GENERAL FRAMEWORK OF COMPRESSIVE SAMPLING AND ITS APPLICATION TO RADAR SIGNALS

A Dissertation Submitted in Partial Fulfillment of the Requirements

For the Degree of

Master of Technology

In

Electronics & Communication Engineering

Under the Supervision of

Prof. Dr. Sunil Vidya Bhooshan

(HOD)

By

Prabhat Thakur (132014)

Jaypee University of Information Technology
Waknaghat, Solan – 173234, Himachal Pradesh

May 2015

DECLARATION

I certify that

a. The work contained in this dissertation has been done by me under the guidance of my supervisor.

b. The work has not been submitted to any other organization for any degree or diploma.

Date: **PRABHAT THAKUR**

EN. NO. – 132014

Place **Sign______________________**

CERTIFICATE

This is to certify that dissertation entitled **"GENERAL FRAMERORK OF COMPRESSIVE SAMPLING AND ITS APPLICATION TO RADAR SIGNALS"**, submitted by **"PRABHAT THAKUR"** in partial fulfillment for the award of degree of Master of Technology in Electronics & Communication Engineering to Jaypee University of Information Technology, Waknaghat; Solan has been carried out under my supervision.

This work has not been submitted partially or fully to any other University or Institute for the award of this or any other degree or diploma.

Date: **Supervisor:** Prof. S.V. Bhooshan

(HOD)

Place **Sign**______________________

Acknowledgement

First and foremost, I would like to express my deep sense of gratitude to my supervisor **Prof. Dr. Sunil Vidya Bhooshan** for providing excellent guidance during my research and study at Jaypee University of Information Technology, Waknaghat, Solan (H.P). Their perpetual energy, motivation, enthusiasm and immense knowledge inspired me to discipline myself in efficiently executing my multiple responsibilities simultaneously. In addition, they were always accessible and willing to help me with my queries and doubts during my research.

I would like to thank **Mr. Salman Raju Talluri**, who encouraged me to work consistently for my dissertation and helped me at every moment. Due to very friendly nature I always felt free to access him with my problems .With their support, I was able to collect the building data and proceed for my dissertation.

I wish to express my gratitude and high regards to **Dr. Davinder Singh Saini** M.Tech Coordinator of ECE department, JUIT.

I would like to express my profound sense of gratitude to all the faculty members for their valuable suggestion and encouragement throughout my course especially **Mr. Tapan Jain and Prof. Dr. Ghanshyam Singh.**

I would also like to thank my family and my friends especially my colleagues for their valuable suggestions and undue support which has been a constant source of encouragement in all educational pursuits.

Date:

Place Prabhat Thakur

Table of Contents

List of Figures

List of Tables

Abbreviations

ADC	Analog to Digital Converter
AIP	Analog to Information Converter
BPF	Band Pass Filter
CR	Compression Ratio
CS	Compressive Sampling
DC	Direct Current
DSP	Digital Signal Processing
IR	Information Rate
LFM	Linear Frequency Modulation
LFSR	Linear Feedback Shift Register
MLS	Maximal Length Sequence
NR	Nyquist Rate
PN	Pseudo Noise
PRI	Pulse Repetition Interval
RADAR	Radio Detection and Ranging
Rx	Receiver
Rerr	Recovery Error
RIP	Restricted Isometric Property
Tx	Transmitter
SL	Sparsity Level
SNR	Signal to Noise Ratio
UWB	Ultra wideband Signals

Abstract

Today we are moving towards the era of digital devices i.e. we want to replace every analog device with a suitable digital device. Digital devices operate on the digital signals but most of the times origination of the signal is in analog form. So we need to convert analog signal to digital signal before processing by using Analog to Digital Converter (ADC) devices. But analog to digital conversion is bounded by Nyquist criteria i.e. number of samples to represent an analog signal should be at least twice of the maximum frequency of the signal. This bottleneck criterion prevents us to use digital signal processing for the devices which operate on high frequencies like Radar (Radio Detection and Ranging). Compressive sampling allows us to recover the original analog signal with very less number of samples as compared to the number of samples given by Nyquist rate. But compressive sampling can be implemented on the sparse signals. Radar signals can be made sparse in dictionary domain due to prior knowledge of the transmitted waveform at the receiver. So we used compressive sampling technique for radar signals.

In this dissertation, we provided the general framework of compressive sampling for the arbitrary one dimensional and two dimensional signals. We analyzed the effect of sparsity level and noise level on the recovery of an arbitrary signal. Further we provided the framework of compressive sampling for Radar Signals. In radar systems popular waveforms are linear frequency modulated waveforms which are not sparse in frequency domain so we used waveform matched dictionary so that we can apply compressive sampling on radar signals. Further we proved that the elements of waveform matched dictionary shows orthogonal nature with each other which is required. Further we analyzed the effect of sparsity level and noise on the recovery of radar signals.

The purpose of this dissertation is to understand the basics of compressive sampling and how it can be implemented on the signals which are sparse in some domain.

CHAPTER 1

INTRODUCTION

Radar is an electronic system which is used to detect and locate the objects. Objects are called as targets. “Radar” is acronym for radio detection and ranging. Radar’s basic principle relies on the properties of electromagnetic waves as they interface with physical objects. Electronic waves could be reflected by physical objects. This property allows radar to detect the targets by sensing the presence of reflected wave from the target. Initial forms of radar are used to detect the target and to know the distance of target from the radar. The distance of target from the radar is known as “Range” of the target. The process of detecting the target is known as Detection. Advance radar system performs a number of functions beside the detection and the range measurement. As we see in Figure.1.1, radar performs all function by processing the received waveform or echo. Initial radar systems use analog signal processing for functioning. But due to popular nature of digital signal processing, in modern radar systems we are trying to replace analog signal processing with digital signal processing.

Principle: Radar principle is based on electromagnetic waves.

Basic block diagram of radar systems is shown in Figure 1.1. It consists of a transmitter connected to the transmitting antenna to covert voltage current waves into electromagnetic waves and a receiver which is connected to receiving antenna for receiving the echo signal reflected form the target. In general, target is the part of channel between transmitter and receiver.

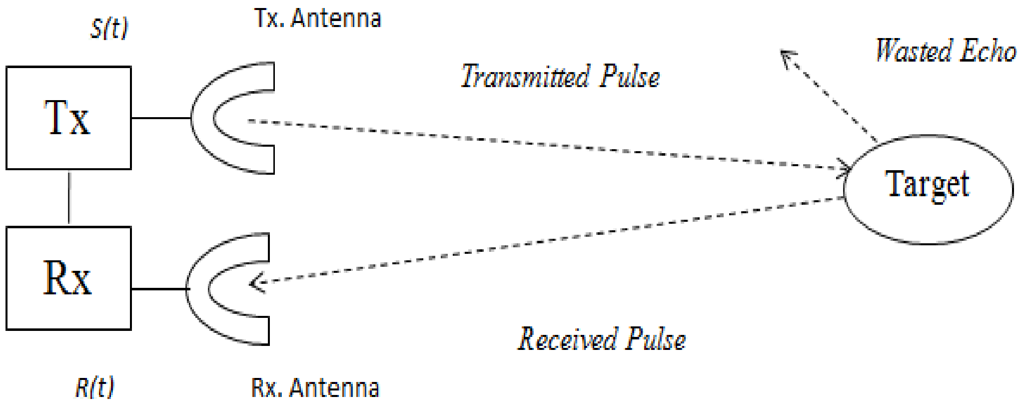

Figure 1.1 Basic block diagram for radar [1]

In the basic radar, waveform *S(t)* is produced at output of the transmitter and transmitting antenna converts this waveform into an electromagnetic waves so that it can travels through channel(free space). Electromagnetic waves travel with the speed of light. Receiving antenna is used to receive the echo signal reflected form the target at distance R from the radar. Total time taken by electromagnetic wave to travel from radar to target and come back from target to radar is called round trip time. Received electromagnetic wave at receiving antenna is converted into received waveform *R(t)*.Receiver circuit conforms the presence of target by observing the presence of receiving waveform *R(t)*. Receiver circuit further process the received waveform *R(t)* to achieve more knowledge about target like range, velocity etc.

Radar basically operates in two modes i.e. Pulsed mode or Continuous mode. In pulsed mode, radar transmits the waveform in the form of pulses and expects the reflected echo from the target in a limited time frame called as pulse repetition interval (PRI).Total time for which we transmit the signal is called as pulse width denoted as τ. If we operate the radar in pulsed mode, it is called as pulsed wave radar.

Radar which uses continuous waveform is called as continuous radar. Pulsed radar is useful when we have to find that how much far the object is i.e. range measurement. To detect the speed of the moving targets we use continuous wave radar. Generally for range measurement and velocity measurement we use two different mode of operation. We can use single mode for determination of both range and velocity measurement if we use waveform modulation. Considering the case of pulsed wave radar, for better range resolution between two targets pulse width should be small whereas for better received signal strength pulse width should be large. But we can't meet with both requirements at same time. We can solve this problem using pulse compression. One technique is to modulate the pulsed waveform with high frequency so that effective pulse width can be reduced to get better resolution when actual pulse width is large to transmit more energy. Thus in radar applications, use of linear frequency modulated (LFM) waveform and phase coded waveform became popular due to the regions discussed above [1-3].

1.1 Motivation

In initial form of the radar, we used analog signal processing to achieve the knowledge about the target. But in modern radar systems we are trying to replace analog signal processing with digital signal processing because of its advantages over analog signal processing. But one major bottleneck for digital signal processing is analog to digital conversion at very high frequencies. Operating frequency band for radar signals is given in table 1.1. Where commonly used bands are L, S, C and X.

Table 1.1 Radar Frequency Bands [1]

Band designation	Nominal Frequency Range
L	1-2 GHz
S	2-4 GHz
C	4-8 GHz
X	8-12 GHz
Ku	12-18 GHz
K	18-27 GHz
Ka	27-40 GHz

Conventional sampling system follows Nyquist Criteria i.e. a signal can be completely represented with its samples if sampling rate is greater than or equal to twice of the maximum frequency present in the signal [4].

$$f_s \geq 2f_m$$

Where $f_s - sampling\ rate$

AND $f_m - maximum\ frequecy\ of\ the\ signal$

So if we sample the signal of 1 GHz using above criteria. So number of samples required $2 \times 10^9 \frac{samples}{sec}$and 1 sample represented with 8 bits. So data rate will be 2 GB/sec. Storage and processing of such large number of samples is very difficult or sometimes infeasible. If somehow we can overcome the Nyquist Criteria on required number of samples for given signal, then above problem can be resolved.

1.2 Problem Statement

We can apply Digital Signal Processing (DSP) on Radar Signals with the help of new born technique known as compressive sampling. Compressive sampling is a technique which allows us to recover original signal by using very less number of samples as compared to number of samples given by Nyquist Criteria if signal is sparse in some domain like time domain ,frequency domain etc.

Quadrature sampling plays important role in radar systems due to required coherence nature to know the velocity of the target. It is also known as “Digital Quadrature Demodulation”. Quadrature sampling is a technique which generates samples of the In Phase and Quadrature Phase components of the signal.

In this dissertation we provided a general framework of compressive sampling and its applications for radar signals considering quadrature sampling. We also considered the effect of noise on the reconstruction of the signal.

1.3 Organization of Dissertation

Complete dissertation is organized as below. In Chapter 1, we provide introduction to radar systems. Further we introduce motivation and problem statement. Chapter 2 provides the general framework of compressive sampling. In chapter 3 we discussed about new architecture of digital quadrature demodulation or quadrature sampler. Chapter 4 provides overview of PN sequences. Chapter 5 is studies on combination of compressive and quadrature sampling. In chapter 6 we discussed simulation results. In chapter 7 we concluded the dissertation.

CHAPTER 2

GENERAL FRAMEWORK OF COMPRESSIVE SAMPLING

2.1 Introduction

Today we are moving towards digital domains, but origination of a signal is in analog domain most of the times. So analog to digital conversion systems are required but these systems follow some criteria i.e. sampling frequency should be greater than twice of the analog signal frequency (Shannon Nyquist Criteria) [4]. But if the frequency of signal will be very high then it will be very inconvenient to use Nyquist criteria because number of samples will be very large. Storage and processing of such large number of samples will be costly or sometimes infeasible.

But if somehow we can overcome a Shannon Nyquist criterion, i.e. if we can reconstruct original signal by using very less number of samples as compared to the Nyquist criteria, then problem of storage and processing of large data can be solved. This problem may be solved by compressive sampling [5-8], a random approach if signal is sparse in some domain.

Compressive Sampling uses very less number of samples as compared to Shannon Nyquist rate which reduces the hardware and software loads and then signal is recovered by using various recovery mechanisms [8-16]. Compressive sampling uses a random matrix to form out linear random projections of signals with most of the desired information. It is possible due to two properties of signal i.e. sparsity and incoherence [17]. Sparsity refers to the property of signal according to which information present in signal is very less as compare to the bandwidth occupied by the signal. Incoherence is property of sparse signal to get transform into desired domain. Desired domain is the domain in which signal is sparse. If signal will be more sparse i.e. low sparsity level, its reconstruction will be better as compared to less sparse signal. Basically incoherence refers to not coherent i.e. the dictionary(domain) elements should be independent to the sampling matrix.

The chapter is organized as follows. In section 2.1 we discussed about introduction. Section 2.2 provides some background on Compressive Sampling, Mathematical Model and Signal Reconstruction by solving Optimization problems. In Section 2.3 we use compressive sampling for signal and image compression and its successful reconstruction. Section 2.4 presents logic behind two dimensional signal compression and recovery. In section 2.5 we consider effect of noise. Section 2.6 deals with the simulations and results whereas in section 2.7 we conclude the chapter.

2.2 Background

2.2.1 Compressive Sampling

Emerging theory of compressive sampling (CS) allows us to project random measurements of signal of interest so that we can sample the signal at information rate rather than its ambient data rate. This reduces the number of samples to represent a signal. Reduced number of samples can be stored easily and processing of such small number of samples can be performed efficiently. But to apply compressive sampling on the signal, signal should be sparse and incoherent.

Sparse Signal: A signal is said to be sparse if only some of the components have significant magnitude and all other components have insignificant magnitude i.e. closer to zero.

Incoherent Signal: One signal is said to be coherent with respect to other signal if they have no relation.

2.2.2 Mathematical Approach for Compression

Consider a signal r which is sparse. r is said to be sparse if it can be represented as a linear combination of basis functions where some of the coefficient's magnitude is significant and all others have zero magnitude.

$$\boldsymbol{r} = \boldsymbol{\psi c} \tag{2.1}$$

$\boldsymbol{\psi}$– Basis Functions

$\boldsymbol{c}$ -Basis Coefficients

For compressive sampling a random matrix or sampling matrix**Ø** need to project random projections or random measurements

$$\boldsymbol{b} = \emptyset \boldsymbol{r} \tag{2.2}$$

where $\boldsymbol{b}$ – Random Measurement vector

2.2.3 Reconstruction

Now we need to reconstruct back r form b

$$\boldsymbol{b} = \emptyset \boldsymbol{\psi c} \qquad using(2.1) \tag{2.3}$$

By solving above equation we can find basis coefficients **c**. Information of c leads us towards recovery solution

$$\boldsymbol{r} = \boldsymbol{\psi c} \tag{2.4}$$

2.2.4 Optimization Problem Formulation

The equation we have to solve i.e. (2.3) is an underdetermined system as number of equations is less than number of unknowns. So we need to use norm minimization techniques to solve above problem.

Mathematically norm provides total size or positive lengths of all vectors in a vector space or matrices.

Generally Norm n of vector x is defined as: $||x||_n = \sqrt[n]{\sum_i ||x||^n}$ Where, n ∈ R

Frequently using norms are l0, l1, l2 but here we use l1 norm.

l1 norm **:** l1 norm is defined as : $||x||_1 = \sum_i |x|$

l1 optimization problem is formulated as

$$min||\boldsymbol{x}||_1 \quad subject\ to\ \ |\boldsymbol{Ax} = \boldsymbol{b}| \tag{2.5}$$

Above problem can be solved using least square optimization

$$\boldsymbol{x} = \boldsymbol{A}^{+}\boldsymbol{b} \tag{2.6}$$

Where $\boldsymbol{A}^{+} - \text{Psuedoinverse of } \mathbf{A}$

Even though this method is easy to compute but it is not necessary that it provides best solution [18].That is why we use l1 norm optimization.

So our optimization problem is formulated as:

$$min\left|\left|\boldsymbol{c}\right|\right|_1 \quad subject\ to \quad |(\,\emptyset\boldsymbol{\psi})\boldsymbol{c} = \boldsymbol{b}\ | \tag{2.7}$$

2.2.5 l1 Optimization Solution

l1 optimization problems can be solved by using number of alogorithms.one among them is greedy type-orthogonal matching pursuit[19]

2.2.5.1 Greedy Type - Orthogonal Matching Pursuit, Basis Pursuit

Orthogonal matching pursuit is a greedy-type algorithm as it selects the one index regarded as the optimal decision after each iteration.

Basis pursuit is a technique where signal is decomposed into an optimal superposition of dictionary elements. Optimization criteria is l1 norm of coefficients

2.3 One Dimensional Signal Compression and Recovery

Reduced load on hardware and software leads us to use compressive sampling in all possible fields. e.g. Signal compression, Image compression, Speech compression, audio and video compression, wireless sensor networks etc. But here we apply compressive sampling on one dimensional signal.

2.3.1 Signal Compression and Recovery

Many times we deal with one dimensional signals such as audio signals speech signals and we need to sample these signal for performing some digital operation on these signal. Less number of samples can be processed easily with reduced processing time. So we go for compressive sampling of such signals if they are sparse. Complete recovery of signal relies on sparsity level (SL) and compression ratio (CR). Sparsity level is number of components having significant magnitude. Compression ratio is with respect to the ratio that up to what level we have compressed the signal e.g. N/10 where N is total number of samples present in the signal.

If sparsity level is low, recovery will be better.
If compression ratio is more, recovery will be better.

Consider a one dimensional signal r having length n $r_{n\times 1}$

$r_{n\times 1}$can be represented with the help of basis functions and its coefficients

$$r_{n\times 1} = \psi_{n\times n} c_{n\times 1} \tag{2.8}$$

Where $\psi_{n\times n}$ - $n \times n$ matrix of basis function

$c_{n\times 1}$ - $n \times 1$ vector of basis coefficients

For random measurements after random sampling we use measurement matrix $\emptyset_{m*n}$

$$b_{m\times 1} = \emptyset_{m\times n} r_{n\times 1} \tag{2.9}$$

$\emptyset_{m\times n}$ – Measurement Matrix .

$$b_{m\times 1} = (\emptyset_{m\times n}\psi_{n\times n})c_{n\times 1} \qquad using\ (2.8)$$

Above equation needs to solved using l1norm optimization.

l1 norm optimization problem is formulated as :

$$\min||c_{n\times 1}|| \qquad subject\ to \qquad (\emptyset_{m\times n}\psi_{n\times n})c_{n\times 1} b_{m\times 1} \tag{2.10}$$

Solution to above equation is in the form of a vector$c_{n\times1}$. Once coefficient vector $c_{n\times1}$ is available to us, reconstruction becomes possible.

Reconstruction using above solution

$$\widehat{r_{n\times1}} = \psi_{n\times n}c_{n\times1} \quad (2.11)$$

2.3.2. Recovery Error($Rerr$)

Recovery error gives us the error for successful recovery and defined as:

$$Rerr = ||r - \hat{r}||_2 \quad (2.12)$$

2.4 Two Dimensional Signal Compression and Recovery

Two dimensional signal like image can also be compressed using its Fourier or wavelet domain where image shows some sparse nature. Mathematical approach for image remains same as for signals but we choose basis functions either on Fourier or Wavelet domain .Instead of a one dimensional vector we deal with a two dimensional matrix as image is a two dimensional signal.

2.5 Effect of Noise on Recovery Error

Noise is an undesired signal that may affect the performance of the system. So we analyzed the effects of noise on our system of compressive sampling i.e. how recovery error is going to vary with respect to noise.

For compressive sampling noise may affect the sampled values and mathematical equation for sampled signal will be defined using eq. (2.8)

$$b_{m\times1} = \text{Ø}_{m\times n}r_{n\times1} + n_{n\times1} \quad (2.13)$$

Where $n_{n\times1} - noise\ vector$

Recovery procedure will be same as in eq. (2.10) i.e. we need to solve l1-norm minimization problem

$$\min ||c_{n\times 1}||_1 \qquad such\ that \qquad \left||b_{m\times 1} - (\emptyset_{m\times n}\psi_{n\times n})c_{n\times 1}|\right|_2 \leq \varepsilon \qquad (2.14)$$

But due to addition of noise, values of vector **b** gets change. Thus above problem gets solve with affected value of b. So value of coefficient vector **c** also varies from the desired values. By this way reconstruction or recovery gets affected.

2.6 Simulation and Results Discussion

For implementation of all algorithms, we used Matrix Laboratory on a standard computer.l1- magic toolbox is used to achieve the solution of l1-norm optimization problems.

2.6.1 Signal Reconstruction

We considered a signal in time domain and make it sparse in frequency domain by taking all frequency domain coefficients zero which are below some threshold value. Here threshold value is assumed as one fifth of the maximum amplitude of the coefficients. We used same procedure used for signal compression and recovery of original signal in section 2.3.1. We sampled the signal using sampling rate which is ten times less than Nyquist rate and successfully reconstructed the original signal as shown in Fig. 2.1. Further we sampled the signal with sampling rate twenty times less than the nyquist rate. Original and reconstructed signal is shown in Fig 2.2. Here we varied the sparsity level of signal and analyzed its results on recovery error. In addition to this we also analyzed the effect of variation of compression ratio on recovery error. Compression ratio is defined as ratio of the difference between total number of samples before compression and the total number of samples after compression to the total number of samples after compression and resultant value need to multiply by hundred.

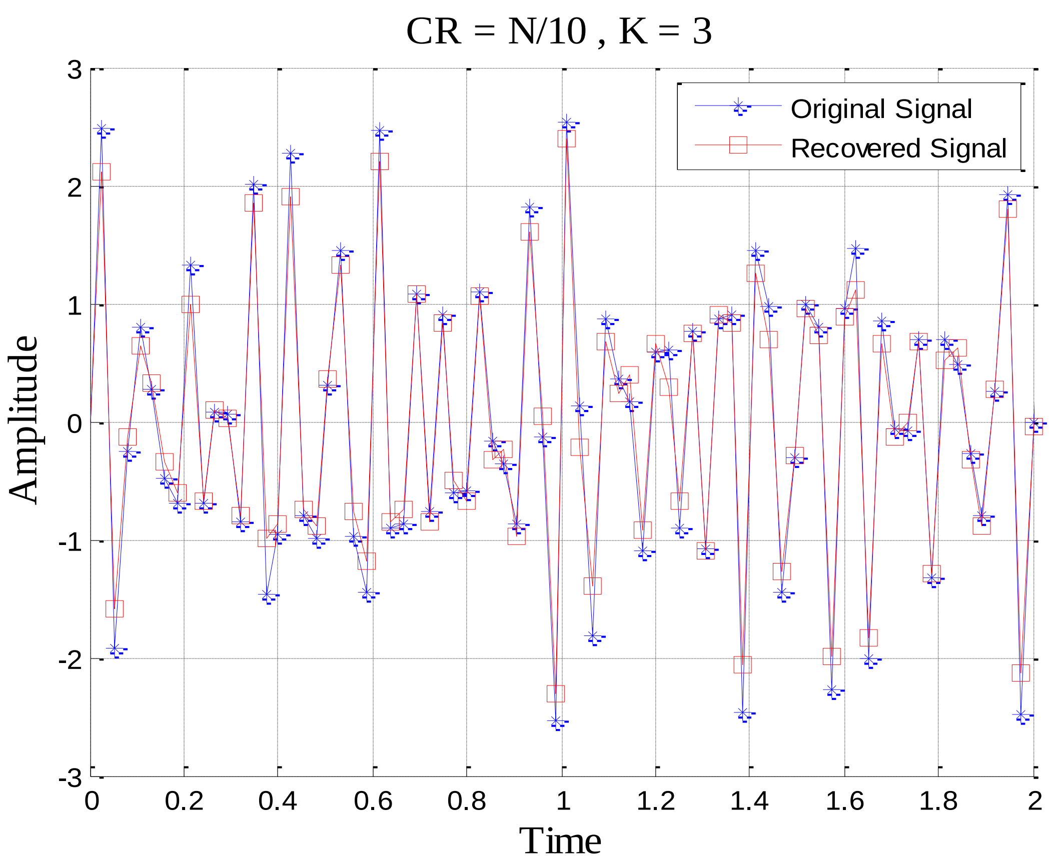

Figure.2.1 Original signal versus Recovered Signal

Fig.2.3. explains the behavior of recovery error with variation in compression ratio for different sparsity levels. Here total number of samples (N) we consider are 751.It is clear that with increase in compression ratio, recovery error increases. If sparsity level will be more, recovery error will be more. So it conforms that the compressive sampling can be implemented on the signals having sparse nature otherwise recovery or reconstruction cannot be done successfully. Recovery error relies on sparsity level and compression ratio. If signal is more sparse, can be compressed more and can be reconstructed successfully.

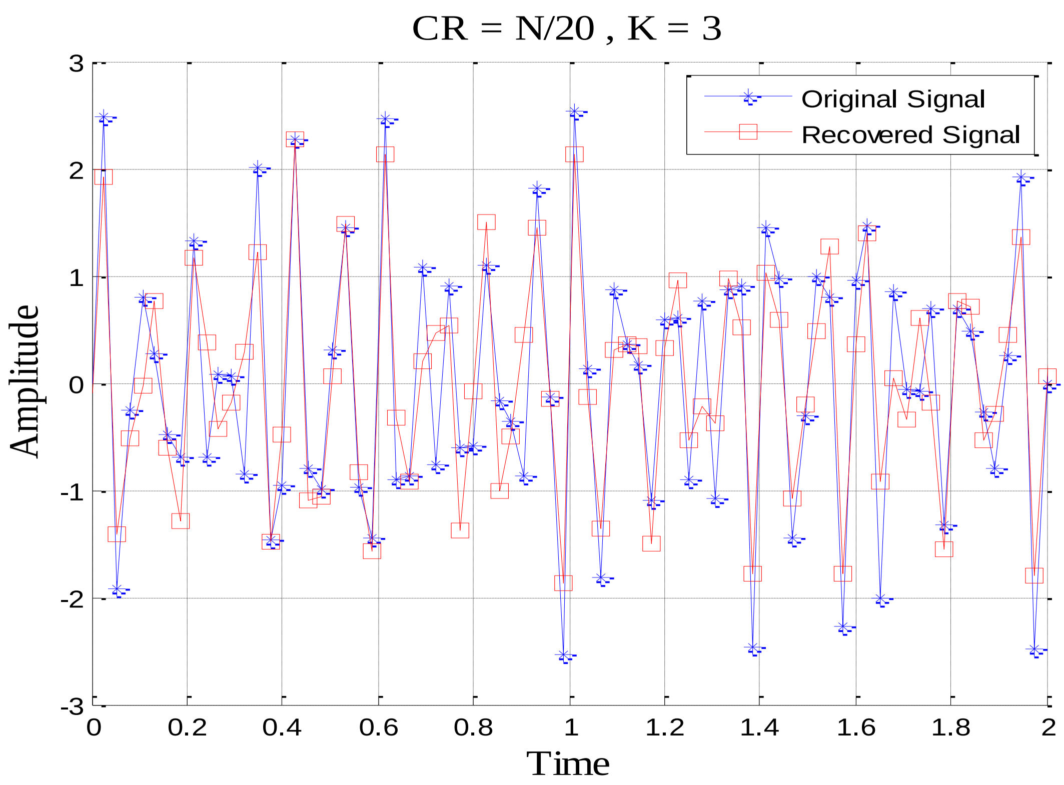

Figure 2.2 Original signal versus Recovered Signal

Table 2.1 Effect of compression ratio and sparsity level on Recovery error

NOS / SL	N/5	N/6	N/7	N/8	N/10	N/12	N/15	N/20
3	0.0928	0.1126	0.1330	0.1596	0.2049	0.2499	0.3285	0.4072
4	0.1340	0.1340	0.1840	0.2440	0.3384	0.4030	0.4813	0.8336
5	0.1866	0.1866	0.2366	0.2766	0.3484	0.3820	0.4229	0.7624
10	0.2338	0.2733	0.3340	0.4058	0.7284	0.9220	1.1342	1.3432

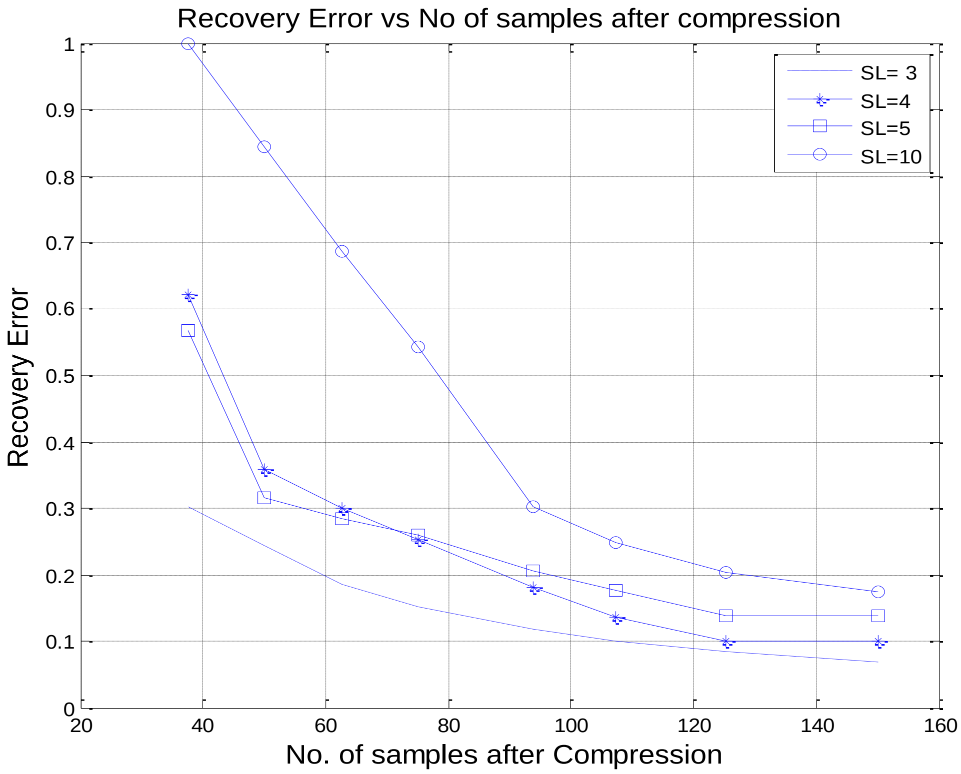

Figure.2.3 Recovery error versus No of samples after compression with different sparsity level

2.6.2 Effect of Noise on Recovery Error:

We have seen in section 2.5 that how noise affects our recovery error. Here we analyzed the effect of noise on the recovery error for different values of sparsity level.

Table 2.2 Effect of noise on Recovery Error for different Sparsity Level

SL=3

SNR3(dB)	49.62	29.62	9.62	3.60	.085	-2.41	-4.3515	10.3721
Rerr3	7.8014	8.01	10.42	13.86	17.684	22.33	26.3561	49.30]

SL=5

SNR5(dB)	51.44	31.44	11.24	5.42	1.90	-0.59	-2.33	-8.55
Rerr5	19.78	19.57	22.57	27.68	31.68	36.78	41.78	58.78

SL=7

SNR7(dB)	54.48	34.01	14.48	8.96	4.94	2.44	0.50	-5.51
Rerr7	28.84	29.01	31.25	32.75	37.13	40.07	43.13	61.06

SL=8

SNR8(dB)	54.97	34.97	14.97	8.95	5.43	2.93	0.99	-5.022
Rerr8	33.15	32.49	32.488	35.06	39.79	43.16	48.10	64.88

SL=10

SNR10(dB)	57.85	37.85	17.85	11.85	8.30	5.81	3.87	-2.14
Rerr10	40.25	40.05	40.49	43.60	46.26	49.70	52.00	70.48

It is observed that as signal to noise ratio (SNR) is going to increase, recovery error decreases for certain threshold value of SNR. Once SNR crosses certain threshold value, *Rerr* becomes constant.

We also analyzed recovery error for different values of sparsity level considering noise effects. We found the expected results i.e. as sparsity level increases, recovery error also increases.

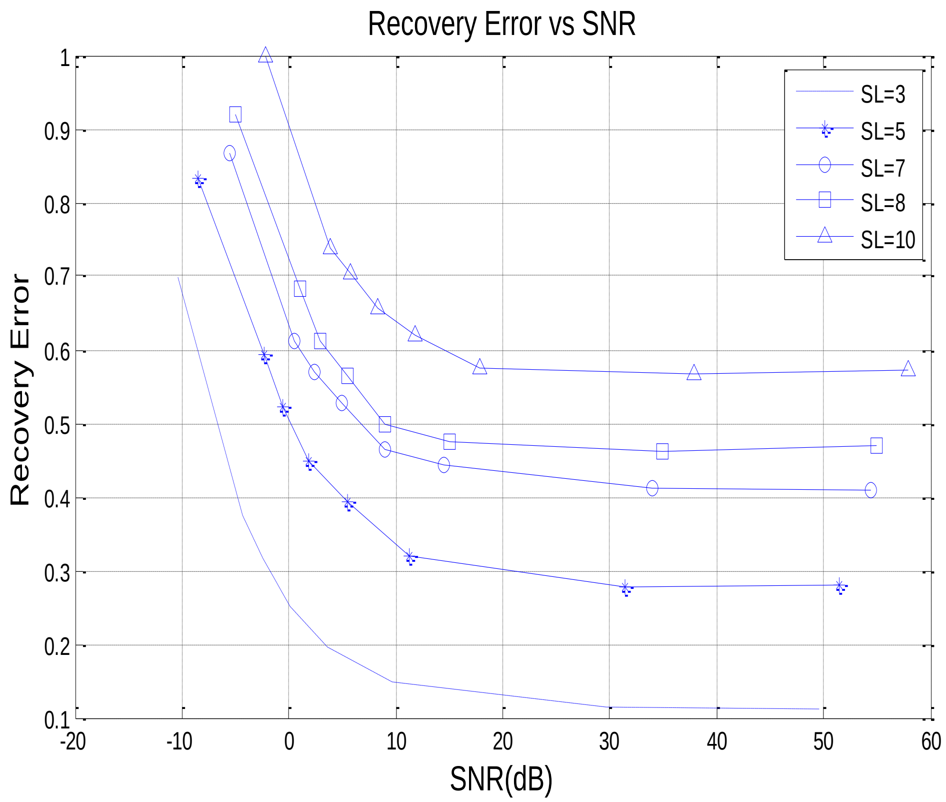

Figure 2.4 Effect of SNR on Recovery Error

2.7 Conclusion

Compressive sampling appears to be a revolutionary technique for data acquisition and successful reconstruction. We implemented this technique for one dimensional signal as well as two dimensional signal and successfully recover them from compressive random measurements. We analyzed recovery error due to variations in sparsity level and compression ratio and assured that successful reconstruction of signal relies on sparsity level and compression ratio. Effect of noise also considered in compressive sampling and we verified that with increase in SNR, Recovery error decrease that is according to our system expectations.

CHAPTER 3

QUADRATURE SAMPLING

3.1 Introduction

Quadrature sampling is a sampling technique through which we gets sampled In phase and Quadrature phase components of a signal with dc level near origin. For a band pass signal, it is initially converted into baseband signal and then In phase and quadrature phase components get derived using complex envelope. Complex envelope of the band pass signal is low pass equivalent of complex equivalent of band pass signal. [20]

A band pass signal can be represented as:

$$s(t) = A(t)\cos\big(w_c t + \emptyset(t)\big) \qquad (3.1)$$

Where $A(t) - represents\ Amplitude\ Modulation$

$w_c \quad - \ represents\ Carrier\ frequency$

$\emptyset(t) \ - \ responsible\ for\ phase\ or\ frequency\ modulation$

Complex equivalent of band pass signal is given by

$$s(t) = A(t)\exp\big(\,j\big(w_c t + \emptyset(t)\big)\,\big) \qquad (3.2)$$

Complex envelope or low pass equivalent of band pass signal is achieved by demodulating the signal with respect to carrier frequency.

$$s_c(t) = A(t)\exp(j(\text{Ø}(t))) \tag{3.3}$$

In phase and Quadrature phase components can be derived from the complex envelope by considering real and imaginary parts. Real part is considered as In phase whereas imaginary part as Quadrature phase component.

$$s_I(t) = real\big(s_c(t)\big) = A(t)\cos\big(\text{Ø}(t)\big) \tag{3.4}$$

$$s_Q(t) = Imag\big(s_c(t)\big) = A(t)\sin\big(\text{Ø}(t)\big) \tag{3.5}$$

Where $s_I(t) - \ In\ phase\ component$

$s_Q(t) - \ Quadrature\ phase\ component$

To get digital versions of In phase and Quadrature phase components $s_I(t)$ & $s_Q(t)$ need to sample using Nyquist sampling rate.

$$s_I[n] = s_I(nT_s) \tag{3.6}$$

$$s_Q[n] = s_Q(nT_s) \tag{3.7}$$

Where $s_I[n] \ - \ Digital\ In\ phase\ \ Component$

$s_Q[n] - \ Digital\ Quadrature\ phase\ Component$

This complete process of translating the received band pass signal to baseband signal and deriving In phase and Quadrature Components is also known as Quadrature demodulation.

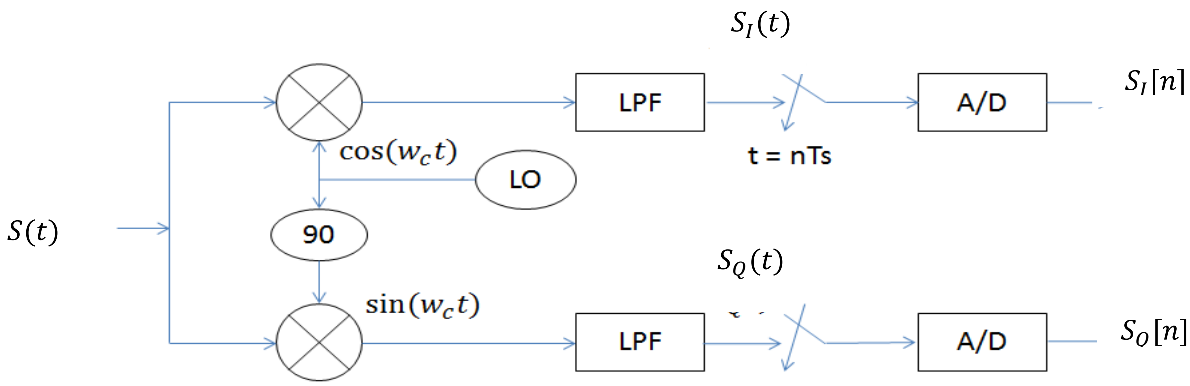

Figure 3.1 Basic Architecture of Digital Quadrature Demodulator

Here system multiplies given signal $s(t)$ with $\cos(w_c t)$&$\sin(w_c t)$ to take out base band outputs with 90 degree phase difference. Filter output gives us In Phase and Quadrature Phase components. Sampling stage generates samples of In Phase and Quadrature Phase Components

Applications:

- Coherent systems [21]
- Reduce the effect of noise [22]

In phase and Quadrature phase components play a very important role in various applications, like they can be used in phase detection of the given signal.

$$s_{phase} = tan^{-1}\left(\frac{r_q[n]}{r_I[n]}\right) \tag{3.8}$$

Where $s_{phase} - phase\ of\ s$ signal.

In case of coherent systems, phase of every waveform or signal should be known and related so that phase difference can be used for certain calculations.

Noise is an undesired signal which degrades the performance of system by affecting desired signal. Noise is independent for real and imaginary parts. So noise can affect real and imaginary parts of desired signals independently. Due to presence of in phase and quadrature phase components simultaneously effect of noise can be reduced on the signal of we consider less affected part.

3.2 Drawbacks of Basic Approach

This approach to take out In phase and Quadrature phase components is very popular and extensively used in different applications. But one serious bottleneck with this approach is imbalance problem due to occurrence of severe phase and gain errors. These errors have been investigated in [23] and phase errors may be up to 2 to 3 degree [24].This high amount of error is intolerable in some applications of signal processing. So other quadrature demodulation approaches are required that can overcome these problems.

3.3 New Architecture of Quadrature Sampling

A band pass signal can be represented by

$$s(t) = A(t)\cos\big(w_c t + \emptyset(t)\big) \tag{3.9}$$

$$s(t) = A(t)\cos(w_c t)\cos\big(\emptyset(t)\big) - A(t)\sin(w_c t)\sin\big(\emptyset(t)\big) \tag{3.10}$$

$$s(t) = I(t)\cos(w_c t) - Q(t)\sin(w_c t) \tag{3.11}$$

Where $I(t) = A(t)\cos\big(\emptyset(t)\big)$ &

$Q(t) = A(t)\sin\big(\emptyset(t)\big)$

Given that the signal bandwidth is much smaller than the carrier frequency, the input signal is usually translated to a certain IF frequency before processing. This down conversion step involves mixing the analog input with a local oscillator at IF frequency and then low pass filtering to remove the image frequency band. A properly selected sampling rate f however, can exploit the frequency domain periodicity of the discrete Fourier transform so that the down conversion can be included in the sampling process, thereby eliminating the mixing and low pass filtering.

Let signal bandwidth is B, highest frequency is f_H and lowest frequency is f_L.

$[B = f_H - f_L]$

After sampling, *R(f)* will convolve with periodic impulses.

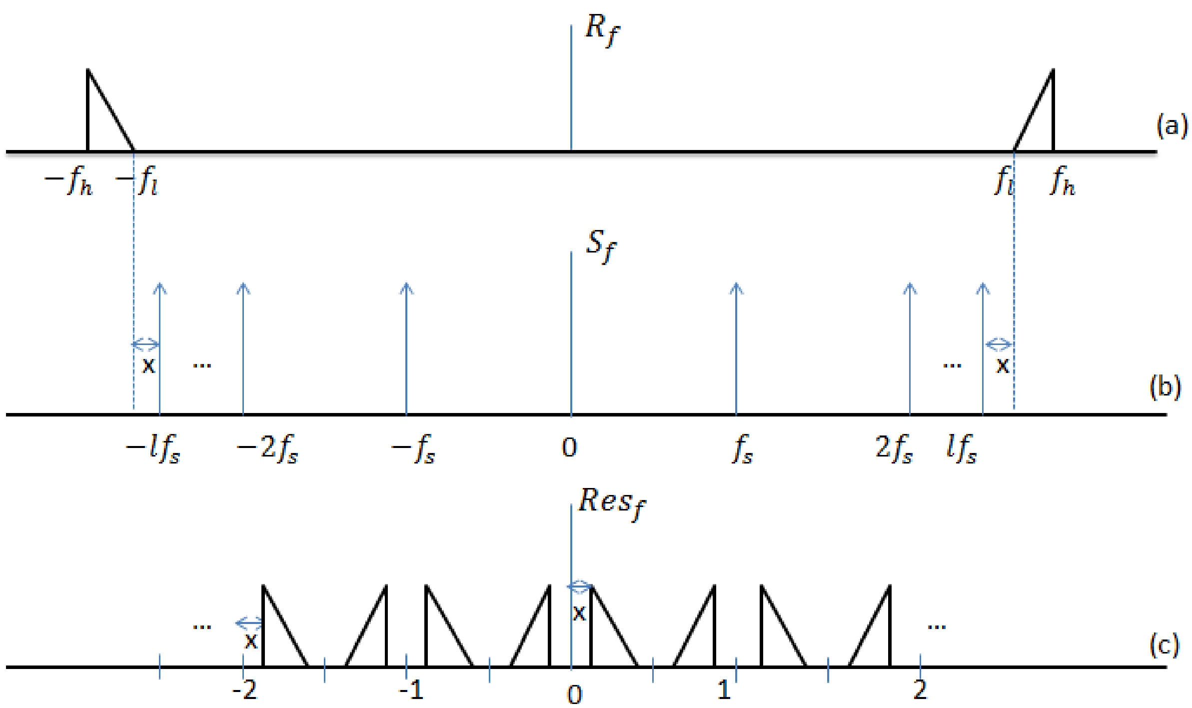

Figure 3.2(a) Input Signal Spectrum (b) Sampling waveform Spectrum (c) Resultant Signal Spectrum

To avoid aliasing $f_s > 2f_H$ such that

$$2B \leq f_s \leq \frac{f_l}{l} \tag{3.12}$$

From the figure $$f_s = 2B + 4X = \frac{f_l - X}{l} \tag{3.13}$$

$$(4l + 1)X = f_l - 2Bl \tag{3.14}$$

$$X = \frac{f_l - 2Bl}{(4l + 1)} \tag{3.15}$$

From above equations $\left(l \leq \ floor(\frac{f_l}{2B}) \right)$

From Equation (3.13) & (3.15)

$$f_s = \frac{4f_l + 2B}{4l + 1} \tag{3.16}$$

$$f_c = \ f_l + \frac{B}{2} \tag{3.17}$$

Now

$$s(t) = A(t)\cos(w_c t + \emptyset(t))$$

Can be sampled at $t = nT_s = n/f_s$

$$s(n) = A(n)\cos(2\pi\, n\, \frac{f_c}{f_s} + \emptyset(n)) \tag{3.18}$$

$$s(n) = A(n)\cos\left(2\pi n\left(l + \frac{1}{4}\right) + \emptyset(n)\right) \tag{3.19}$$

$$s(n) = A(n)\cos\left(\frac{n\pi}{2} + \emptyset(n)\right) \tag{3.20}$$

$$s(n) = I(n)\cos\left(\frac{n\pi}{2}\right) - Q(n)\sin\left(\frac{n\pi}{2}\right) \tag{3.21}$$

Substitute n=1, 2, 3,……..

$$s(n) = \begin{cases} (-1)^{\frac{n}{2}} I(n) & n - even \\ (-1)^{n + \frac{1}{2}} Q(n) & n - odd \end{cases} \tag{3.22}$$

From (3.22) it is clear that at specified value of n either *I(n) or Q(n)* is directly available from *s(n)* . But to detect other component at same time further processing is required.

That is multiply *s(n)* with $-2\sin\left(\frac{n\pi}{2}\right)$

$$\text{So } s_1(n) = -2A(n)\sin\left(\frac{n\pi}{2}\right)\cos\left(\frac{n\pi}{2} + \text{Ø}(n)\right) \tag{3.23}$$

$$s_1(n) = -A(n)\sin\big(n\pi + \text{Ø}(n)\big) + A(n)\sin\big(\text{Ø}(n)\big) \tag{3.24}$$

&

$$s_2(n) = -A(n)\sin\big(n\pi + \text{Ø}(n)\big) \tag{3.25}$$

$$s_3(n) = A(n)\sin\big(\text{Ø}(n)\big) \tag{3.26}$$

From above equations it is clear that

$$s_2(n)\ \ is\ centered\ at\ a\ frequency\ \ \frac{1}{2}$$

$$s_3(n) \text{ is centered at a frequency } 0 \text{ which } \ Q(n)$$

So by passing $s_1(n)$through LPF of cut of frequency ¼ Hz a down sampling by 2 gives samples of Quadrature Phase Components. Thus complete structure for Digital Quadrature Demodulation is presented here

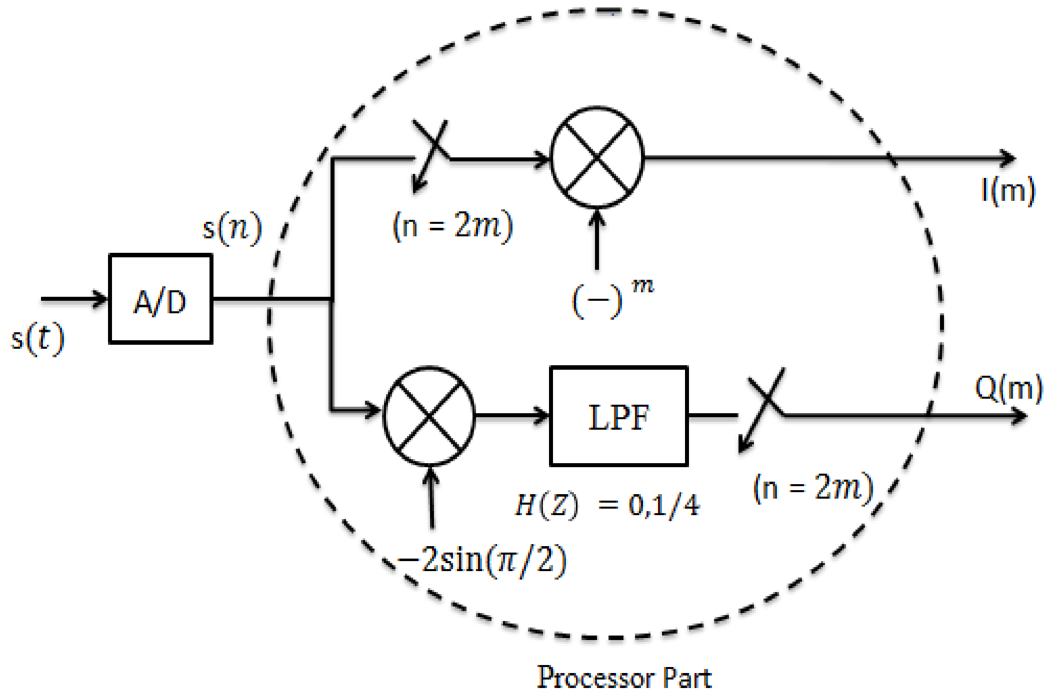

Figure 3.3 Digital Framework of Quadrature Sampling

3.4 Conclusion

We have studied about quadrature demodulation and analyzed the basic architecture used for quadrature demodulation. We find that there are some practical limits to produce the exact 90 degree phase difference which is very much required. We find new architecture in literature and analyze that it directly operates digital inputs and can provide us sampled in phase and quadrature phase components. New architecture is better as compared to basic architecture because only one ADC is required in new architecture whereas in basic architecture two ADC are required. Second benefit of using new architecture is that no need to generate two signal with 90 degree phase difference.

CHAPTER 4

PN SEQUENCES

4.1 Introduction

A pseudo random noise (PN) sequence is defined as a coded sequence of 1s or 0s with certain auto correlation properties. These sequences are called as pseudorandom noise because these sequences appear to be unpredictable to an outsider but are generated by deterministic means with some properties in mind. Pure random sequences are never periodic i.e. their bit pattern does not repeat itself. A pseudo random binary sequence is a semi random sequence in the sense that it appears random within the sequence length as it fulfills the requirements of randomness, but the entire sequence repeats indefinitely.

A PN sequence can be considered as an ideal test signal because it simulates the random characteristics of the signal and can be easily generated. 'Maximum Length Sequence' is a typical sequence among all PN sequences. It represents an ordinary using periodic sequence. A PN sequences can be generated using phase shift register with m flip-flops if desired length of sequence is m. PN sequences may also be aperiodic and such sequences are known as Barker sequences. But Barker sequences are limited due to their length up to thirteen [3,4,25,26,].

4.2 Generation of PN Sequences

Typically PN sequences are generated by using linear feedback shift register (LFSR) as shown in Fig. 4.1

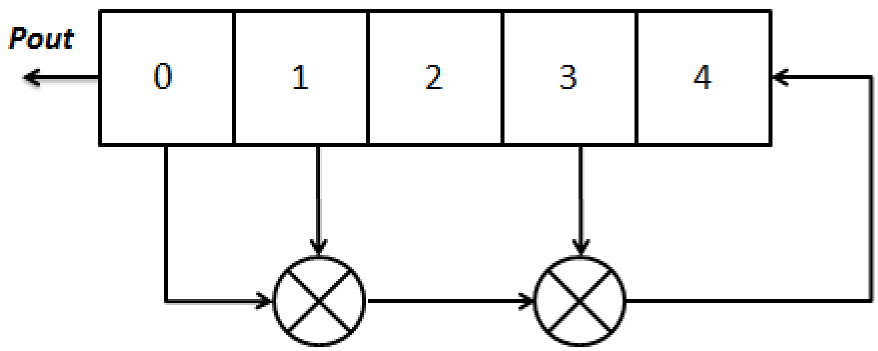

Figure 4.1 PN Sequence Generator

The PN sequence generated by using LFSR has periodic nature and maximum length

$$N_{pn} = 2^{m_{pn}} - 1$$

Where $m_{pn} - Number\ of\ staged\ in\ the\ feedback\ shift\ register.$

The sequence available from LFSR with maximum possible period is called a Maximal length sequence (MLS).

4.3 Properties of PN Sequences(MLS)

1) **Balance Property:** Number of binary one's is one more than the number of binary zero's in the maximal length sequence per period of sequence.
2) **Run Property:** Among all runs, one-half runs are of length one, one –fourth runs are of length two and so on.
3) Co-relation Property: For maximal length sequence auto correlation function is periodic and binary valued.

4.4 Frequency Spectrum of PN Sequences

PN sequences seem like noise signals and exhibits the spectrum similar to the noise signal. PN sequences have wideband spectrum. We analyzed the frequency spectrum for different PN sequences by considering them continuous function of time as shown in Fig.1.2. From Figure 1.2, It is clear that spikes appear over the entire spectrum. Further spectrum appears to be periodic over the frequency given by reciprocal of the minimum width of the pulse. This periodic nature appears because frequency transform of the rectangular pulse is *Sinc* function.

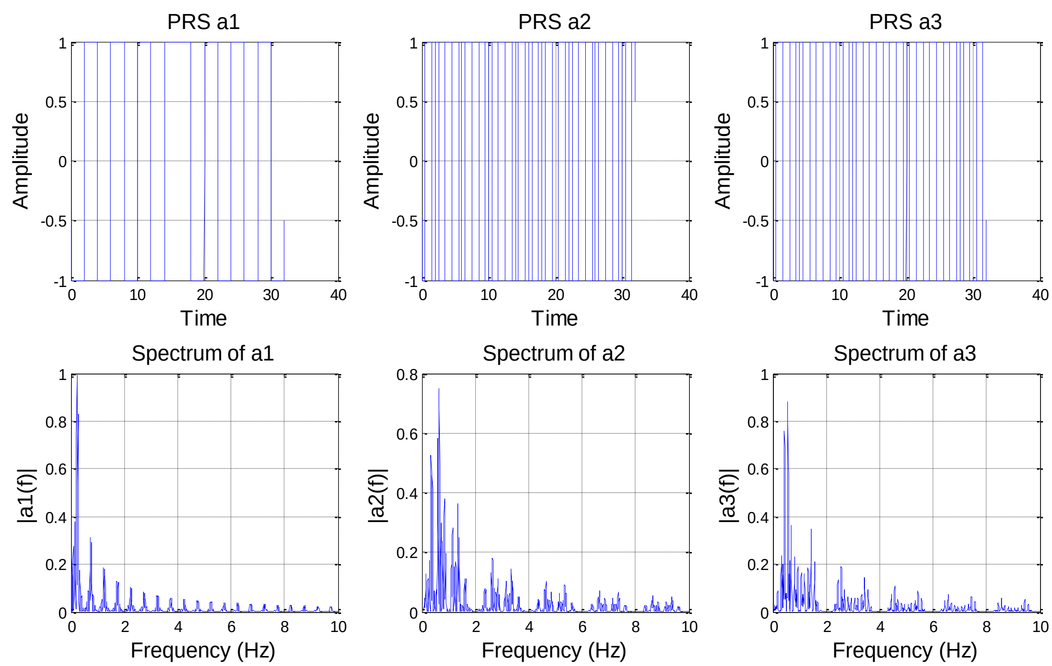

Figure 4.2 PN sequences with their Magnitude Spectrum

4.5 Effect of PN Sequence Multiplication on the Spectrum of an Arbitrary Signal

Here we try to analyze the effect of PN sequence multiplication on the Spectrum of an arbitrary signal. From Fig.1.3 it is clear that as signal is multiplied with a PN sequence with wide spectrum. The spectrum of resultant signal is shifted version of original spectrum. Thus we can say that information about signal gets smear up over the complete spectrum of PN sequence after multiplication with PN sequence. In Fig.4.3 we analyze the affect on signal with small frequency.

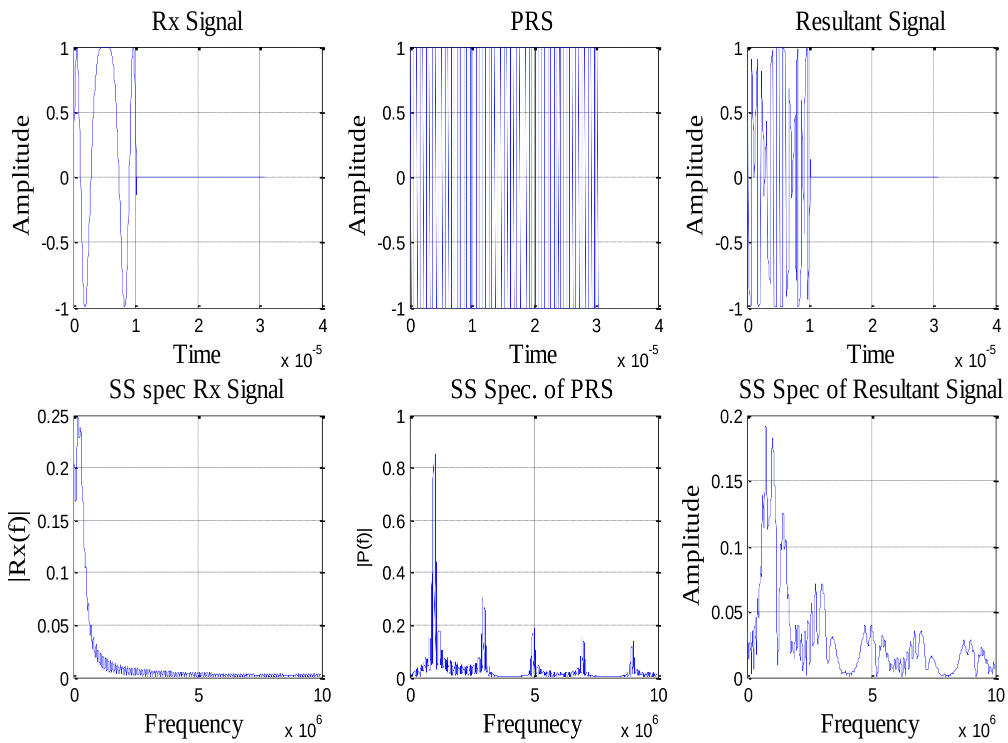

Figure 4.3 Effect of PN Sequence on the Magnitude Spectrum of Signal

4.6 Conclusion

In this chapter we discussed some basics of PN sequences and their properties .we analyzed the spectrum of the PN sequences. Further we consider an arbitrary signal and analyze its spectrum after multiplication with PN sequence. PN sequences shows periodicity over the frequency given by reciprocal of the minimum width of the pulse of sequence. This periodic nature appears due to the sinc nature of Fourier transform of the rectangular pulse.

CHAPTER 5

QUADRATURE COMPRESSIVE SAMPLING

5.1 Introduction

We have seen in chapter 2 that compressive sampling is a very effective tool to reduce the number of samples for representation of an analog signal if signal is sparse. From literature it is also clear that quadrature sampling plays important role in radar systems. Now challenge is that how to use both techniques in radar systems simultaneously. General framework of compressive sampling for ultra wide band signals is given in [27]. The output of the random demodulator [28-31] is in sampled form i.e. a digital signal. The technique of Quadrature sampling illustrates in chapter 3 uses digital signals as input and generate In phase and quadrature phase components in digital form. So if output of random demodulator applies to quadrature sampler, we get digital In Phase and Quadrature phase components in compressed form. Recovery mechanism needs to use to recover original In Phase and Quadrature Phase components.

We have analyzed that to apply compressive sampling theory on a signal it need to follow three key elements

1. Find the space in which signal has sparse representation.
2. Obtain random measurements as samples of sparse signal.
3. Reconstruct the original signal from the samples using optimization techniques.

As the number of measurements required for exact recovery relies on the sparsity level of signal representation in certain space. Thus choice of basis functions is the premise of compressive sampling theory. But bound on basis functions is that they should be orthogonal. Rauhut et al. [32] proved that signal which is sparse in redundant dictionary can still be sampled based on the CS theory. A basis or dictionary for representation of sparse signal plays important role so we want best basis so that our signal bear more sparsity. Shi et al. [33] formulate a waveform matched dictionary for radar signals on the bases of prior knowledge of transmitted waveform. This enables us to use compressive sampling for ultra wideband radar signals.

So initially we will discuss about construction of waveform matched dictionary in section 5.2 and then complete framework of quadrature compressive sampling in section 5.3

5.2 Construction of Waveform Matched Dictionary

Here we use waveform matched rules to construct the dictionary for ultra wide band signals (UWB). Frequency modulated waveform is very popular waveform used in radar applications and represented as

$$s(t) = A(t)\cos\left(w_c t + \emptyset(t)\right) \tag{5.1}$$

Where $A(t) - enevelope\ of\ the\ waveform$

In case radar system, the receiver is aware of the transmitted waveform. The transmitting waveforms are known and received waveform can be modeled as sum of various scaled and time shifted versions of transmitted waveform for stationary targets when all noise and interference effects are consider be zero. Thus knowledge of transmitted waveform act as prior knowledge required to form the dictionary. Using above considerations, we can construct a matched dictionary for radar echo signals. If *s(t)* be the transmitted signal and τ be the Nyquist sampling interval of its echo signal in theory. All time-shifted versions of transmitted waveform at integral multiples of Nyquist sampling interval τ form a redundant dictionary. Formed dictionary can be represented as ψ.

$$\psi_n(t) = s(t - n\tau_0) \tag{5.2}$$

Where τ_0 = sampling interval, T- Total observing interval.

n∈ 1, 2,….,N [N=T/τ_0]

This dictionary is complete for radar echo signals and every element of dictionary has waveform similar to transmitted waveform i.e. why dictionary is called as waveform matched dictionary waveform.

Consider the noise free environment for radar systems. When transmitted signal is $s(t)$ at the transmitter, then received echo signal at the receiver can be described as:

$$r(t) = \sum_{n=1}^{K} v_n \psi_n(t) \qquad K \neq 0$$

Where $\boldsymbol{v}$ – showing coefficient vector.

$$\boldsymbol{\psi_n}(t) - Basis\ function\ vector$$

K is the number of targets present. Thus our signal became K sparse in dictionary domain. If we will consider received signal with zero and non-zero coefficients.

$$r(t) = \sum_{n=1}^{N} v_n \psi_n(t) \qquad (5.3)$$

Where number of non-zero coefficients in coefficient vector $\boldsymbol{v}$ areK and number of zero coefficients are N – K. i.e. sparsity level in radar echo receiver is equal to the number of targets. Here $v_n$represents the amplitude of the nth target. For real radar scenario large number of elements in vector v will be zero because of limited number of target presence. The position of the target echoes can be determined by using non-zero elements of the vector v and this is required information to calculate the range of the target. Therefore it is clear that matched dictionary guarantee a successful sparse representation of echo signal and can also make echo detection easier.

5.3 Framework of quadrature compressive sampling

Basic Block Diagram is shown in Fig. 5.1

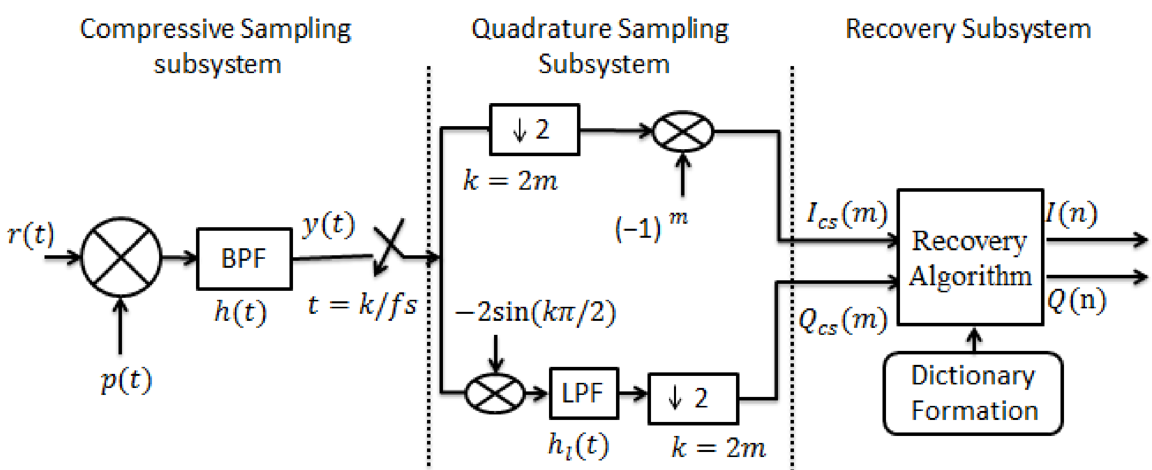

Figure 5.1 Quadrature compressive sampling system

Complete process consists of three main subsystems named as:

- Compressive Sampling Subsystem

- Quadrature sampling subsystem
- Recovery Subsystem

5.3.1 Compressive Sampling Subsystem

Compressive Sampling Subsystems works similar to random demodulator but here we use band pass filter instead of low pass filter since signal in processing is band pass one with known if frequency instead of low pass signal. Complete operation of compressive sampling subsystem is explained below.

The received signal $r(t)$ mixes with a random binary signal $p(t)$

$$p(t) = \varepsilon_k \quad , \quad t \in \left[\frac{k}{B_p}, \qquad \frac{(K+1)}{B_p}\right] \qquad where\ k = 0,1,2 \dots \dots \dots$$

$\varepsilon_k = +1\ or\ -1$ and $B_p \geq B$

$p(t)$is known as chipping sequence which alternates between two values +1 and -1. The rate of alteration is equal to or greater than the Nquist rate $1/\tau_0$ of the baseband signal. The effect of mixing is that it spread the frequency content of the baseband signal on the complete spectrum of $p(t)$. Analog band pass filter with impulse response $h(t)$ is centered at frequency f_0 with band width $B_{cs} \ll B$. Thus output of the filter $y(t)$ has a bandwidth B_{cs} centered at f_0.

Convolution of filter's impulse response $h(t)$ with resultant of multiplication of received signal $r(t)$ with random signal $p(t)$

$$y(t) = r(t)p(t) * h(t) \tag{5.4}$$

$$y(t) = \int_{-\infty}^{\infty} r(t-\tau)p(t-\tau)h(\tau)d\tau \tag{5.5}$$

Complex envelope of signal y(t) can be represented as

$$S_{cs}(t) = \int_{-\infty}^{\infty} r(t-\tau)p(t-\tau)h(\tau)e^{-j2\pi f_0 \tau} d\tau \qquad (5.6)$$

Where f_0 - Center frequency of BPF

$S_{cs}(t)$ - Complex envelope of y(t)

In phase and Quadrature phase components in compressed form directly taken out

$$I_{cs}(t) = Re\{S_{cs}(t)\} \qquad (5.7)$$

$$Q_{cs}(t) = Im\{S_{cs}(t)\} \qquad (5.8)$$

Highest and lowest frequency components of $y(t)$ are

$$f_H = f_0 + B_{cs}/2$$

$$f_L = f_0 - B_{cs}/2$$

Second step now is to sample the signal $y(t)$ using band pass sampling theorem []. Bandwidth of $y(t)$ is B_{cs} which is defined as $B_{cs} = f_H - f_L$. So sampling frequency is chosen as explained in chapter 3

$$f_s^{cs} = \frac{4f_l + 2\,B_{cs}}{4l+1} \qquad (5.9)$$

Where $\left(l \leq floor(\frac{f_l}{2\,B_{cs}}) \right)$

Minimum value of sampling frequency can be achieved if f_l is divisible by $2B_{cs}$ i.e. the value $\frac{f_l}{2B_{cs}}$ is an integer.

$$y(kT_{cs}) = \int_{-\infty}^{\infty} r(kT_{cs}-\tau)p(kT_{cs}-\tau)h(\tau)d\tau \qquad (5.10)$$

$$y[k] = \int_{-\infty}^{\infty} r(kT_{cs}-\tau)p(kT_{cs}-\tau)h(\tau)d\tau \qquad (5.11)$$

Where k = 0,1,2,3,4…………

So output of our compressive sampling subsystem is $y[k]$, where number of samples to represent the signal are very less as compared to number of samples if we directly sample the $r(t)$.

5.3.2 Quadrature sampling subsystem

We have seen in chapter 3 that how quadrature sampling plays important role in various applications. Further we have seen the basic architecture of quadrature sampler and identified its practical limits. After that we have also confirmed that how new digital architecture can overcome the practical limits of the quadrature sampler. Due to new architecture we became able to combine compressive sampling with quadrature sampling, because output of the compressive sampling subsystem in discrete form and input required for the quadrature sampling subsystem is also discrete.

Quadrature sampling subsystem is used to take out compressive In phase and Quadrature phase components from the applied input $y[k]$. The digital compressive In phase component is obtained by first down sampling $y[k]$ by a factor of 2 and after that multiplying the resultant sequence with $(-1)^m$.

$$I_{cs}[m] = I_{cs}(mT_{cs}) \tag{5.12}$$

Where m = 0, 1, 2, 3, 4…. $\&T_{cs} = 2/f_s^{cs}$

To obtain digital Quadrature phase component, first step is digital demodulation of the sequence $y[k]$ through $-2\sin(k\pi/2)$ and then filter the resultant output through low pass filter. At last down sample the filtering output by a factor of 2 .

$$Q_{cs}[m] = Q_{cs}(mT_{cs}) \tag{5.13}$$

Where m = 0, 1, 2, 3, 4…. $\&T_{cs} = 2/f_s^{cs}$

Thus we get In phase and Quadrature phase components in digital form, $I_{cs}[m]\&Q_{cs}[m]$ and their complex samples $S_{cs}[m] = S_{cs}(mT_{cs})$, of compressive complex envelope $S_{cs}(t)$.

$$S_{cs}[m] = I_{cs}[m] + jQ_{cs}[m] \tag{5.14}$$

Thus at the output of Quadrature sampling subsystem we get compressed and sampled In phase and Quadrature phase components.

5.3.3 Recovery subsystem

The output of the quadrature sampling subsystem is in the form of sampled and compressive In phase and Quadrature phase components. Now we need to recover back original signal from these samples. We have shown the relationship between In phase and Quadrature phase components with complex envelope of the signal. So now if we consider sampled complex envelope of the $S_{cs}(t)$ i.e. $S_{cs}[m]$ and try to formulate the l1 norm minimization problem similar we have solved in eq. (2.10) of chapter 2, then recovery becomes possible. Further by considering real and imaginary parts of recovered complex envelope we can take out recovered In phase and Quadrature phase components.

We know from the literature of compressive sampling that this technique can be implemented if our signal is sparse in some domain. So consider that received signal $r(t)$is sparse in ψ waveform matched dictionary domain as we have discussed in 5.1

$$r(t) = \sum_{n=1}^{K} v_n \psi_n(t) \tag{5.15}$$

Where $\psi_n(t)$ is a set of basis function.

v_n – Basis coefficients.

Here $r(t)$ can be represented as linear combination of basis functions $\psi_n(t)$ with weighted coefficients. Now for recovery we need to formulate the l1 norm minimization problem.

Use the value of $r(t)$from eq. (4.15) and substitute in eq. (4.05)

$$S_{cs}(t) = \sum_{n=1}^{N} v_n \int_{-\infty}^{\infty} \psi_n(t-\tau)p(t-\tau)h(\tau)e^{-j2\pi f_0 \tau} d\tau \qquad (5.16)$$

After sampling $S_{cs}(t)$ by substituting t = mT_{cs}

$$S_{cs}(mT_{cs}) = \sum_{n=1}^{N} v_n \int_{-\infty}^{\infty} \psi_n(mT_{cs}-\tau)p(mT_{cs}-\tau)h(\tau)e^{-j2\pi f_0 \tau} d\tau \qquad (5.17)$$

For the given observation interval T, the number of complex samples of $S_{cs}(t)$ are given by $= T/T_{cs}$. As $B_{cs} \ll B$, so M is much less than BT.

$$S_{cs}[m] = \sum_{n=1}^{N} v_n \int_{-\infty}^{\infty} \psi_n(mT_{cs}-\tau)p(mT_{cs}-\tau)h(\tau)e^{-j2\pi f_0 \tau} d\tau \qquad (5.18)$$

Now we need to define the measurement vector $\widetilde{S_{cs}}$and measurement matrix $\widetilde{M}$ in time domain as

$$\widetilde{S_{cs}} = [\ S_{cs}[0],\ S_{cs}[1] \dots \dots \dots .\ S_{cs}[M-1]]' \qquad (5.19)$$

$$\widetilde{\mathrm{M}} = \left[\widetilde{M}_{mn}\right] \in \mathrm{C}^{M\times N}$$

Eq. (4.18) can be represented in matrix form

$$\widetilde{S_{cs}} = \widetilde{\mathrm{M}}\widetilde{V} \qquad (5.20)$$

Where $\widetilde{M}_{mn}$=$\int_{-\infty}^{\infty} \psi_n(mT_{cs}-\tau)p(mT_{cs}-\tau)h(\tau)e^{-j2\pi f_0 \tau} d\tau$

$$\widetilde{V} = [v_1 , v_2, v_3, v_4 , \dots \dots \dots v_n]'$$

$\widetilde{\mathrm{M}}$ is $M \times N$ *matrix* &$\widetilde{V}$is $N \times 1$ *vectror*

Our complete system i.e. quadrature compressive sampling system is described by the matrix $\widetilde{\mathrm{M}}$ .For recovery of complex envelope $s(t)$ of the received signal $r(t)$ we need to reconstruct the sparse coefficient vector $\widetilde{V}$ from eq. (5.20). We can solve eq. (5.20) directly $M \geq N$. But in the given observation interval $T, M < N$. So eq. (5.20) shows an underdetermined system which has infinite solution. But this eq. can be solved if vector $\widetilde{V}$ shows sparse nature. From literature of compressive sampling, it is stated that if matrix $\widetilde{\mathrm{M}}$ satisfies restricted isometric property (RIP), then we it becomes possible for us to reconstruct the *K* sparse vector $\widetilde{V}$.

RIP Condition[34]*:* Given matrix $\widetilde{\mathrm{M}}$ with parameter φ_k satisfies restricted isometric property if following inequality holds for every k sparse $\tilde{V}$

$$(1-\varphi_k)||\tilde{V}||_2^2 \leq \left|\left|\widetilde{\mathrm{M}}\tilde{V}\right|\right|_2^2 \leq (1+\varphi_k)\left|\left|\tilde{V}\right|\right|_2^2 \qquad (5.21)$$

The RIP ensures that the measurement matrix try to preserves the norm of any *K*- sparse vector living in $\mathbb{C}^N$ when mapping it to a *M* dimension space (*M*<*N*).Thus the information of *K*-sparse signal is preserved so reconstruction becomes possible.

If the matrix $\widetilde{\mathrm{M}}$ satisfies RIP, the sparse coefficient vector $\tilde{V}$ can be reconstructed by solving the l1-norm optimization problem given below.

$$\min ||\tilde{V}||_1 \qquad \textit{such that} \qquad \widetilde{S_{cs}} = \widetilde{\mathrm{M}}\tilde{V} \qquad (5.22)$$

This complete scenario is defined for noise free case. But in actual scenario noise exists and affects the output of the system. So for noisy environment measurement vector can be defined as

$$\widetilde{S_{cs}} = \widetilde{\mathrm{M}}\tilde{V} + \tilde{n}_{cs} \qquad (5.23)$$

Where $\tilde{n}_{cs}$ shows compressive samples of additive noise in received signal $r(t)$.So for reconstruction in noisy environment we need to solve following l1-norm minimization problem.

$$\min ||\tilde{V}||_1 \qquad \textit{such that} \qquad \left|\left| \widetilde{S_{cs}} - \widetilde{\mathrm{M}}\tilde{V} \right|\right|_2 \leq \varepsilon \qquad (5.24)$$

Where - ε is a small parameter given by the noise environment.

Problems of optimization defined in eq. (5.22) and (5.24) can be solved by using greedy algorithms or convex optimization algorithms. For our simulations we use convex optimization algorithms.

CHAPTER 6

SIMULATION AND RESULTS ANALYSIS

We consider the case of quadrature compressive sampling for radar signals. We have seen in Chapter 1 that for radar systems we use linear frequency modulated waveform typically because it can avoid the Doppler and Range ambiguities (LFM). So we use LFM waveform to analyze how quadrature compressive sampling.

6.1 Waveform Generation

The complex baseband signal of the LFM waveform can be represented as [1]

$$s(t) = rect\left(\frac{t - \frac{Tp}{2}}{Tp} \right) \exp\left(j\pi\mu\left(t - \frac{Tp}{2}\right)^2 \right)$$

Where $B\ - Bandwidth\ of\ LFM\ Signal$

$$Tp\ - Pulse\ width$$

$$u = \frac{B}{Tp}$$

$$rect\left(\frac{t}{Tp}\right) = \begin{cases} 1 & -\frac{Tp}{2} \leq t \leq \frac{Tp}{2} \\ 0 & else \end{cases}$$

That is it represents a rectangular pulse as shown in Fig. 6.1.

For our experiments, we consider the signal parameters as

$$B - 100\ MHz$$

$$Tp\ - 10.24\ \mu s\ \&\ T\ - 20.48\ \mu s$$

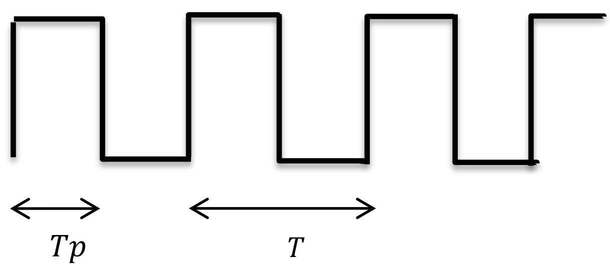

Figure 6.1 Rectangular Pulse

In this scenario targets are assumed to exit in the interval $(T - Tp, T)$.So dictionary size is take as $N = floor(B(T - Tp))$. The target in environment can be anywhere i.e. random so time delay t_k for k^{th}target is chosen randomly from the set *SET1*.

$SET\ 1 = (\tau\ ,2\tau, 3\tau, \dots \dots . . N\tau)$; Where $\tau\ = 1/B$

The gain coefficients v_k of the dictionary domain are randomly distributed between (0, 1]. Sampling rate is considered with respect to bandwidth of the filter B_{cs} . Absolute value of LFM transmitted waveform is as shown in Fig. 6.2

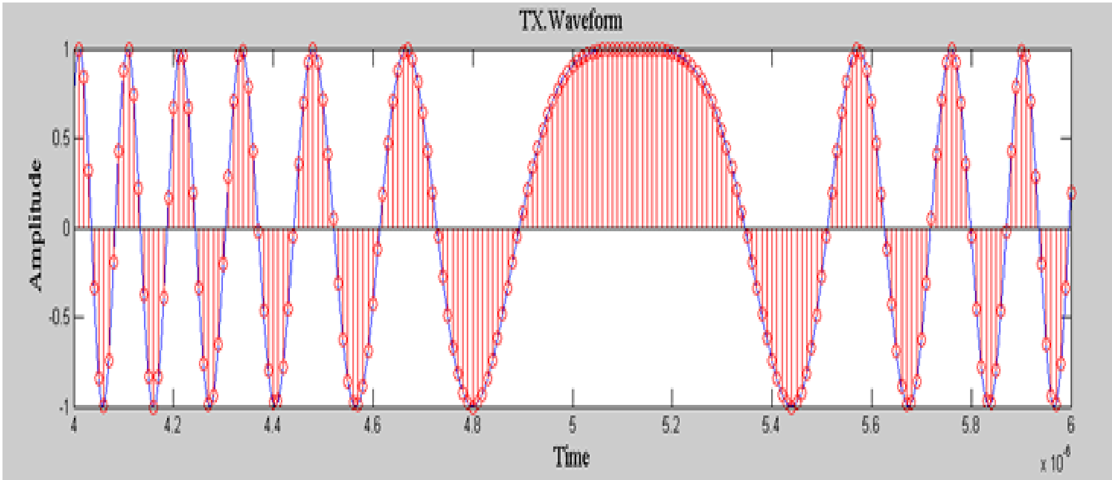

Figure 6.2 Transmitted Waveform (Real Part)

6.2 Dictionary Generation

We have seen in chapter 5 that received waveform $r(t)$ is sum of time shifted and scaled versions of the transmitted waveform $s(t)$. So dictionary $\psi_n(t)$that we have created is combination of all possible time shifted versions of transmitted waveform and time shift is provided by nyquist interval τ. Scaling to the shifted version is given by random vector $\widetilde{V}$. We have also seen in literature of compressive sampling that the domain or dictionary in which signal is sparse should maintain Orthogonality. So we verify whether our dictionary is orthogonal or not.

For a two functions to be orthogonal inner product should be zero and for a matrix to be orthogonal multiplication of a matrix with its transpose should be identity matrix.

Our Dictionary have N number of elements and every element should be orthogonal to each other.

$$\psi_n(t) = s(t - n\tau_0) \quad , \qquad \text{where } \tau_0 = \text{ sampling interval}$$

$$n = 1,2\ldots.N \quad [\, N=T/\tau_0 \,]$$

We sample the dictionary elements by replacing t with lT_{nq}.

$$\psi_n[l] = \psi_n(lT_{nq})$$

$\psi_n[l]$is a matrix and we need to prove Orthogonality for this matrix. We can also prove Orthogonality in other transform domain of transformation is orthogonal because Orthogonality preserves in transform domain if transform is orthogonal [35] .So we take DTFT of $\psi_n[l]$ as Z and find the gram matrix.

$$z_{ln} = \frac{1}{\sqrt{N}}\psi_n\left(e^{j\left(\frac{2\pi}{N}l\right)}\right)$$

$$G = Z * Z'$$

If G matrix is an Identity (Diagonal) matrix Z matrix will be orthogonal

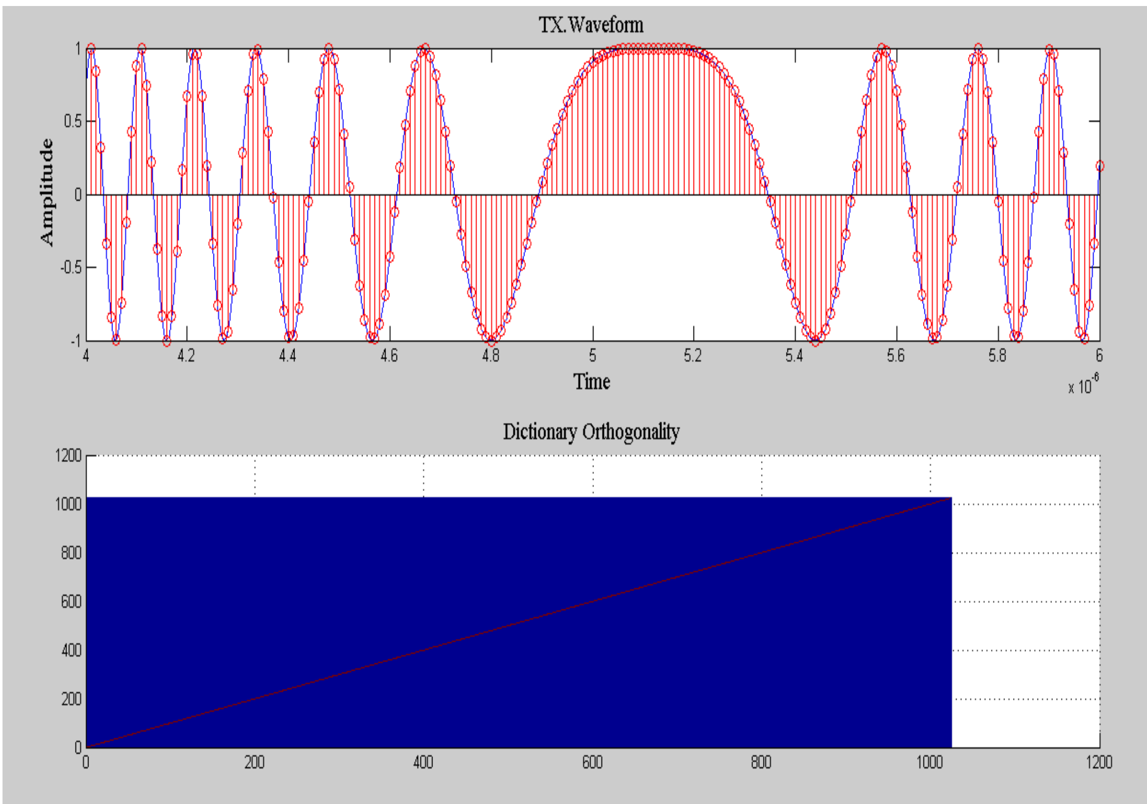

Figure 6.3 Transmitted Waveform (Real Part) and Gram matrix

It is clear from Fig. 6.3 that our dictionary is almost orthogonal because Gram matrix is identity matrix.

6.3 Signal Reconstruction

We have seen in section 5.2 that how to apply quadrature compressive sampling on the given signal. Initially by passing through compressive sampling system and then from quadrature sampler we need to formulate an underdetermined problem that can be solved using l1 norm minimization.

Formed problem is given in eq. (5.20) $\widetilde{S_{cs}} = \widetilde{M}\widetilde{V}$

Solution to this problem is

$$\min\left|\widetilde{V}\right| \quad such\ that \quad \widetilde{S_{cs}} = \widetilde{M}\widetilde{V}$$

We have shown here received and reconstructed signal at different sparsity level when total number of coefficients in dictionary domain are 1024.

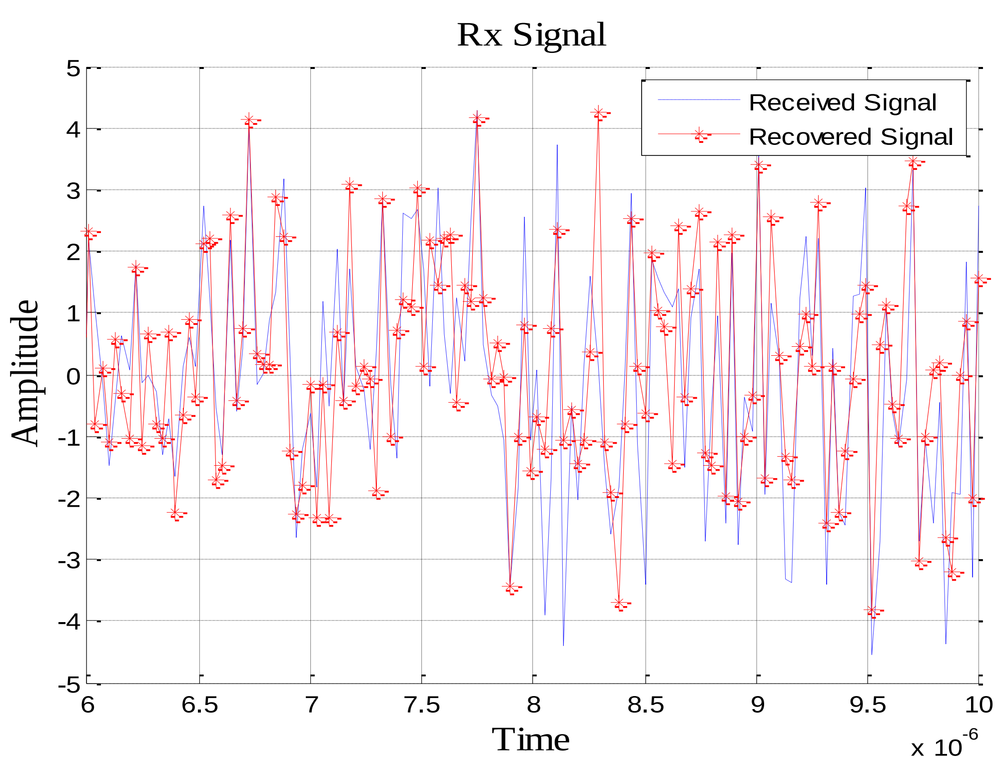

Figure 6.4 Received versus Recovered Signal for SL = 26

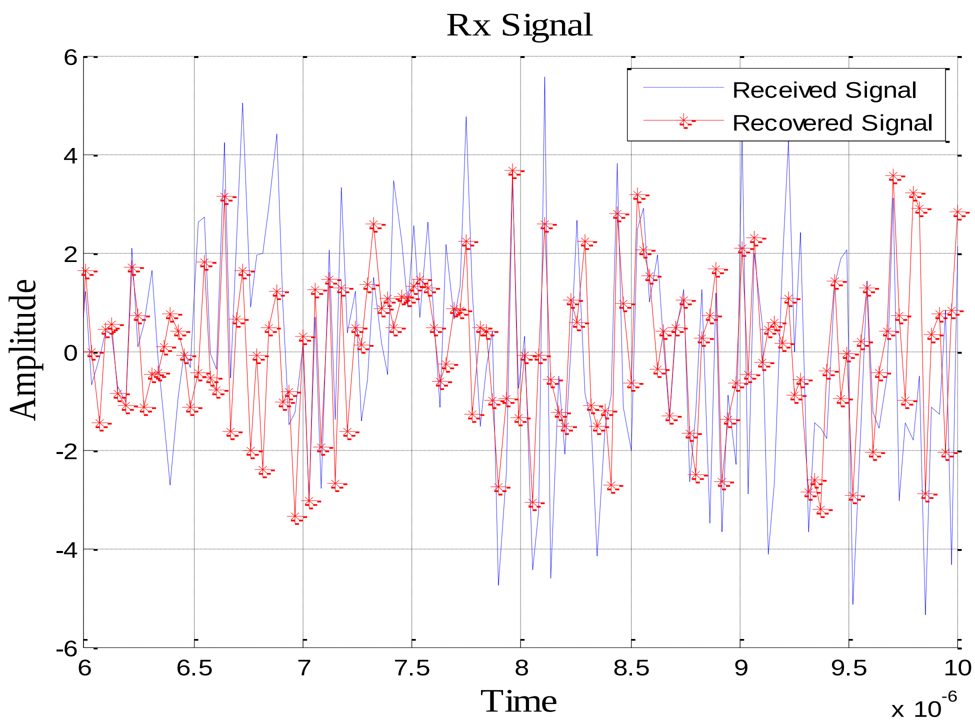

Figure 6.5 Received versus Recovered Signal for SL = 35

Fig.6.4 corresponds to sparsity level 26 where Fig. 6.5 corresponds to sparsity level 35. It is clear from above figures that recovery is better in case of sparsity level 26 as compare to sparsity level 35.

6.4 Effect of variation of sparsity level on Recovery Error

We have analyzed the effect of variation of sparsity level on recovery error for different bandwidths as shown in Fig. 6.6. In Table 6.1 value of Recovery error is given with variation in sparsity level for different bandwidth of the filter. Fig. 6.6 clearly shows that for large bandwidth recovery error is more as compared to recovery error for small bandwidth. We have discussed in chapter 5 that because compression is decided by the bandwidth of the filter i.e. less compressed signal can be recovered better as compare to more compressed signal. It is clear from the Fig. 6.6 that as sparsity level increases Recovery error also increases. This statement follows the general theory of compressive sampling i.e. if signal will be more spars, recovery will be better. We also analyzed the recovery error with variations in sparsity level for different bandwidths of the filter.

Table 6.1 Effect of Bandwidth and Sparsity Level variation on Recovery Error

B / SL	1MHz	2 MHz	3MHz
103	2.44	2.30	1.97
86	2.13	1.9	1.72
69	1.94	1.58	1.13
61	1.71	1.43	0.3384
52	1.54	1.25	0.0070
47	1.41	1.02	1.31e-4
41	1.31	0.0232	4.41e-6
37	1.23	5.57e-4	4.40 e-6
35	1.17	9.93e-5	3.48e-6
30	1.05	4.84e-7	3.48e-6
26	0.8532	3.25e-7	3.48e-7
23	0.0254	1.055e-7	2.0e-7
21	0.0002	7.25e-8	5.2e-8

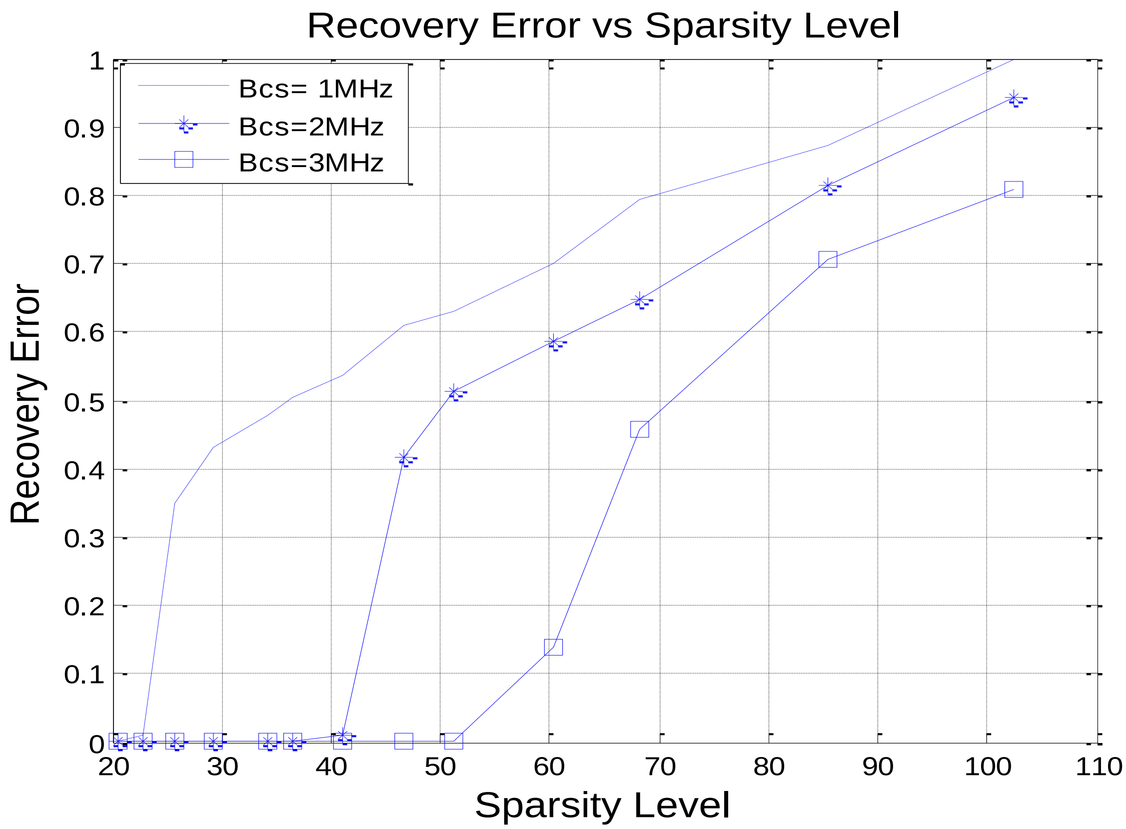

Figure 6.6 Recovery Error versus Sparsity Level

6.5 Effect of SNR on Recovery Error

We have seen in section 5.4 that how noise affects our recovery. Here through simulations we saw that how recovery error varies with change in signal to noise ratio (SNR).

Table 6.2 Effect of Noise on Recovery Error with different Sparsity Level

SL=21

SNR1(dB)	12.47	8.94	6.45	4.512	2.92	1.59	0.4302	-0.59	-1.5	-2.3
Rerr1	9.29	13.84	18.9	23.53	29.08	34.0	38.79	44.47	48.29	53.7

SL=23

SNR2(dB)	14.74	11.29	8.72	6.78	5.19	3.85	2.69	1.67	0.76
Rerr2	9.87	14.90	20.29	25.95	33.02	38.01	43.81	49.86	55.97

SL=25

SNR3(dB)	14.73	11.29	8.72	6.77	5.19	3.97	2.69	1.65	0.75
Rerr3	11.57	17.36	25.05	29.32	35.67	42.08	51.37	57.98	64.09

SL=30

SNR4(dB)	15.05	11.53	9.03	7.10	5.51	4.17	3.01	1.99
Rerr4	23.71	35.171	46.74	63.17	74.78	87.26	100.0112	112.63

Fig. 6.7 is showing the relation between recovery error and signal to noise ratio for different sparsity level. From here it is clear that with increase in SNR recovery error is going to decrease. One more relation is also verified here i.e. change in recovery error with sparsity level. If sparsity level is more recovery error will be more i.e. for better recovery sparsity level should be more.

6.7 Effect of Chipping Sequence Multiplication in Compressive Sampling Subsystem on Recovery

We have used chipping sequence *p(t)* in compressive sampling subsystem in section 5.2. We use this sequence so that frequency content of the incoming signal *r(t)* smears up on entire spectrum of the chipping sequence *p(t)* and to avoid the information loss in next stage i.e. low pass filtering. Here we analyzed that how much effect of chipping sequence on recovery.

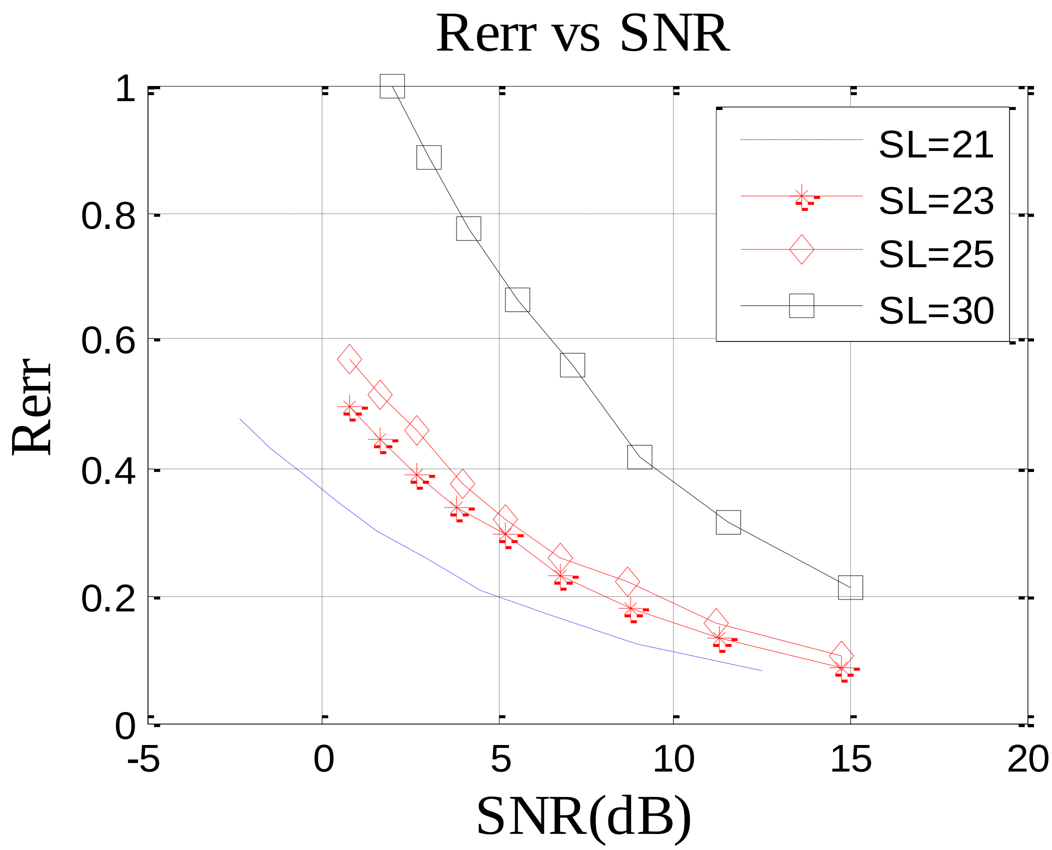

Figure6.7 Recovery Error versus SNR

From Fig 6.8 and Fig. 6.9, it is clear that if we use chipping sequence then recovered signal try to approach the received signal. But we don't use chipping sequence it is not following the received signal. From this it is clear that chipping sequence plays important role.

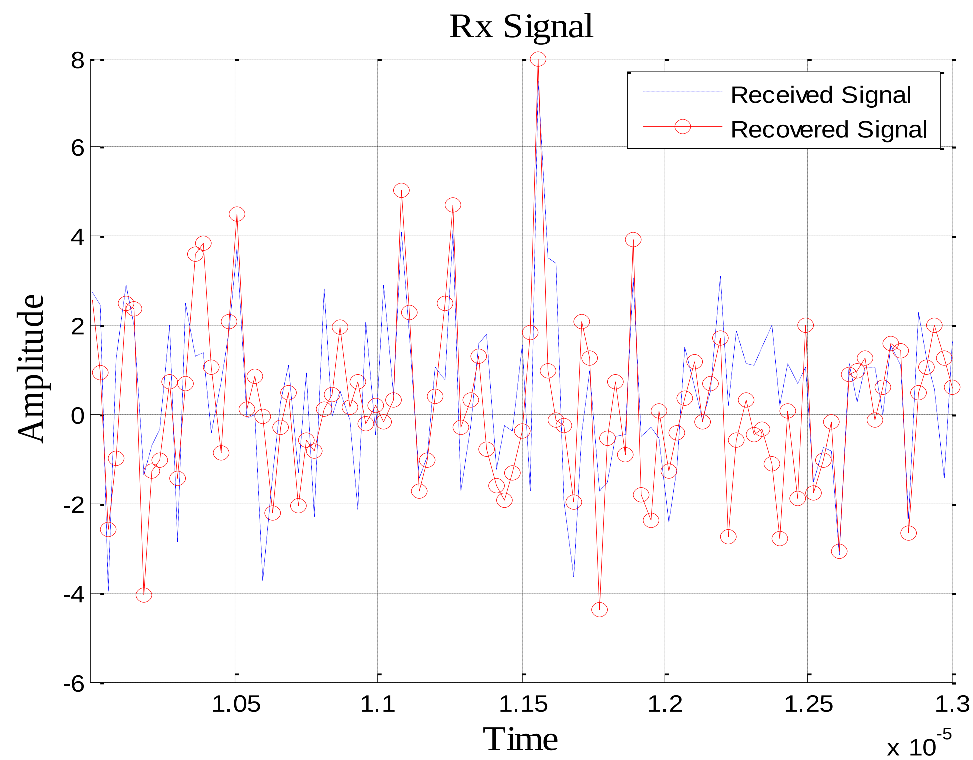

Figure 6.8 Received versus Recovered Signal using PN Sequences

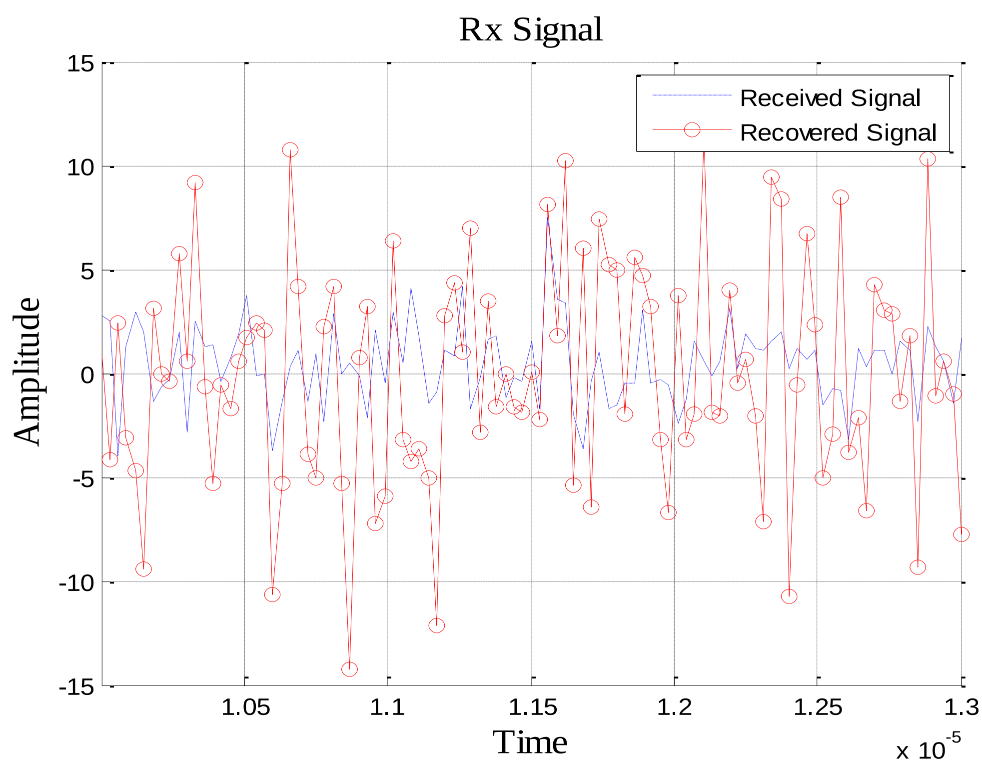

Figure 6.9 Received versus Recovered Signal without using PN Sequences

6.8 Conclusion

In this chapter we simulated the quadrature compressive sampling system for radar signals. We generated dictionary for radar signals and show that dictionary elements are orthogonal to each other that was actual requirement for our dictionary. Further we apply LFM signal on quadrature compressive sampling system and analyze the effect of sparsity level and noise. Our results conforms that recovery is better for more sparse signal as compare to the less sparse signal. Noise also affects performance of the system. For large vale of SNR recovery error is less i.e. recovery is better.

CHAPTER 7

CONCLUSION AND FUTURE SCOPE

In this dissertation we have analyzed that Compressive sampling appears to be a revolutionary technique for data acquisition and successful reconstruction. We implemented this technique for one dimensional signal as well as two dimensional image signal and successfully recover them from compressive random measurements. We analyzed recovery error due to variations in sparsity level and compression ratio and assured that successful reconstruction of signal relies on sparsity level and compression ratio. Effect of noise also considered in compressive sampling and we verified that with increase in SNR, Recovery error decrease that is according to our system expectations.

Further we simulated the quadrature compressive sampling system for radar signals. We generated dictionary for radar signals and show that dictionary elements are orthogonal to each other that was actual requirement for our dictionary. Further we applied LFM signal on quadrature compressive sampling system and analyze the effect of sparsity level and noise. Our results conforms that recovery is better for more sparse signals as compared to the less sparse signal. We also performed simulations by considering noise effects. Noise also affects performance of the system. For large vale of signal to noise ratio(SNR) recovery error is less i.e. recovery is better.

Future Scope:

We have analyzed throughout this dissertation that compressive sampling appears to be a very effective tool to move from analog domain to digital domain by overcoming the nyquist criterion. So we can use compressive sampling for high frequency signals for which we can't use sampling due to nyquist criterion. Further sampling rate of normal frequency signals can be decreased through this technique so that storing and hardware cost can be reduced.

Today we are dealing with online activities like video conferencing, video streaming etc. Discussed approach can't handle the streaming signal. Because we have considered that our signal is static and represented in the form of basis functions. One approach to handle streaming signal is given in [36-37]. So by using this technique we can use streaming signals on quadrature compressive sampling system.

References

[1] Peyton Z. Peebles,Jr.*, "Radar Principles",* John Wiley and Sons ,New Delhi, 2011.

[2] Merrill I. Skolnik, "*Introduction to Radar Systems*", 3rd ed.,Tata McGraw-Hill, New York, 2001.

[3] Bassem R. Mahafza, "*Radar Systems Analysis and Design Using MATLAB*", 2nd ed.,Chapman and Hall/CRC, New York, 2005.

[4] Simon Haykin, "*Communication Systems",* 4th ed.,John Wiley and Sons, NewYork ,2001.

[5] D. Donoho, "Compressed sensing," *IEEE Trans. Inform. Theory*, vol. 52, no. 4, pp. 1289 -1306, Apr. 2006.

[6] E. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inform. Theory*, vol. 52, no. 2, pp. 489–509, 2006.

[7] E.J.Candes, "Compressive sampling,"*in Proceedings of the International Congress of Mathematicians*, vol. 3, pp. 1433–1452, 2006.

[8] J.A.Tropp and A.C. Gilbert, "Signal recovery from partial information via orthogonal matching pursuit," *IEEE Trans. Inform. Theory*, vol. 53, no. 12, pp. 4655- 4666, 2007

[9] Emmanuel Cand`es and Terence Tao. Decoding by linear programming. *IEEE Trans. Inform. Theory*,vol 51,no 12,pp.4203–4215, December 2005

[10] Scott Shaobing Chen, David L. Donoho, and Michael A. Saunders, "Atomic decomposition by basis pursuit*". SIAM J. Sci Comp*., 20(1):33–61, 1999.

[11] D. Needell, J. A. Tropp, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," *Appl. Comput. Harmon. Anal.*, vol.26, no.3, pp.301-321, 2009.

[12] W. Yin, S. Osher, D. Goldfarb, and J. Darbon, "Bregman iterative algorithm for l1-minimization with applications to compressive sensing," *SIAM J. Imaging Sci.*, vol.1, no.1, pp.143-168, 2008.

[13] S. R. Becker, E. J. Candès, and M. C. Grant, “Templates for convex cone problems with applications to sparse signal recovery,” *Mathematical Programming Computation*, vol.3, no.3, pp.165-218, 2011.

[14] M. Fornasier, “Numerical methods for sparse recovery,” *Radon Series Comp. Appl. Math*, vol.9, pp.1-110, 2010.

[15] E. van den Berg and M. P. Friedlander, “Probing the Pareto frontier for basis pursuit solutions,” *SIAM J. Sci. Comput.*, vol.31, no.2, pp.890-912, 2008.

[16] E. van den Berg and M. P. Friedlander, “Sparse optimization with least-squares constraints,” *SIAM J. Optim.*, vol.21, no.4, pp.1201-1229, 2011.

[17] E. Candès and J. Romberg, “Sparsity and incoherence in compressive sampling,” *Inverse Prob.*, vol. 23, no. 3, pp. 969–985, 2007.

[18] Prabhat Thakur,”General framework of compressive Sampling and its applications for signal and image compression” in *Proc. of International conference on Wireless Networks and Embebded Systems(WECON)*,Punjab, India, 2015.

[19] T. Tony Cai , Lie Wang “Orthogonal matching pursuit for sparse signal recovery with noise”, *IEEE Trans. on inform. Theory,* vol. 57,no. 7,pp. 4680-4688, july 2011.

[20] K. C. Ho, Y. T. Chan, and R. Inkol, “A digital quadrature demodulation system,” *IEEE Trans. Aerospace and Electronic Systems*, vol. 32, no. 4, pp. 1218-1227, 1996

[21] http://www.jhuapl.edu/techdigest/TD/td1803/guest.pdf

[22] Tse D Viswanath P, “ Fundamentals of Wireless Communication”,Cambridge University Press,U.K.,2005.

[23] Roome, S. J. (1989) , “Analysis of quadrature detectors using complex envelope notation”, *lEEE Proceedings*, 136, Pt. P,2 (Apr. 1989), 95-100.

[24] Waters, W.M., and Jarrett, B. R., “Bandpass signal sampling and coherent detection”, *IEEE Transactions on Aerospace and Electronic Systems,* AES.l8, 4,pp. 731-736,1982.

[25] Thand B Welch, Cameron H.G. Wright ,Micheal G. Morrow, “ *Real time digital signal processing from MATLAB to C with the TMS320C6x DSPs* ”, 2nd ed. ,CRC press, New York,2012

[26] Andrew J.Viterbi , " CDMA-principles of spraed spectrum",Addison-Wesley Logman Inc.,Massachusetts,1995

[27] Feng Xi, Shengyao Chen and Zhong Liu, "Quadrature compressive sampling for radar echo signals," *in International Conference on Wireless Communications and Signal Processing (WCSP),* pp. 1-5, Nov 2011

[28] J. Laska, S. Kirolos, M.F. Duarte, T. Ragheb, R. Baraniuk, and Y. Massoud, "Theory and implementation of an analog-to-information converter using random demodulation," in *Proc. IEEE Int. Symp. OnCircuits and Systems (ISCAS)*, New Orleans, Louisiana, 2007, pp. 1959-1962.

[29] J. A. Tropp, J. N. Laska, M. F. Duarte, J. K. Romberg, and R. G. Baraniuk, "Beyond Nyquist: efficient sampling of sparse bandlimited signals," *IEEE Trans. Inform. Theory*, vol. 56, no. 1, pp.520-544, 2010.

[30] J. Laska, S. Kirolos, Y. Massoud, R. Baraniuk, A. Gilbert, M. Iwen, and M. Strauss, "Random sampling for analog-to-information conversion of wideband signals," in *Proc. IEEE Dallas Circuits and Systems Workshop (DCAS)*, Dallas, Texas, 2006, pp.119-122

[31] Pawel J. Pankiewicz, Thomas Arildsen, Torben Larsen, " Sensitivity of the random demolation framework to filter tolerances " in *Proc.19th European Signal Processing Conference (EUSIPCO),* Barcelona, Spain,pp.534-538, 2011.

[32] H. Rauhut, K. Schnass, and P. Vandergheynst, "Compressed sensing and redundant dictionaries," *IEEE Trans. Inform. Theory*, vol.55,no. 5,pp. 2210-2219,2008

[33] G. Shi, J. Lin, X. Chen, F. Qi, D. Liu, and L. Zhang, "UWB echo signal detection with ultra-low rate sampling based on compressed sensing," *IEEE Trans. Circuits and Systems II: Express Briefs*, vol. 55, no. 4, pp. 379-383, 2008.

[34] E. Candès, "The restricted isometry property and its implications for compressed sensing," *Comptesrendus Mathematigue*, vol.346, no.9-10, pp.589-592, 2008.

[35] http://in1.csie.ncu.edu.tw/~chia/Course/LinearAlgebra/sec5-3.pdf

[36] P. T. Boufounos and M. S. Asif, "Compressive sensing for streaming signals using the streaming greedy pursuit," in *Proc.Military Commun. Conf. (MILCOM)*, San Jose, CA, USA, 2010, pp. 1205–1210.

[37] M. Salman Asif and J. Romberg, “Sparse recovery of streaming signals using l1-homotopy,” *IEEE Trans. on Signal Processing*, vol. 62, no. 16, pp. 4209-4223, 2014

LIST OF PUBLICATIONS

1. Prabhat Thakur, “General framework of compressive Sampling and its applications for signal and image compression” in *Proc. of International conference on Wireless Networks and Embedded Systems(WECON),*Punjab, India, March 2015.