 0.2 \exp\left\{-\frac{1.6a'_k}{(2^q - 1)x}\right\} = 0.2 \exp\left\{-a_k / x\right\} \tag{6.78}$$

where $\quad a'_k \quad \gamma_{s,k}|\rho_k|^2 \Big/ [\gamma_{s,k}(1-|\rho_k|^2)+1] \quad , \quad x \quad \|(\hat{H}[t-t_d])^{\dagger}\|_F^2 \quad , \quad a_k \quad 1.6a'_k \Big/ (2^q - 1)$

$$x = \sum_{k=1}^{K} 1 \Big/ (\lambda'_k)^2 = \sum_{k=1}^{K} 1 \Big/ \lambda_k \tag{6.79}$$

where λ'_k is a singular value of $\hat{H}[t-t_d]$ and $\lambda_k=(\lambda'_k)^2$. So $\rho_{b,k}$ depends on the square λ_k of each singular value of $\hat{H}[t-t_d]$ [3], [11],[12]. The elements of $\hat{H}[t-t_d]$ are i.i.d zero-mean complex Gaussian random variables with unit variance, so the joint probability density function (PDF) of the square λ_k, denoted by $f(\lambda_1, \lambda_2, \ldots \lambda_K)$, can be written as follows

$$f(\lambda_1,\lambda_2,\ldots,\lambda_k)=e^{-\sum_{k=1}^{K}\lambda_k}\prod_{k=1}^{K}\frac{\lambda_k^{M_{T_x}-K}}{(K-k)!(M_{T_x}-k)!}\prod_{k<j}^{K}(\lambda_k-\lambda_j)^2 \qquad (6.80)$$

Therefore the average BER, denoted by $\tilde{\rho}_{b,k}$, can be obtained as

$$\tilde{\rho}_{b,k}(a_k,K,M_{T_x})=\int_0^{+\infty}\int_0^{\lambda_1}\ldots\int_0^{\lambda_{K-1}}0.2\exp\left(-a_k\Big/\sum_{k=1}^{K}\frac{1}{\lambda_k}\right)f(\lambda_1,\lambda_2,\ldots,\lambda_K)d\lambda_K\ldots d\lambda_2 d\lambda_1 \qquad (6.81)$$

The BER function $\tilde{\rho}_{b,k}$ depends on three parameters including a_k, K and M_{Tx} [12].

7.2.2 BER analysis using MGF

From (6.19)

$$\gamma_k=\bar{\gamma}_k\,\|H\|_F^2$$

Thus, the SNR distribution of above expression can be expressed as

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}_k}\exp\left(-\frac{\gamma_k}{\bar{\gamma}_k}\right) \qquad (6.82)$$

Then, the MGF of SNR γ with probability distribution function $f_\gamma(\gamma)$ is

$$M_\gamma(s) = \int_{-\infty}^{\infty} e^{\gamma_k s} f_\gamma(\gamma)d\gamma \qquad (6.83)$$

$$M_\gamma(s) = E\left[\exp(-s\gamma_k)\right] = \prod_{k=1}^{K}\frac{1}{1+s\bar{\gamma}_k\lambda_k} \qquad (6.84)$$

After calculating the MGF, pair wise error probability can be expressed in terms of MGF as follows

$$P_k = \int_0^{\infty} Q(\sqrt{2\gamma_k})\, f_\gamma(\gamma)\, d\gamma = \int_0^{\infty} Q(\sqrt{2\gamma_k})\, \frac{\exp\left(-\frac{\gamma_k}{\bar{\gamma}_k}\right)}{\bar{\gamma}_k}\, d\gamma \tag{6.85}$$

Now, by writing the conditional PEP and taking average over SNR γ to get

$$P\,(d|\,H) = E_\gamma[P\,(d|\,H, \gamma)]$$

$$= \frac{1}{\pi} \int_{\theta=0}^{\pi/2} \prod_{i=1}^{M_R} \prod_{j=1}^{M_T} \prod_{k=1}^{K} \int_{\gamma=0}^{\infty} \exp\left(-\frac{\gamma_{i,j}(k)}{2\sin^2\theta}\right) d\gamma_{i,j}(k)\, d\theta \tag{6.86}$$

Here the inner integral is the MGF of SNR γ, $M_\gamma(s) = E[e^{s\gamma}]$ evaluated at $s=1/\sin^2\theta$. Now we can write

$$P\,(d|H) = \frac{1}{\pi} \int_{\theta=0}^{\pi/2} \prod_{i=1}^{M_R} \prod_{j=1}^{M_T} \prod_{k=1}^{K} M_{\gamma_{i,j}(k)}\left(-\frac{d}{\sin^2\theta}\right) d\theta \tag{6.87}$$

BER expression can be calculated using (6.87)

$$\rho_b = \sum_{L=1}^{d} \frac{L}{d} A_d P(d\,|\,H) \tag{6.88}$$

Here $1 \le L \le d$, d is hamming weight, A_d is multiplicity of the symbol with hamming weight d and P (d|H) is PEP of SNR γ [3]. Thus, the BER expression is evaluated using MGF and PEP. The MGF based approach is used to evaluate PEP and then PEP is used to analyze and to get the expression of BER.

6.3 Simulation Model

For a comprehensive assessment of multi-antenna techniques, it is mandatory to consider the performance at system level, since many effects of spatial processing, like multi-user decoding, the impact of spatially-colour interference, and the benefits of interference management techniques are not tractable at the link level. In this section, we will investigate the performance of MU-MIMO techniques with precoding and different decoding techniques.

Major requirements for the next generation of wireless systems include among others high performance, robustness and adaptability to a wide range of scenarios and terminal classes.

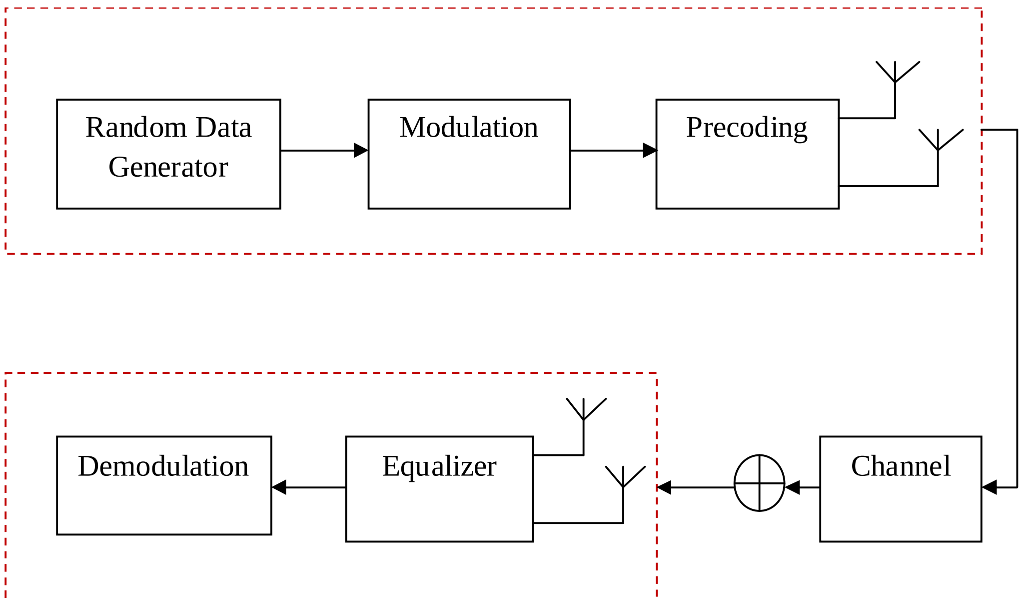

Figure 6.1: MATLAB Simulation Model

Fig. 6.1 shows the MATLAB simulation model which contains-

- **Random Data Generator-** To generate random binary data, random data generator is used. It generates data serially. Data stream generated by random data generator represents the data information to be transmitted.

- **Modulation-** This block does the modulation on the transmitted data stream. I have used BPSK, QPSK and 16-QAM modulation technique. This block does mapping of data to symbol using constellation.

- **Precoding-** When symbol is mapped accordingly with modulation scheme then the stream is fed to precoding block which format the transmitted signal in such a way that the effect of channel in the transmitted signal get reduced if the transmitter side knows the channel state information.

- **Fading Channel-** Precoded data then pass through the channel and experiences various channel effects.

- **Equalization-** This technique is used independently or in random to improve receive signal quality. Equalization is used at receiver to mitigate the effect of channel on received signal by knowing channel response.

- **Demodulation-** This block does demodulation of the signal output form equalizer block corresponding to the modulation scheme used at the transmitter side.

Some of the goals for the future wireless systems are:

• Improved BER performance with reduced computational complexity

• Improved spectral efficiency and increased user peak data rate

• Increased range or coverage in a cost-efficient manner

• Enhanced interference management

• Adaptivity to scenario and channel conditions.

Channel knowledge is typically described with two sorts of measures; channel state information and channel quality indicators (CQI). The term CSI usually refers to knowledge of the complex valued radio channel, while CQI, on the other hand, is rather a real valued measure of the quality of the channel, for example an SINR after receiver processing that may be used to adapt the code rate, modulation order, and spreading at the transmitter. The amount of channel knowledge dictates which methods are applicable and the potential benefits of spatial processing techniques.

MU-MIMO precoding and decoding facilitates the simultaneous transmission of multiple data streams (SMUX) to multiple users (SDMA) which results in a significant throughput improvement. In the previous chapter, we have introduced several linear and non-linear decoding techniques.

Non-linear decoding techniques provide higher diversity than linear techniques at high SNRs. However, the point where non-linear decoding techniques become better than linear depends on the specific antenna configuration of the system, e.g., the number of antennas at the base station and the number of user terminals and antennas at the user terminals.

Linear decoding techniques can achieve the sum-rate capacity bound of the broadcast channel when the number of users in the system is large and appropriate spatial scheduling of users is performed or when the total number of antennas at the user terminals is greater than the number of antennas at the base station.

Together with a lower computational complexity this renders linear decoding techniques more favourable for practical implementation than nonlinear decoding techniques but to achieve improved BER performance, non-linear decoding algorithms are preferable.

6.4 Simulation parameter

TABLE 6.1 SIMULATION PARAMETERS

No. of Transmitting Antenna	4
No. of Receiving Antenna	1
No. of Users	4
Modulation Scheme	16-QAM
Decoding Scheme	ZF, MMSE, ML,SD
Spectral Efficiency	4 bits/sec/Hz

The environment specific characteristics that will be used later for system level investigations are given in the Table 6.1, [70]. The channel is modelled using the parameters reported in [34].

6.5 Simulation Results

In this section, simulation results are plotted on the basis of different modulation schemes for MU-MIMO system. BER performance results are also plotted depending on no. of users and different equalizers.

6.5.1 Simulation results with different equalizers

In this section, we will focus on the system level performance of both linear and non-linear decoding techniques.

Fig. 6.2 shows BER performance for Zero forcing decoding technique. Table 6.1 shows the simulation parameters used in this simulation. Here, I have analyzed the BER performance for four users equipped with single antenna. Data generated by random data generator is modulated with 16-QAM modulation scheme. Fig. 6.2 shows that with the use of zero forcing equalizer, 10^-4 BER can be achieved at an SNR of around 26 dB.

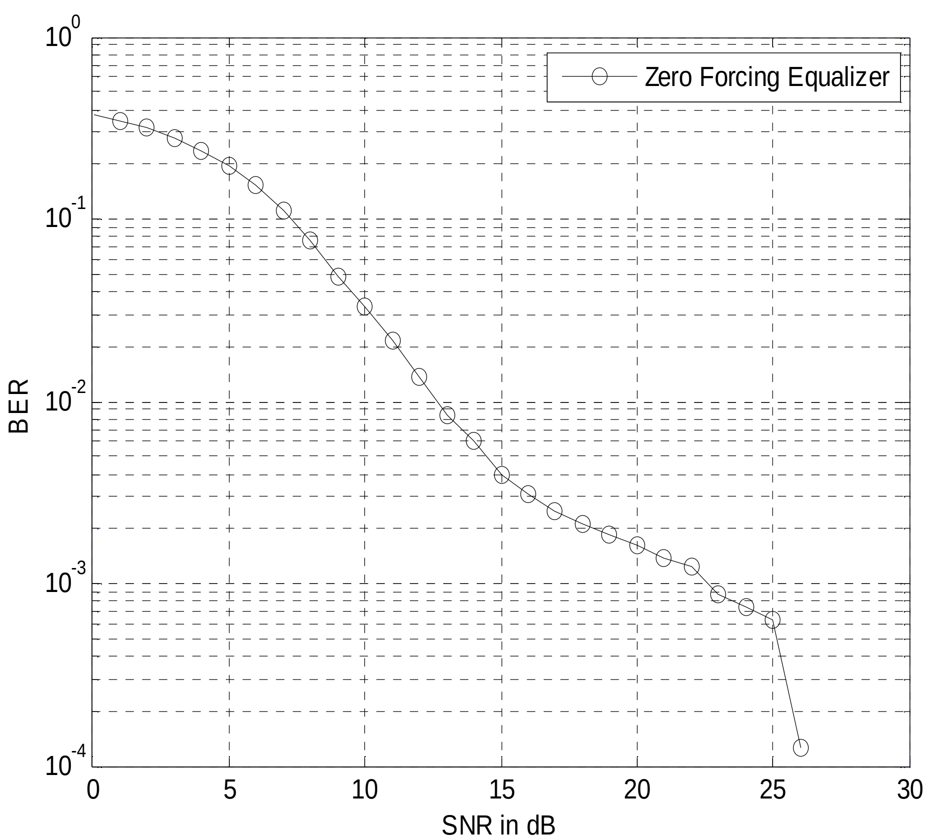

Figure 6.2: BER analysis for Zero-Forcing Decoder with four users

Fig. 6.3 shows BER performance for MU-MIMO system which is precoded with dirty paper coding technique and decoded with Zero forcing technique. Table 6.1 shows the simulation parameters used in this simulation. Here, I have analyzed the BER performance for four users equipped with single antenna. Data generated by random data generator is modulated with 16-QAM modulation scheme. Fig. 6.3 shows that with the use of Dirty paper coding and zero forcing decoding, 10^-4 BER can be achieved at an SNR of around 17.6 dB. Thus, it can be concluded that combination of ZF with DPC improves the system performance.

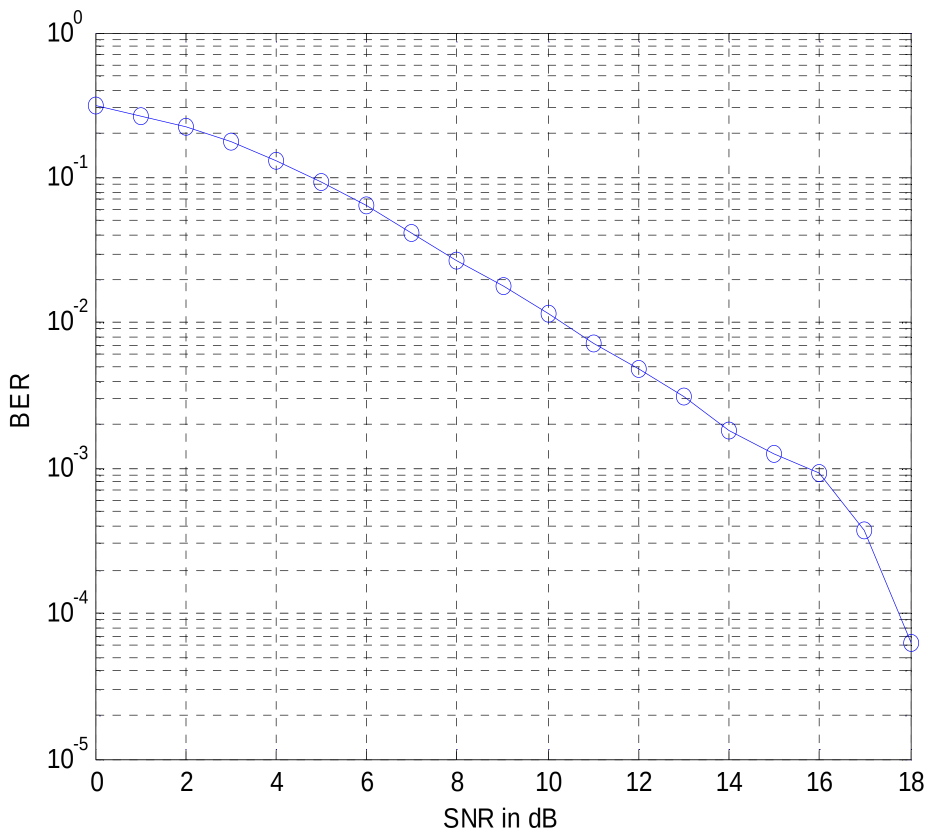

Figure 6.3: BER analysis for DPC-ZF Multi-User (4) System

Fig. 6.4 shows BER performance for MU-MIMO system which is decoded with minimum mean square error (MMSE) technique. Table 6.1 shows the simulation parameters used in this simulation. Here, I have analyzed the BER performance for four users equipped with single antenna. Data generated by random data generator is modulated with 16-QAM modulation scheme. Fig. 6.4 shows that with the use of MMSE decoding technique, 10^-4 BER can be achieved at an SNR of around 17.6 dB, same as combination of dirty paper coding and zero forcing decoding techniques. Thus, it can be concluded that combination of ZF with DPC gives the same performance as MMSE.

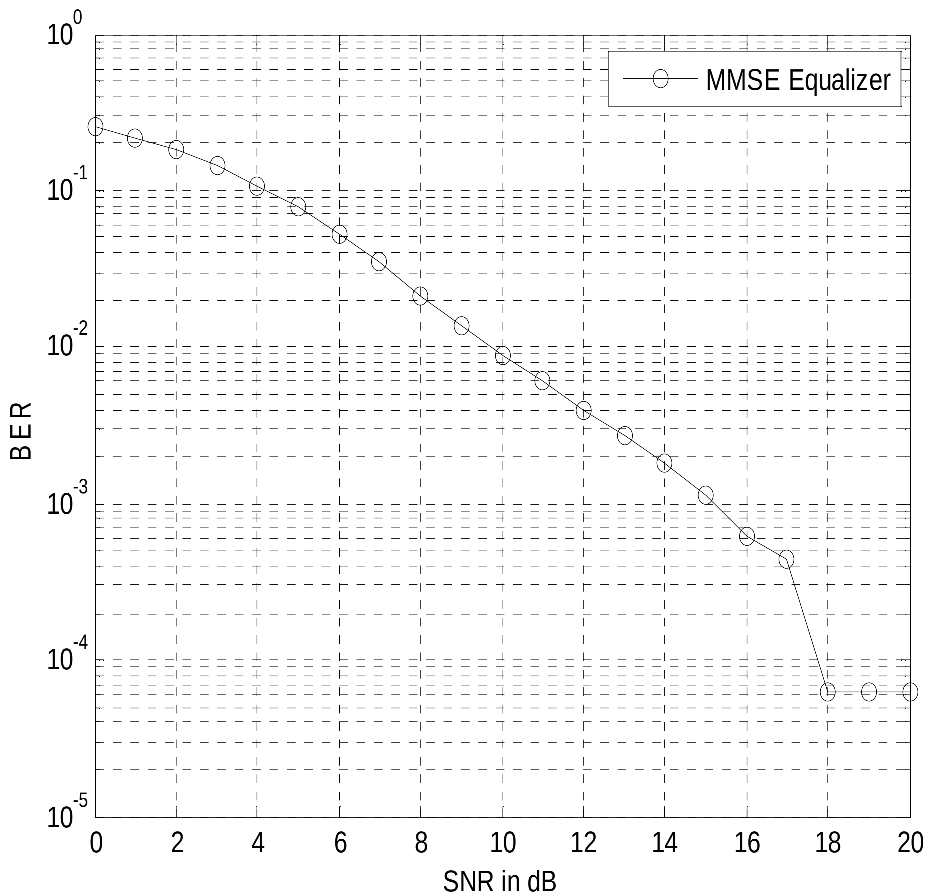

Figure 6.4: BER analysis for MMSE Decoder with four users

Fig. 6.5 shows BER performance for MU-MIMO system which is decoded with maximum likelihood (ML) technique. Table 6.1 shows the simulation parameters used in this simulation. Here, I have analyzed the BER performance for four users equipped with single antenna. Data generated by random data generator is modulated with 16-QAM modulation scheme. Fig. 6.5 shows that with the use of ML decoding technique, 10^-4 BER can be achieved at an SNR of around 10 dB, which is far better than the linear decoding techniques and the combination of dirty paper coding and zero forcing decoding techniques. Thus, it can be concluded that non-linear decoding techniques provides better performance than linear. Simultaneously, ML has large computational complexity.

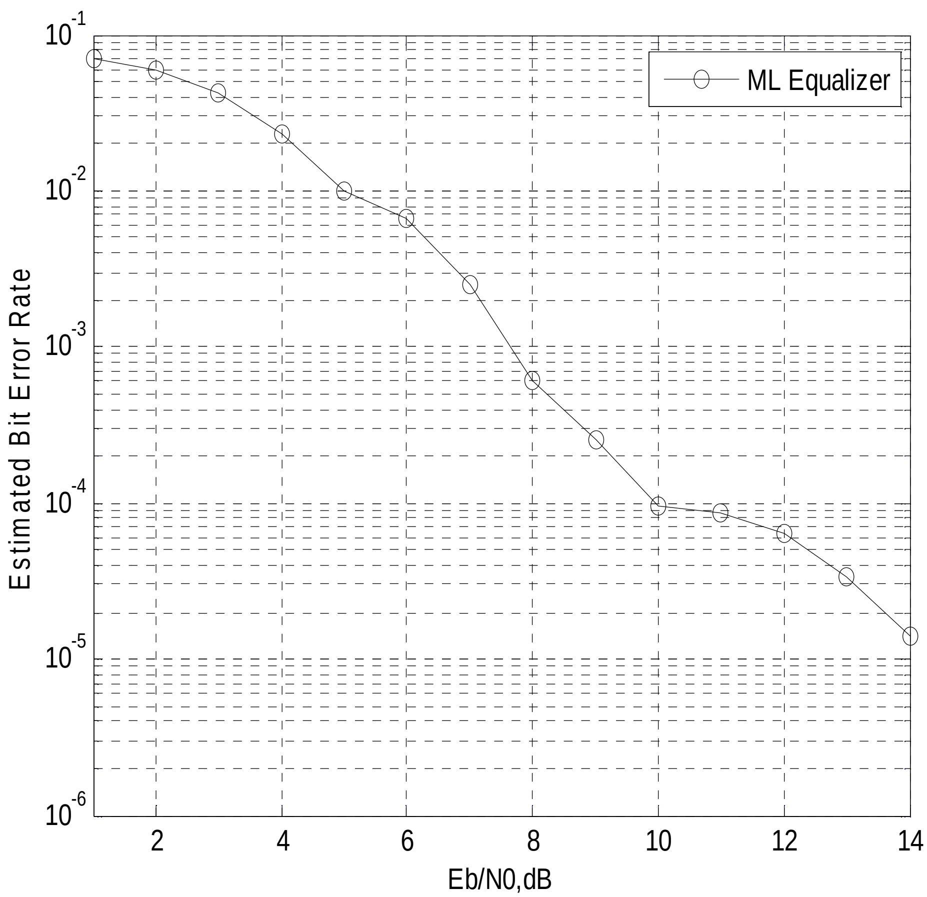

Figure 6.5: BER analysis for ML Decoder with four users

Fig. 6.6 shows BER performance for MU-MIMO system which is decoded with sphere decoding (SD) technique. Table 6.1 shows the simulation parameters used in this simulation. Here, I have analyzed the BER performance for four users equipped with single antenna. Data generated by random data generator is modulated with 16-QAM modulation scheme. Fig. 6.6 shows that with the use of SD technique, 10^-4 BER can be achieved at an SNR of around 9 dB, which is far better than the linear decoding techniques and the combination of dirty paper coding and zero forcing decoding techniques. Thus, it can be concluded that non-linear decoding techniques provides better performance than linear. Simultaneously, SD gives slightly better performance than ML. It also mitigate the drawback of ML. SD has very less computational complexity.

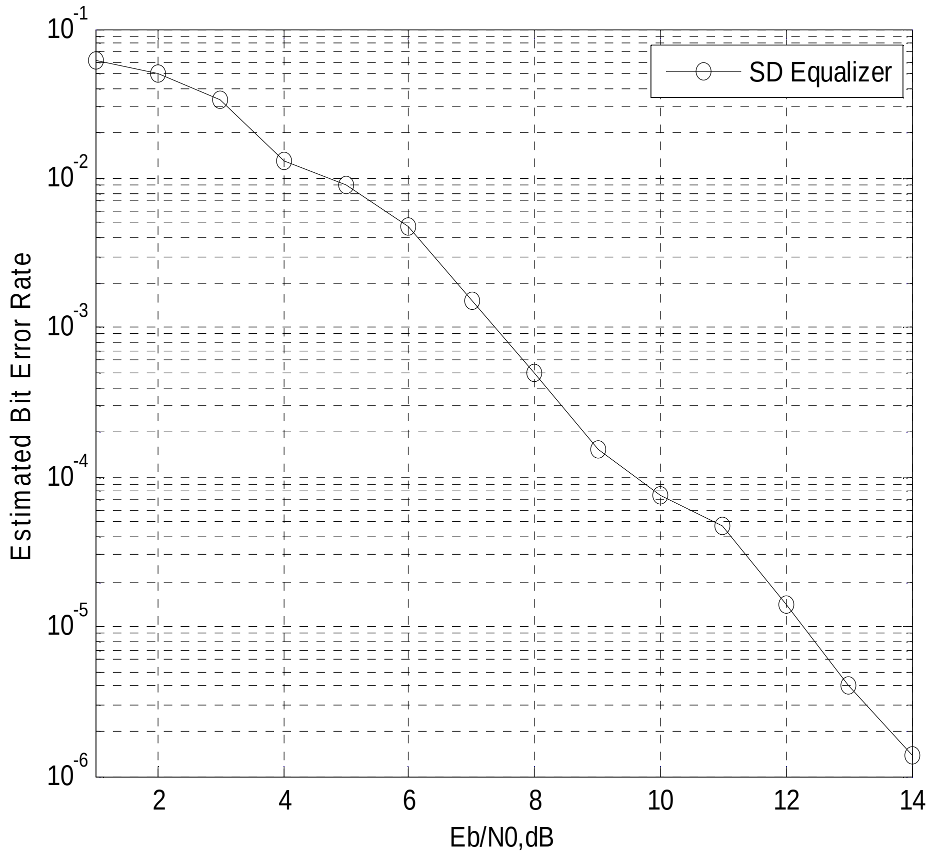

Figure 6.6: BER analysis for Sphere Decoder with four users

Table 6.2 shows the simulation results of all the decoding techniques.

TABLE 6.2: COMPARISON OF SIMULATION RESULTS

Decoding Technique	BER	SNR (in dB)
Zero- Forcing	10^-4	26
Zero- Forcing with DPC	10^-4	17.6
MMSE	10^-4	17.6
Maximum Likelihood	10^-4	10
Sphere Decoder	10^-4	9

6.5.2 Simulation results with various parameter:-

In this section, simulation results are plotted on account of different modulation schemes in both i.i.d and spatially correlated Rayleigh channel. BER performance results are also simulated depending on no. of users, diversity order and different equalizers.

Fig. 6.7 demonstrates BER performance analysis in i.i.d Rayleigh channel with two users under different modulation schemes like BPSK, QPSK and 16-QAM etc. From this Fig., we get BER of 10^{-5} at SNR of about 6.8, 7.4 and 8.2 dB in BPSK, QPSK and 16-QAM respectively. Simulation results show that BER performance is better with BPSK as compared to QPSK and 16-QAM modulation scheme.

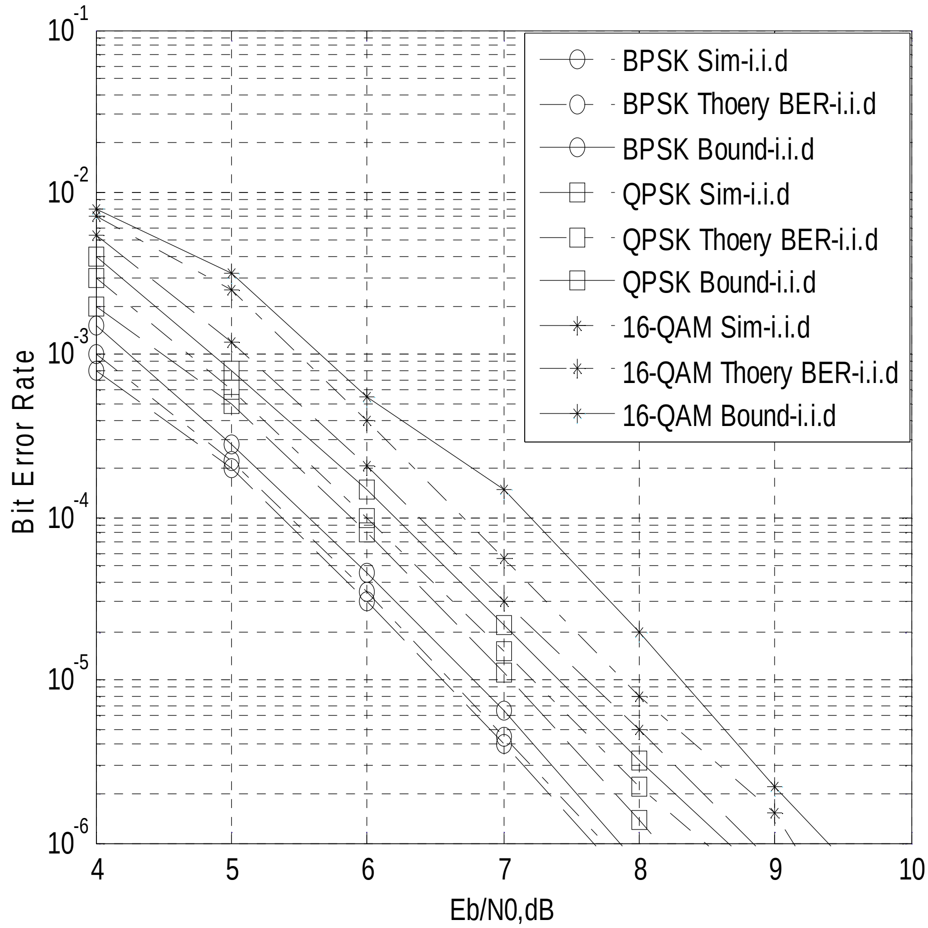

Figure 6.7: BER analysis with different modulation schemes in i.i.d Rayleigh channel with two users

Fig. 6.8 shows BER performance scrutiny in spatially correlated Rayleigh channel with two users. From this Fig., we get BER of 10^{-5} at SNR of about 7.5, 8.2 and 8.9 dB in BPSK, QPSK and 16-QAM respectively. Results in Fig. 6.8 are of same pattern as of Fig. 6.7. On comparing results of Fig. 6.7 and 6.8, it can be concluded that the BER performance of i.i.d is better than the spatially correlated Rayleigh channel for each modulation scheme.

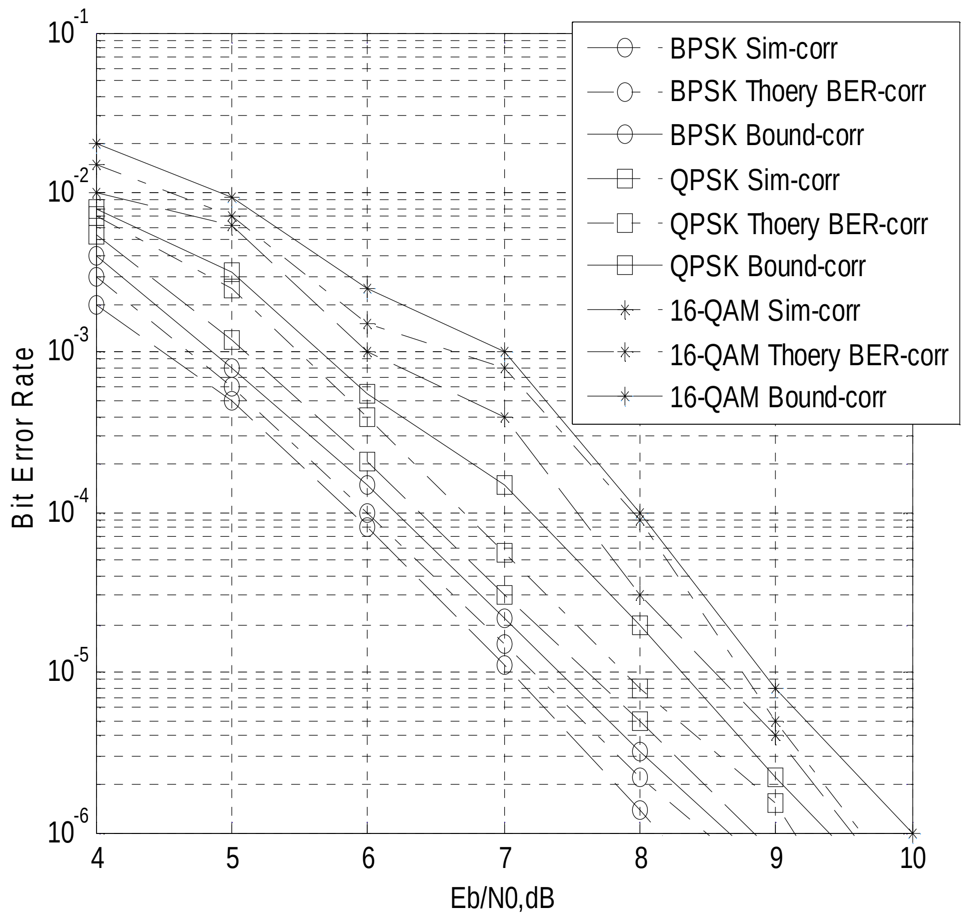

Figure 6.8: BER analysis with different modulation schemes in spatially correlated (ρ=0.7) Rayleigh channel with two users

Fig. 6.9 shows the BER performance for multi-user environment. Simulation results are plotted for single, 2-user and 4-user. From this Fig., we get BER of 10^{-5} at SNR of about 7.9, 6.7 and 5.6 dB in single, 2-user and 4-user respectively. Simulation results in Fig. 6.9 illustrate that BER performance improves as the no. of users increases.

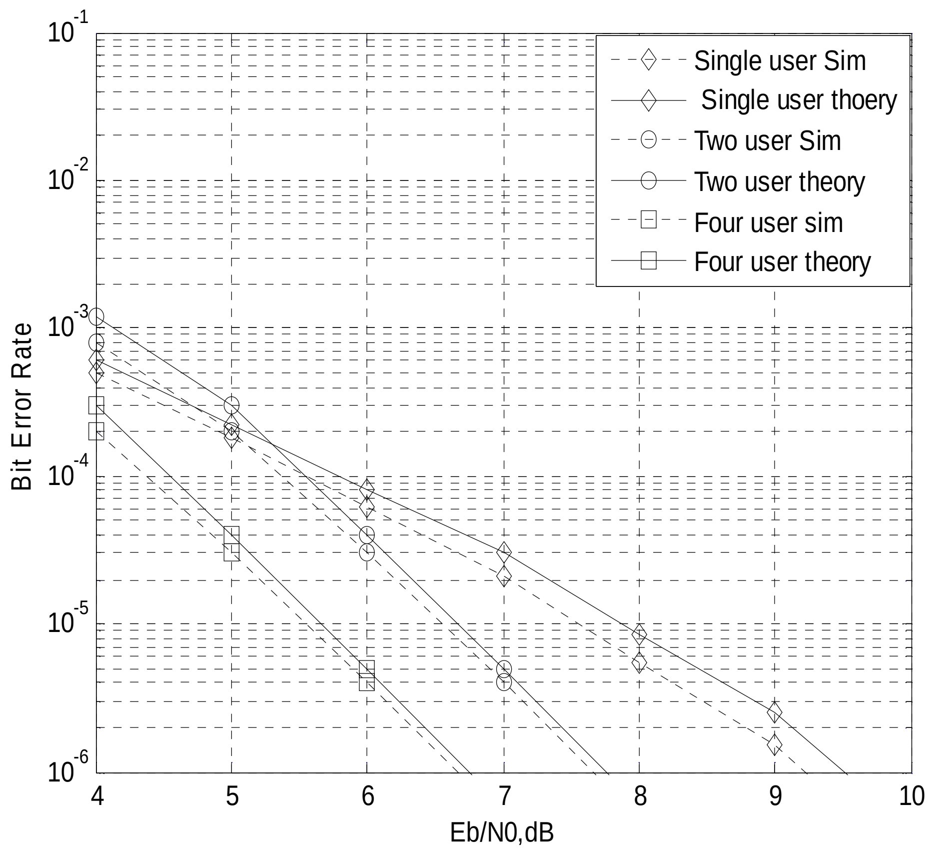

Figure 6.9: BER analysis with multiple users

Fig. 6.10 shows BER performance analysis for different equalizers (ZF, MMSE, ML and SD-equalizer) with two users. From this Fig., we get BER of 10^{-5} at SNR of about 9.6, 8.9, 6.7 and 6.4dB in ZF, MMSE, ML and SD-equalizer respectively. Simulation results show that BER performance is approximately similar in ML and SD- equalizer. Further it can be concluded that BER performance is improved in SD-equalizer than ML, MMSE and ZF. Simulation result also shows that MMSE has better BER performance than ZF-equalizer.

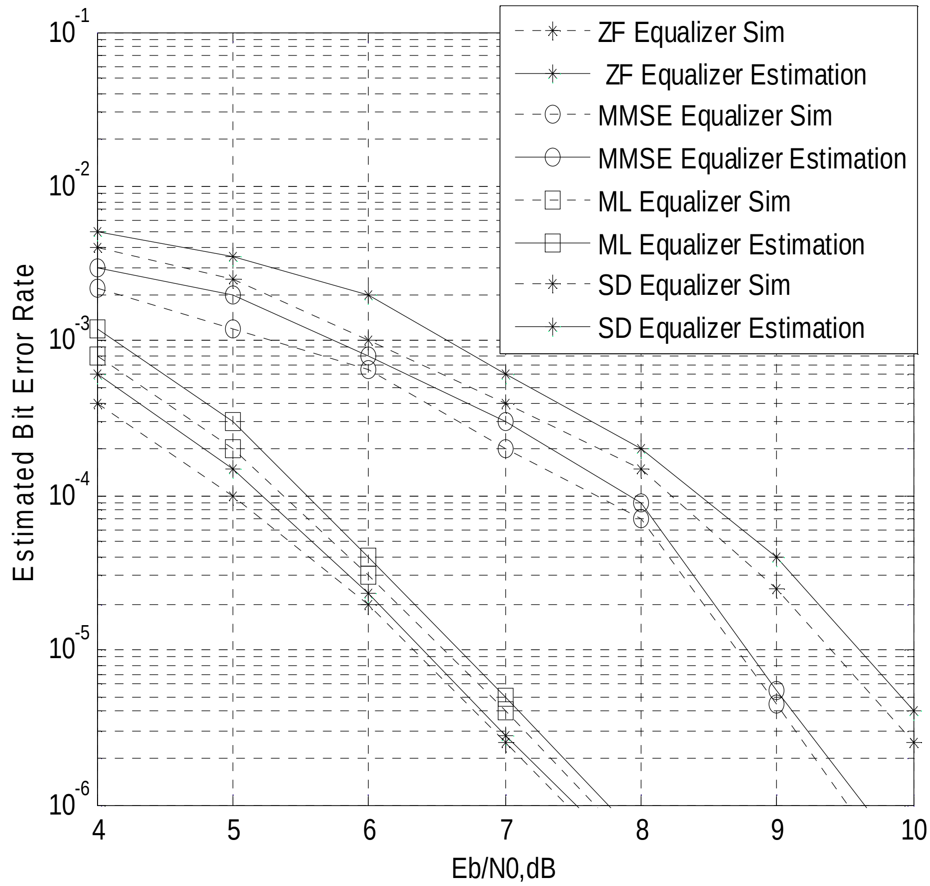

Figure 6.10: BER analysis with different equalizers for two users

It can also be concluded that all the theoretical results are approximately similar to simulated results.

Chapter 7
Conclusion and Future Scope

In this thesis, the problem of designing multi-user MIMO processing techniques has been addressed. The goal was to define one technique that can target several optimization criteria, a technique that will be able to adapt to different qualities of channel state information and that can combine instantaneous and long-term channel state information.

Information theoretic results have shown that the linear increase of capacity in a multiuser MIMO system is possible to obtain only by spatially multiplexing users and by sending multiple data streams to each user. It is known that the maximum sum-rate capacity in a multi-user uplink system is achieved by MMSE decoding and successive interference cancellation. In order to achieve high data rates foreseen by the information theoretic investigations in a multi-user downlink system, it is necessary to use ”dirty paper” codes (DPCs). DPCs are in general very complex and almost impossible to implement. MMSE suffers from a performance loss when users are equipped with more than one antenna.

The other way to improve the system performance is to perform joint processing over a group of multi-user MIMO channels in different frequency and time slots. Joint processing of MIMO channels yields maximum diversity regardless of the level of multi-user interference. As these techniques rely on the fact that there is either instantaneous or long-term channel state information (CSI) available at the base station to perform precoding and decoding, it is very important to investigate the influence of the transceiver front-end imperfections and channel estimation errors on their performance. The CSI at the transmitter can be acquired either through feedback of the channel coefficients or by using the estimates of the channel transfer function and the reciprocity principle. In a TDD system the reciprocity principle allows us to use the estimates of the channel on the uplink to perform precoding on the downlink. This significantly reduces required feedback needed to acquire the channel state information at the transmitter. However, in this case the problem of the influence of the radio front-end characteristics on the performance of the system arises. This is a consequence of the fact that the transfer functions of the transmit RF chain and the receive RF chain in general are not identical. In order to cope with the RF front-end impairments and to meet the conditions so that reciprocity principle holds, it is necessary to either perform calibration or use reciprocal transceivers. We have shown in our simulations that channel estimation errors result in an SNR loss and that by using the self-calibration methods reported in the literature only at the base station, the influence of the calibration errors on the system performance is almost negligible.

Another important issue is the complexity of the multi-user MIMO precoding and decoding techniques. RF complexity is directly related to the number of antennas and the number of separate RF chains. The baseband complexity is related to the relative energy and cycle count required for one run of the multi-user MIMO processing algorithm.

Non-linear decoding techniques provide higher diversity than linear techniques at the high SNRs. However, the point where non-linear decoding techniques become better than linear techniques depends on the specific antenna configuration of the system, e.g., the number of antennas at the base station and the number of user terminals and antennas at the user terminals. Linear decoding techniques can approach the sum-rate capacity bound of the broadcast channel when the number of users in the system is large and appropriate spatial scheduling of users is performed or when the total number of antennas at the user terminals is greater than the number of antennas at the base station. Furthermore, linear decoding techniques can adapt from instantaneous CSI to the long-term CSI and allow the combination of instantaneous and long-term CSI unlike non-linear decoding techniques which require the exact CSI in order to be able to pre-subtract the non-causal interference.

Together with a lower computational complexity this renders linear decoding techniques more favourable for practical implementation than non-linear decoding techniques but linear decoding techniques are only concerned about channel effect optimization, whereas non-linear decoding techniques are adaptive in nature and these decoding techniques consider channel effect optimization as well as inter symbol interference (ISI). On the other hand, linear decoding techniques have less computational complexity than non-linear decoding techniques.

System level investigations have shown that MU-MIMO precoding and decoding techniques provide several times higher data rates than SISO systems with only slightly increased pilot and control overhead. The biggest problem is the influence of user mobility, and calibration and channel estimation errors on their performance. A straightforward way to reduce the sensitivity of these techniques to real-life impairments is to deploy more antennas at the base station or to jointly process the signal from a group of spatially distributed antenna arrays. Distributed antenna arrays are capable of providing very large spectral efficiencies with reduced sensitivity to the real-life impairments at the cost of increased complexity.

This thesis also endows with SINR, MGF and PEP based BER performance analysis for MU-MIMO system. The analysis is done for both i.i.d and spatially correlated channels. As anticipated BER performance is better using i.i.d channels than spatially correlated. Further, the MU-MIMO system is analysed by varying number of users, applying different modulation schemes and with different equalizers.

Finally, this chapter is wrapped up that MU-MIMO system offers best results for 4 users with BPSK modulation under i.i.d channels. Performance of MU-MIMO system is better with SD than the linear (ZF, MMSE) and non-linear (ML) equalizers. Work can be extended to build up new-fangled algorithms which can further trim down system complexity and provide improve BER.

At the end, it can be concluded that multi-user MIMO processing is capable of providing high spectral efficiencies and thus will be an important part of the next generation wireless systems. However, special care must be taken during the system design, in order to account for the deployment scenario characteristics, hardware imperfections and limitations that might reduce their gain.

Appendix A

Derivation of (33)

We prove-

$$\arg\min_{\bar{x}} \| \bar{Y} - \bar{H}\bar{x} \|^2 = \arg\min_{\bar{x}} (\bar{x} - \hat{\bar{x}})^T \bar{H}^T \bar{H} (\bar{x} - \hat{\bar{x}}) \tag{33}$$

Where $\hat{\bar{x}} = (\bar{H}^T \bar{H})^{-1} \bar{H}^T \bar{Y}$, which is the unconstrained solution of the real system. It shows that the ML solution can be determined by the different metric $(\bar{x} - \hat{\bar{x}})^T \bar{H}^T \bar{H} (\bar{x} - \hat{\bar{x}})$. Consider the following expansion-

$$\| \bar{Y} - \bar{H}\bar{x} \|^2 = \| \bar{Y} - \bar{H}\bar{x} - \bar{H}\hat{\bar{x}} + \bar{H}\hat{\bar{x}} \|^2$$

$$= (\bar{Y} - \bar{H}\bar{x} - \bar{H}\hat{\bar{x}} + \bar{H}\hat{\bar{x}})^T (\bar{Y} - \bar{H}\bar{x} - \bar{H}\hat{\bar{x}} + \bar{H}\hat{\bar{x}})$$

$$= \left\{ (\bar{Y} - \bar{H}\hat{\bar{x}})^T + (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x})^T \right\} \left\{ (\bar{Y} - \bar{H}\hat{\bar{x}}) + (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x}) \right\}$$

$$= \left\{ (\bar{Y} - \bar{H}\hat{\bar{x}})^T (\bar{Y} - \bar{H}\hat{\bar{x}}) \right\} + \left\{ (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x})^T (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x}) \right\}$$
$$+ \left\{ (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x})^T (\bar{Y} - \bar{H}\hat{\bar{x}}) \right\} + \left\{ (\bar{Y} - \bar{H}\hat{\bar{x}})^T (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x}) \right\}$$

Where $\left\{ (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x})^T (\bar{Y} - \bar{H}\hat{\bar{x}}) \right\} = \left\{ (\bar{Y} - \bar{H}\hat{\bar{x}})^T (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x}) \right\} = 0$

Thus $\| \bar{Y} - \bar{H}\bar{x} \|^2 = \left\{ (\bar{Y} - \bar{H}\hat{\bar{x}})^T (\bar{Y} - \bar{H}\hat{\bar{x}}) \right\} + \left\{ (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x})^T (\bar{H}\hat{\bar{x}} - \bar{H}\bar{x}) \right\}$

As we know $\hat{\bar{x}} = (\bar{H}^T \bar{H})^{-1} \bar{H}^T \bar{Y}$

$$\| \bar{Y} - \bar{H}\bar{x} \|^2 = \left\{ (\bar{Y} - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T\bar{Y})^T (\bar{Y} - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T\bar{Y}) \right\} + \left\{ (\hat{\bar{x}} - \bar{x})^T \bar{H}^T \bar{H} (\hat{\bar{x}} - \bar{x}) \right\}$$

Since $(\bar{Y} - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T\bar{Y}) = \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-1} \right\} \bar{Y}$

$$\bar{Y}^T \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T \right\}^T \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T \right\} \bar{Y} = \bar{Y}^T \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-T}\bar{H}^T \right\} \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T \right\} \bar{Y}$$

$$= \bar{Y}^T \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T - \bar{H}(\bar{H}^T\bar{H})^{-T}\bar{H}^T + \bar{H}(\bar{H}^T\bar{H})^{-T}\bar{H}^T\bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T \right\} \bar{Y}$$

$$= \bar{Y}^T \left\{ I - \bar{H}(\bar{H}^T\bar{H})^{-1}\bar{H}^T \right\} \bar{Y}$$

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Publications:-

- Pallvi Chawla and Bhasker Gupta,"**BER Analysis of Single/Multi-User LTE and LTE- A Systems,**" published in 4th IEEE International Advance Computing Conference – IACC, 2014 .

- Pallvi Chawla and Bhasker Gupta**, "SINR, MGF and PEP based BER Performance Analysis of Multi-User MIMO Systems,"-** published in IEEE International Conference on Recent Advances in Engineering and Computational Sciences - RAECS, 2014.

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Career Objective

To be associated with a reputed organization that utilizes my professional ability in terms of technical and analytical skill and helps me in enhancing my current skillset to dynamically contribute towards personal and organizational growth.

Education

Degree	College/University	%/CGPA	Passing Year
M.Tech. (Electronics And Communication)	**Jaypee University of Information Technology Waknaghat, Solan (H.P.)**	**8.6**	**2014 (Pursuing)**
B.Tech. (Applied Electronics And Instrumentation)	**ARYA Institute Of Engineering and Technology, Jaipur (Rajasthan)**	**74%**	**2010**
Class XII	**R.B.S.E. , Ajmer, Rajasthan**	**86%**	**2006**
Class X	**R.B.S.E. , Ajmer, Rajasthan**	**92%**	**2004**

Research Papers Accepted/ Published

- Pallvi Chawla and Bhasker Gupta,"**BER Analysis of Single/Multi-User LTE and LTE- A Systems,**" published in 4th IEEE International Advance Computing Conference – IACC, 2014 .
- Pallvi Chawla and Bhasker Gupta**, "SINR, MGF and PEP based BER Performance Analysis of Multi-User MIMO Systems,"-** published in IEEE International Conference on Recent Advances in Engineering and Computational Sciences - RAECS, 2014.

Experience

- Worked as a Lecturer at BMIT Group Of Institutions, Jaipur from August 2010 to January 2011.
- Worked as a Teaching Assistant at Jaypee University of Information Technology from July, 2012 to till date.

Summer Internship/Industrial Training

- Summer Training from CETPA INFOTECH PVT. LTD., ROORKEE in Embedded System and C language
  Duration of course: - One month (From 15th May 2009 to 16th June 2009)
- Industrial visit at Modern Insulator and LIPI Data, Udaipur.

Technical Skills

Programming Languages**: C, C++**

Software knowledge: **Embedded system (KIEL compiler), VERILOG, LABVIEW, PSPICE, MATLAB, VHDL (XiLinx ISE 8.1/8.2i), Auto-CAD, SHARP-GUI, CCS.**

Projects

Major Project	
Implementation of MU-MIMO system with Sphere Decoder	**July2013- May 2014**
Implemented CDMA system using MATLAB	March 2013
Implemented VoIP using Asterisk	September 2012
Automation system- Automatic room light and security system	May 2010

Minor Project	
Implementation of Wireless Sensor Network using MATLAB	December 2013
Implemented BJT amplifier and Multi-vibrator using PSPICE	April 2013
Implemented Arithmetical Unit (add, subtract, multiply, divide, comparator) using Verilog	November 2012
Implemented shift register using VHDL	April 2010
Digital Heart Beat Counter	December 2009

Training Project	
Automatic temperature sensor Metro Train Prototype Light Follower Robot	May-June 2009

Awards, Scholarships, Distinctions and Honors

- Secured 95% in GATE 2011 and getting scholarship through GATE scores from July 2012 to May 2014.
- Recently participated in workshop on failure management by Bell-Labs, organized at Jaypee University of Information Technology in 2012.
- Certification in Embedded system course and C & C++ language course as summer training program in 2009.
- Participated in conference on ‘Innovative Development in Electronics Arena’’ sponsored by IEEE-MTTS in 2009.
- Participated in All India Council For Technical Education And Management Development Academy in 2008.
- Certification in Infosys Campus Connect Life Skills Program.
- Certification of foundation course in AUTO-CAD at CADD Centre Training Services, Jaipur during January 2007.
- Received a Merit Scholarship given by Rajasthan State Govt. for session 2004-06.
- Received “GARGI AWARD” given by Rajasthan State Govt. in year 2005.
- Got 1st position in Class X at District level and 16th position at state level in 2004.

Extra-curricular Activities

- Participated in Badminton and achieved 3rd position at state level in 2001.
- Organized college fest “Shradhanjali” in 2008.

Personal Information

- Gender : Female
- Date of Birth : 23-06-1988
- Languages Known : Hindi, English, Punjabi
- Hobbies : Playing Badminton
  Solving Sudoku
  Listening to music