

# **SOIL-TRACK INTERACTION MODEL TO STUDY GROUND VIBRATIONS FOR HIGH SPEED TRAINS**

Thesis Report Submitted in Partial Fulfillment of the Requirement for the Degree  
of

Master of Technology

In

**Structural Engineering**

by

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**CERTIFICATE**

This is to certify that the project thesis work titled “**SOIL-TRACK INTERACTION MODEL TO STUDY GROUND VIBRATIONS FOR HIGH SPEED TRAINS**” submitted in partial fulfillment for award of Master’s Degree (Structural Engineering) submitted in Civil Department of Jaypee University of Information Technology, Waknaghat is a faithful record of research work carried out by Anshul Rana (152653) under my guidance and supervision.

The assistance and help received during the course of investigation and source of literatures have been fully acknowledged.

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## ABSTRACT

In the following thesis work ground induced vibrations generated by high speed trains are investigated, by implementation of three dimensional dynamic soil track interaction model. As per previous researches it is seen that ground vibrations induce significant increase in vibration level when these high speed trains move at critical speed on tracks. Therefore critical speed is such speed at which resonance occurs between the high speed moving train and Rayleigh wave of sub grade soil.

Critical speed has been considered as one of the most significant factors affecting high speed rail safety and serviceability and impeding increases in train speed. In this research work a three dimensional dynamic track model is modeled in order to determine ground response generated by high speed trains. This three dimensional track model is based on sophisticated track model, a theoretical model of track and layered sub grade soil. Simulation by commercial Finite element software (ABAQUS) is done for the verification of three dimensional track model. After the verification work effects of critical speed on ground induced vibration has been checked out.

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# Chapter 1

## INTRODUCTION

### 1.1 Background

It's been 150 years that railway has been an integral part of our public transportation. Emergence of high speed trains is one of the most significant technological advances in the transportation industry over the last part of 20<sup>th</sup> century and beginning of 21<sup>st</sup> century. High speed trains are growing rapidly due to their environmental friendly technologies and time efficient distance travel. It is about ten thousand miles of distance of high speed lines have been constructed. Shanghai Maglev in China can run up to a high speed of 267.8 m/s is World's Fastest Train.

As the speed of trains continues to increase number of potential problems arise. The train induced ground vibration is one of the concerns of these problems. It may contribute to passengers discomfort or could be causing serious damage to track and train. In addition, ground induced vibration by high speed train is a major environmental issue as such vibration can cause annoyance to the inhabitants of surrounding buildings in the form of vibrations or noises, and may affect nearby hospitals, historical sites and high tech industries. Therefore prediction of high speed train induced ground vibration is becoming increasingly important and so much attention is given to this subject.

It is well known that when any train runs at along a track, its dynamic load can be transmitted to track structure and subgrade via the wheel rail contact, and can also excite vibrations of both the track and ground which can in in reverse influence the vibration of the vehicle. Train , the track and ground are essentially coupled with each other which indicates that it is necessary to investigate the high speed train induced ground vibrations from integrated system point of view(Zhai et al.,2009).

Many researchers have studied the phenomenon of trains running at critical speed. In 1927 a theoretical model of rail as a beam which is supported as a track structure and ground revealed that dynamic amplification occurs when train reaches at critical speed (Timoshenko,1927). For normal soils critical speed is assumed around 500m/s. Krylov (1994) proposed that train will encounter sound barrier on reaching the velocity of Rayleigh surface waves propagating in the ground. Above phenomenon can be compared with Mach effect of supersonic jets and Cherenkov radiation of light. The velocity of Rayleigh waves travelling in soft ground soil is 90 to 130 m/s which is usually reachable by high speed trains (HST). Furthermore for peat, marine clays, and other soft clays the velocity of Rayleigh waves travelling in ground is much lesser i.e. 30 to 40 m/s. Rayleigh waves have a speed slightly less than shear waves which depends on soil properties. Ground medium which have low shear velocity requires more attention as they are prone to increased ground vibration generated by high speed trains. These ground induced vibrations could lead to deterioration of track structure and even derailment. However, it is

expensive and perilous to run a train at critical speed to test the vibration of the sub grade soil. Therefore, significant previous research has been focused on modeling critical speed effects. How to simulate the vehicle, track and ground system is the core research in this field, as well as the methods to couple the vehicle, track and ground models. The models, according to the literature reviewed, can be divided into three categories. One is the ground vibration generated by moving rail loads, which is the simplest. The moving rail load usually is modeled as a moving point load. The rail, modeled as a beam, just lies on the ground surface without track. A second category of model, which is extensively used, couples a track with the previous model. The models coupled with track model can simulate the realistic rail/wheel dynamic effect and are always more accurate than the first category of models.

Recently, many researchers focus on coupling the vehicle, track structure and ground together to form an integrated system (Zhai et al. 2010). The vehicle model is a complicated system with car mass, bogie mass and wheel, all coupled by a spring/dashpot system. For the track system, the rail is usually considered as an Euler-Bernoulli beam. The sleepers can be modeled as discretely supported or as a continuously supported system. Green's function is widely used to calculate the ground surface vertical displacement. The contact between wheel and rail can be assumed using Hertzian nonlinear elastic contact theory or other contact theories. For track models, ballast is usually modeled as another spring/dashpot system, sometimes the ballast mass is also considered in calculation. But in my thesis work train-track interaction model referred to as the Sandwich Dynamic Track Model is introduced. As later described, FEM is used to simulate the behavior of the ground. Also, to verify the proposed 3D dynamic track model, commercial software (ABAQUS) is introduced. The purpose of the verification is to increase the confidence in the proposed model. Train load in ABAQUS is modeled as a sequence of point loads running on the rail at a constant speed. In addition, the location and influence depth of critical speed effects are also studied.

## 1.2 DEFINITIONS

### 1.2.1 Rail

Rail is an important longitudinal steel track component put on the top of the rail track and is used to support and guide the vehicle by providing smooth running surfaces. It can accommodate the wheel loads and distributes these loads over the sleepers or supports. The horizontal transverse forces on the rail head can be transferred to sleepers and lower track components. Also, the rail enables the vehicle to move in a stable direction. There are many types of rail with regards to its profile, including flat-bottom rail, non-standard profile, grooved rail, block rail and crane rail. Flat-bottom rail is the standard profile used as a general rule in conventional track.

There are many types of rails including flat bottom rails, non standard profile, grooved rail, block rail and crane rail as shown in Figure 1. Flat bottom rail is a standard profile and is used as general rule in conventional track and is introduced in this thesis.

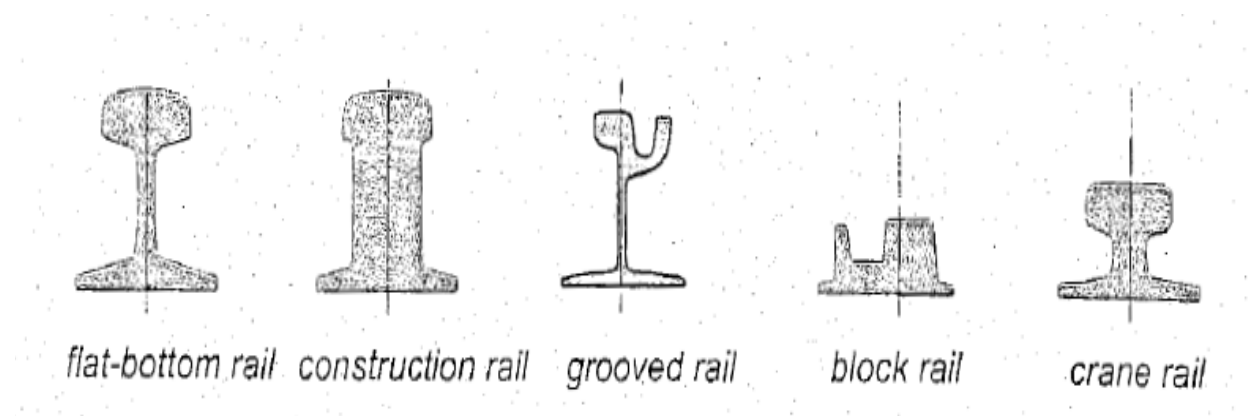


Figure1. Rail Profile Types (Coenraad,2001)

### 1.2.2 Railpad

The function of rail pad is to provide an absorbing component between the steel rail and sleepers through transferring the rail load to sleepers and screening out the high frequency force. Also, rail pad can make a more stable track and significantly lengthen the life of wood sleepers. In addition, rail pads are embedded under rails acting as electrical insulation. The railpad shown in Figure 2 below.

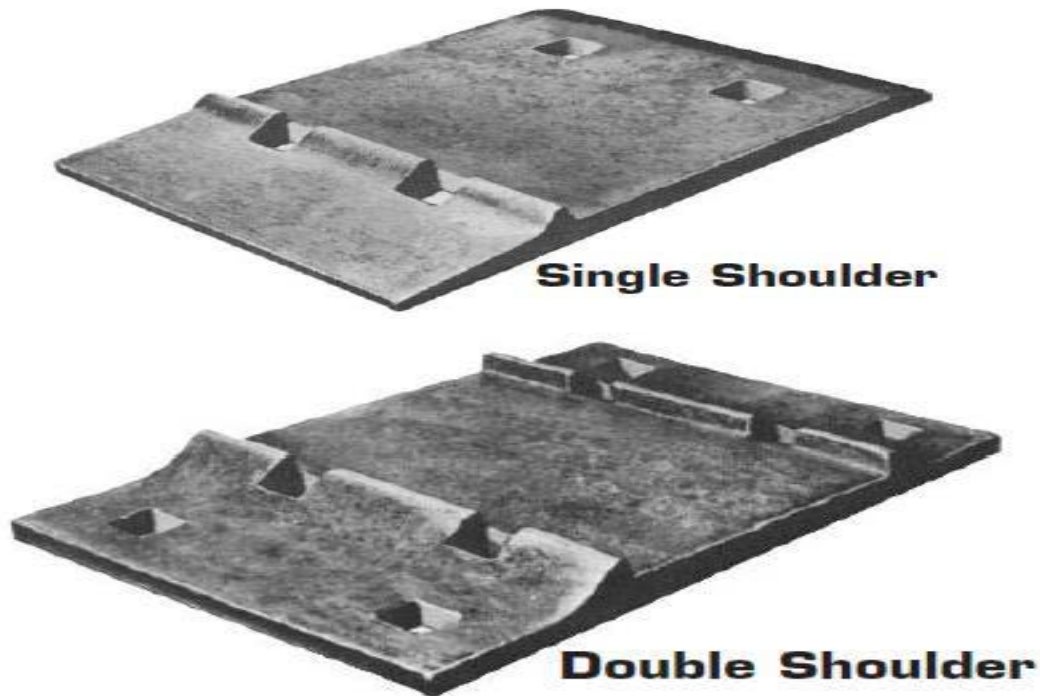


Figure 2. Railpad (Unitrac Railroad Materials Inc.)

### 1.2.3 Sleepers

The sleepers are rested on the transverse direction of the track, which is vertical to the moving direction of the vehicle. The function is to maintain track gauge and fasten the rails to be aligned. Also, it can be considered as electrical insulation for the rails. Sleepers transmit the train loading to the lower track structure. The available materials for sleepers can be wood, steel and concrete. Timber and concrete ties are widely used and steel ties are limitedly used.

### 1.2.4 Ballast

Ballast is a layer that is formed by crushed granular material and placed on the top of the sub ground. Ballast bed can absorb considerable compressive stresses, but not tensile stresses. Also, it has a large bearing strength in the vertical direction, but it is reduced in the lateral direction. The thickness of the ballast bed is typically about 50 cm from the top of the ballast or 25-30 cm from the lower side of the sleeper. The main functions of ballast are to: 1) distribute the stresses transmitted by sleepers; 2) drain rainwater; 3) resist transverse and longitudinal shifting of track; 4) attenuate train vibration significantly.

### 1.2.5 Train and Track Model

Typical train model is shown in Figure 3 below. It has a secondary suspension and primary suspension which both are modeled as spring/dashpot system. Bogie is located between the secondary suspension and primary suspension.

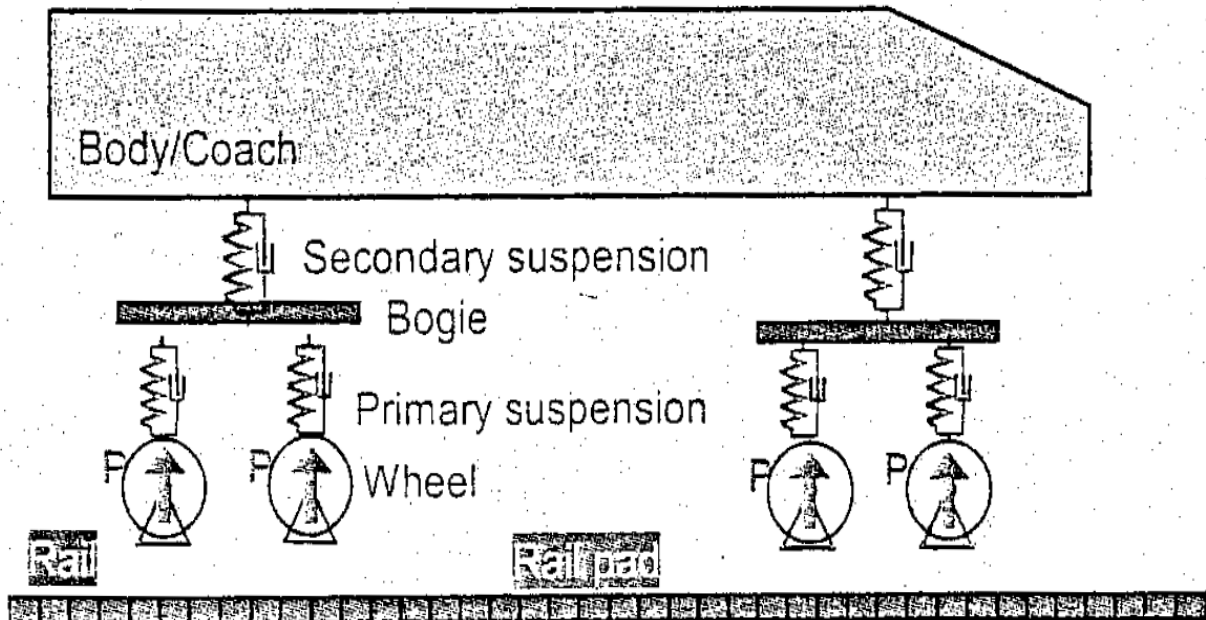


Figure 3 Train Model (Coenraad,2001)

Integrated Track model is shown in Figure 4 below that includes all track components rail, railpad, sleeper, ballast and sub grade.

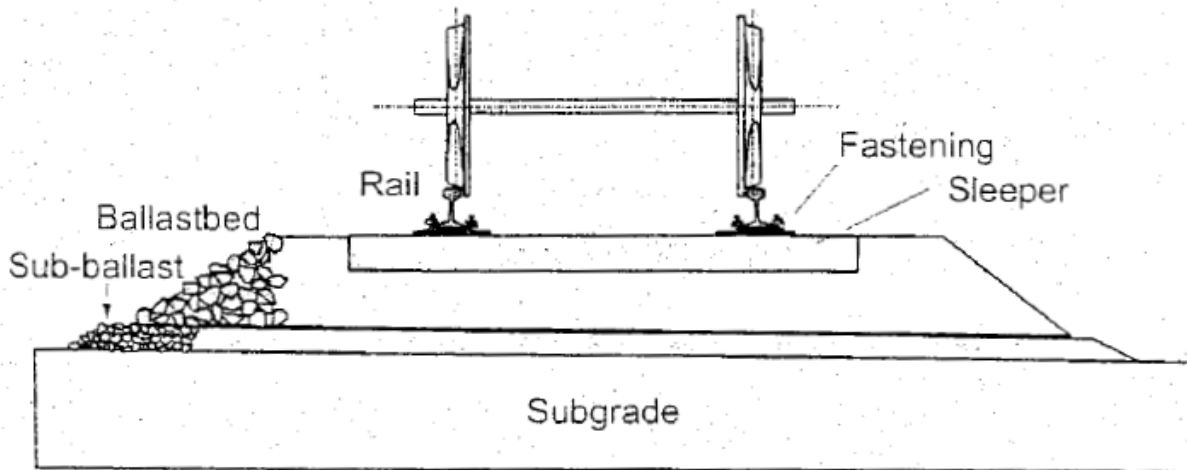


Figure 4 Track Structure (Coenraad,2001)

### 1.2.6 High Speed Train

High Speed Train has been categorized differently by different countries and institutions. Various categories are listed below.

INSTITUTION	MINIMUM SPEED (standard)
EUROPEAN UNION	Upgraded Track:200 km/hr New Track: 250 km/hr
FEDERAL RAILROAD ADMINISTRATION	177 km/hr
U S DEPTT. OF TRANSPORTATION	201 km/hr
CONGRESSIONAL RESEARCH SERVICE	240 km/hr

Table 1 Different Standard for High Speed Trains

### 1.3 Problem Statement

High Speed Trains could induce greater increase in vibration level when runs at a or more than the critical speed. The critical condition may be defined by the resonance between the Rayleigh wave of sub grade soil and velocity of moving train. This can significantly increase the risk of operation of high speed trains. Therefore it is important to predict this phenomenon and further methods of control. Field measurements are not possible at this level of research as they are time consuming and inefficient and also requires big amount of funds. In order to overcome this problem a three dimensional track soil model is much needed.

## Chapter 2

### GOALS AND OVERVIEW OF APPROACH

The goal of this thesis work involves two steps. First step is to develop a three dimensional track interaction model that is able to predict critical effect induced by high speed train. Second step is to use this model accurately in order to predict ground induced vibrations.

In order to obtain proposed model it is composed of track components and sub grade soil. A three dimensional finite element method is introduced to simulate sub grade soil. After this the investigation is carried out to check the effects of various parameters involved in model, these parameters include soil properties, track properties and loadings. All of the development and application of this model is done in commercial software ABAQUS. Critical speed effect is achieved if first step is achieved. The location and depth of critical layer which is major cause for critical speed are explored. Technical flow chart is shown in Figure 5 below.

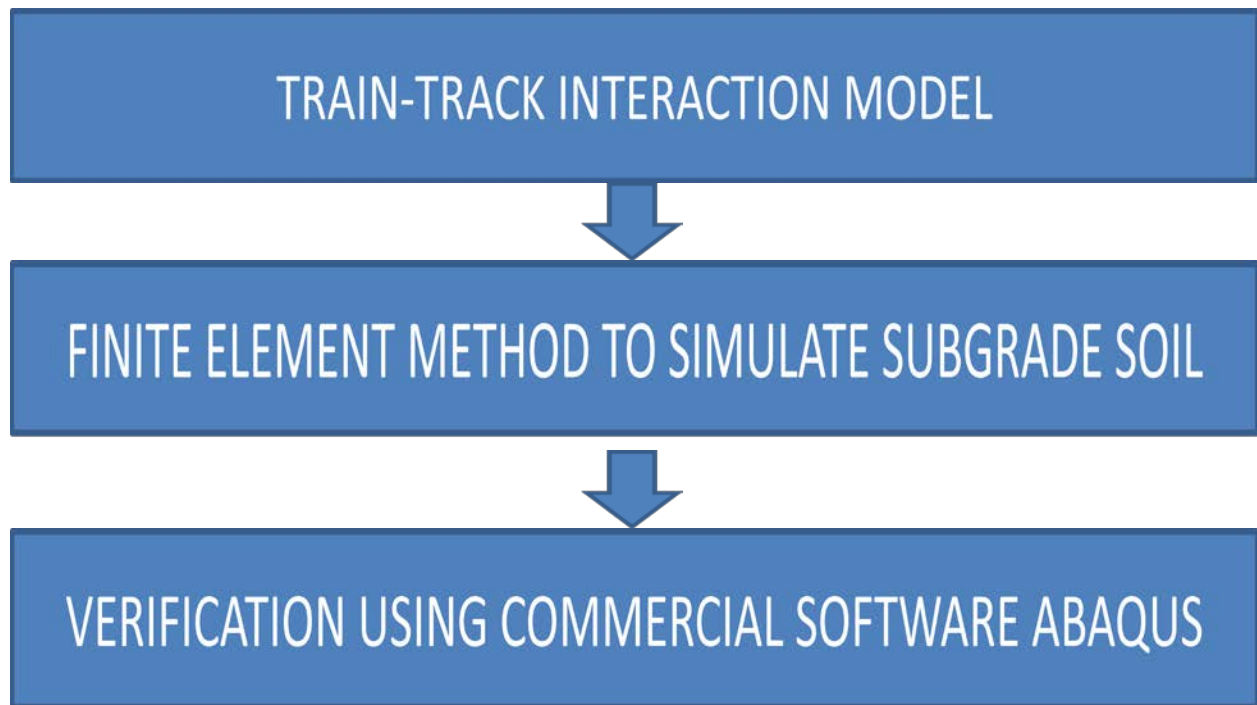


Figure 5 Flow Chart



## Chapter 3

### LITERATURE REVIEW

#### 3.1 Rail Track Model

The function of rail track models is to interrelate each component in the track structure in order to simulate the integrated properties in determining the reactions of the moving train load. Rail track models which include all the track components enable us to predict the track performance more effectively and precisely.

As an important track component, rail is used to support and guide the vehicle by providing smooth running surfaces. The forces of a train on the top of the rail are distributed over sleepers. Lateral loading, from train wheels, uniformly distributed on rails, and longitudinal loading, which is generated by braking and acceleration, are also passed on to the track structure. In simulation, the rails are usually simplified as two mathematical models: Euler-Bernoulli Beam (E-B beam) and Rayleigh-Timoshenko Beam (R-T beam). E-B beam only considers bending behavior of rails. R-T beam theory includes not only bending, but also shear deformation of the beam. Train-induced vibrations with frequency less than 500 Hz carry higher energy. As a result, the vibration dissipates at a lower rate and its impact is felt farther from the source. Also, it was found that when the frequencies of train loading are less than 500 Hz, shear deformation of the rail can be neglected (Dahlberg, 2003). Hence, E-B beam theory is sufficient to simulate the ground vibration induced by High Speed Trains.

Rail pad is an absorbing component between the steel rail and sleepers. The functions of rail pad are transferring the rail load to sleepers, and screening out the high frequency force. Usually, rail pads are embedded under rails acting as electrical insulation and as a protective layer for sleepers. The rail pads also affect the dynamic behavior of the track. Pairs of springs and dashpots are introduced to simulate the effect of rail pads. The sleepers are placed in the transverse direction to the track, or, in other words, perpendicular to the movement direction of the vehicle. Their function is to maintain track gauge and fasten the rails to be aligned for both construction and operation of rail track. Sleepers also transmit the train loading to the lower track structure. Ballast denotes a layer of crushed stone of uniform size, on which the sleepers are resting. It is granular material used to provide support for sleepers and fill the spacing between sleepers. The granular material is hard to simulate. The objective for this research focuses on overall track performance, not each track component.

So, the ballast is also simulated as a spring/dashpot system without its mass. Many researchers have been working on track models. Two-dimensional models are suitable for study of vertical track performance, but they ignore the transverse cross-section of the track. Three-dimensional models are rapidly being developed because they can provide more detailed performance of the track and responses from all directions. However, three dimensional models are usually very time-consuming. As a potential compromise, a 2.5D method is very promising, in that it has three-dimensional motions but only two-dimensional elements. This feature makes it very time-saving.

##### 3.1.1 Two Dimensional Track Models

Many early models are based on a beam-on-elastic-foundation (BOEF) formulation. In this system, rails, simulated as continuous Euler-Bernoulli beams, are placed on elastic spring supports. Thus, the rail reactions in the longitudinal direction are proportional to their deflections. This model which is shown in Figure 6 below has long been accepted and used in modeling rail track, and is the backbone of the subsequent models.

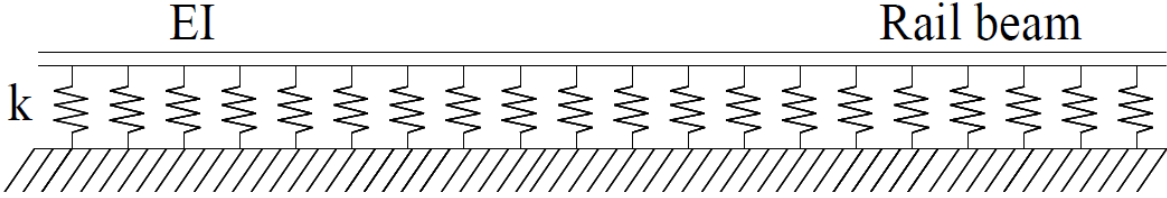


Figure 6 Beam on Elastic Foundation (Coenraad, 2001)

Kaynia et al (2000) developed a 2D model, which evolved from the basic BOEF. Besides the bending rigidity and mass per unit length of embankment, hysteretic damping ratio is also added. The whole track system is bonded to the half-space at discrete points (nodes) along the embankment. The train loads that applied to the nodes with time shifts are combined into one concentrated load at the centerline of the bogie.

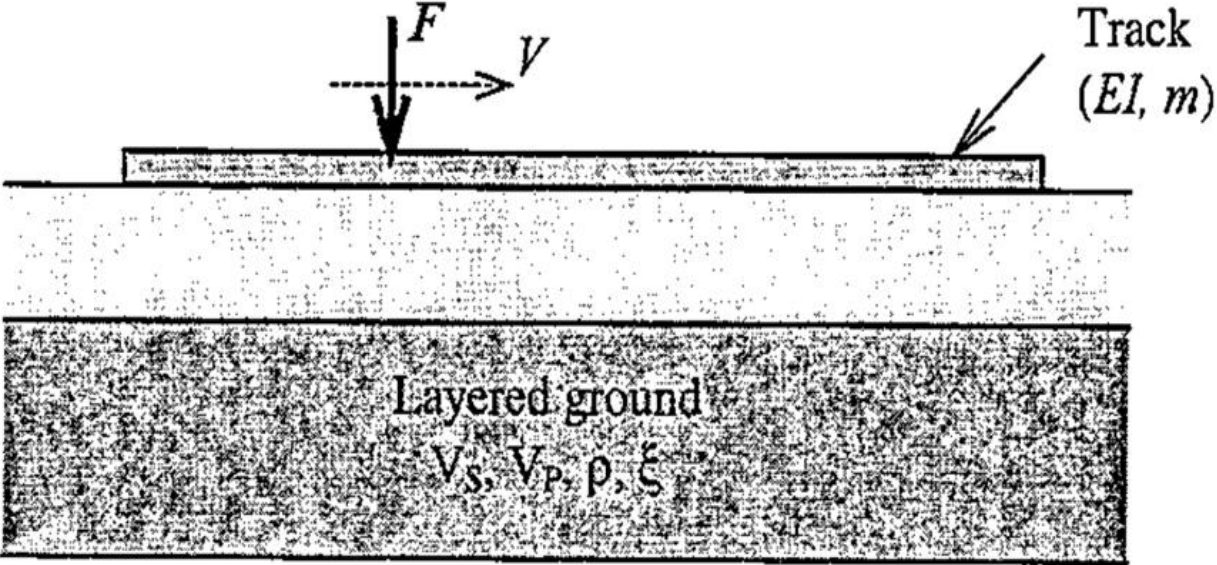


Figure 7 Schematic Representation of embankment ground interaction model (Kaynia, 2000)

Madshus and Kaynia (1999) simulated and analyzed the response of the track-ground system from high-speed train passage by using a computer program, Vibtrain. The ground is modeled as layered half-space by Green’s functions. And the rail/ballast system is taken as a beam by finite

elements. Track and ground are related by enforcing vertical displacement and stress. Train loads are applied to the system through delaying the loads point by point according to different train speeds.

J.J. Kalker (1996) introduced a discretely supported beam model, which is shown in Figure 8 below. In his research, the rail is modeled as an Euler beam. The most salient feature of the paper is the irregular discrete support of the sleepers. The vertical displacement of a railway rail due to a travelling vertical point load of variable intensity is calculated.

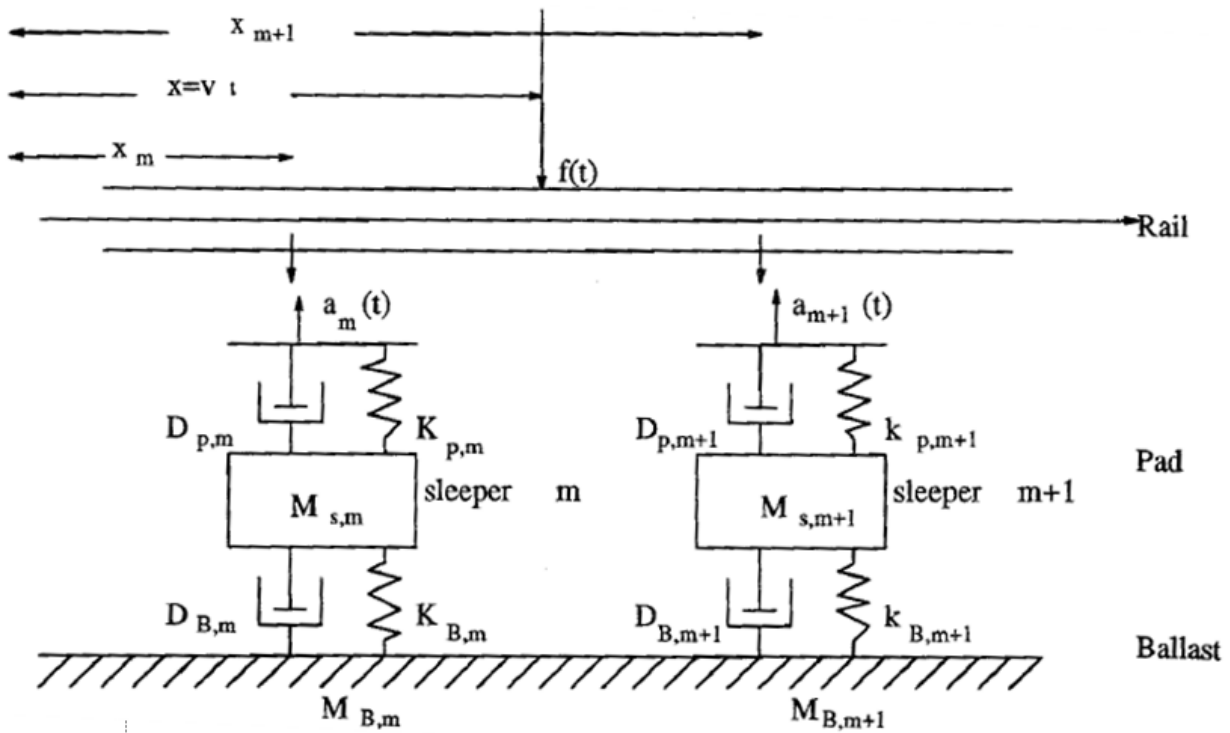


Figure 8 Discretely supported Beam Model (Kalker, 1996)

To introduce ballast effect Huang et al. (2009) introduced a 2D track model, called the Sandwich track model, which is shown in Figure 9 below. The ballast is modeled as discrete masses that are connected to sleepers and the ground with spring/dashpot. Rail, in this Sandwich model, is also modeled as an Euler beam. The rail pad, tie and ballast are all represented by mass and spring/dashpot in this research.

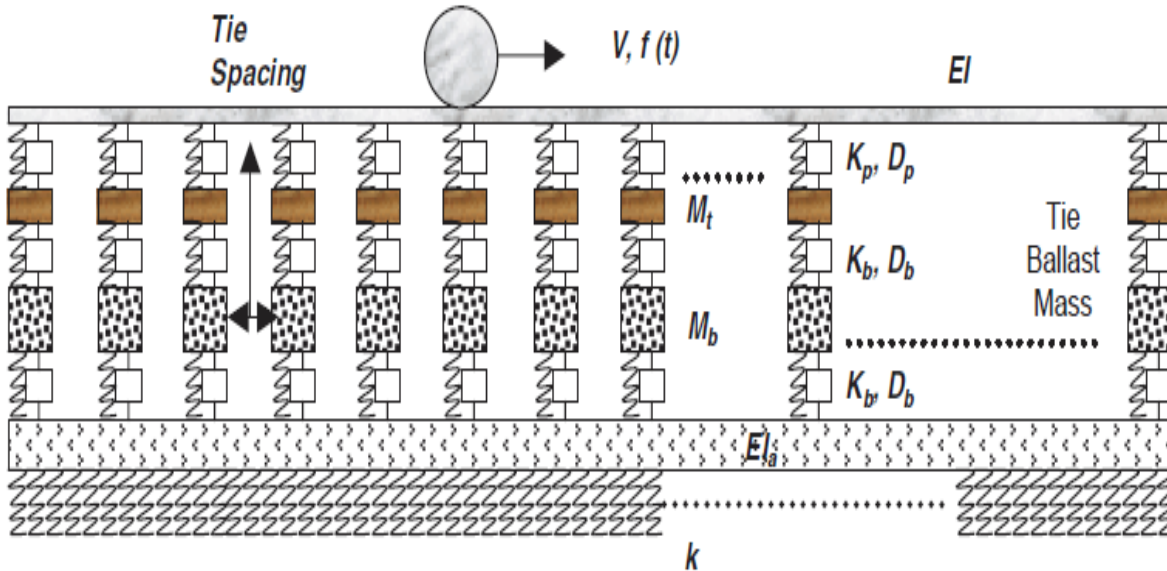


Figure 9 Sandwich Track Model (Huang et al.)

However two dimensional models are insufficient to simulate the ground vibration vertical to the track, thus the Mach radiation effect of soil cannot be detected. Also, most early researchers work on ground-induced vibrations was based on plane strain assumptions, and the beam on elastic foundation system. Hence, strictly speaking, only a more sophisticated three dimensional model is appropriate to investigate the wave propagation in the ground.

### 3.1.2 Three Dimensional Track Models

Due to the limitations of 2D models, many researchers proposed 3D models to simulate the integrated track system to study the ground vibration generated by moving trains.

The track system of Takemiya (2003) shown in Figure 10 below, including rails, sleepers and ballast bed, is also modeled as an Euler-Bernoulli beam on elastic foundation. The Fourier Transform that changes the time-domain problem to a frequency domain problem is applied to obtain the solution for a moving load. The railway track is modeled as an Euler beam resting on ballast, then on the ground. However, the effect of sleepers is not considered in this model.

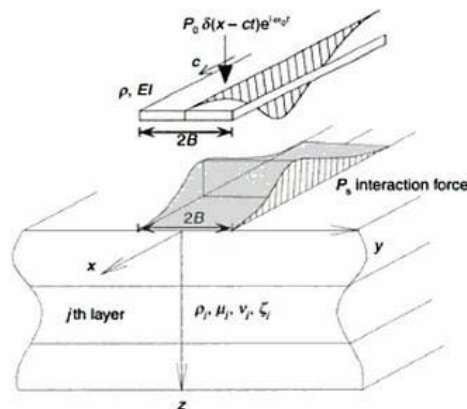


Figure10 Track Ground Interaction (Takemiya, 2003)

Cai (1994) introduced a 3D model with coupling spring/damper elements, representing the mechanism of rail pads, sleepers and ballast bed. The rail can be either modeled as R-T beam or E-B beam to describe the rails which are discretely coupled with sleepers. Another beam element with mass is placed to model the sleepers. The sleeper rests on another spring-damper system, as shown in Figure 11 below.

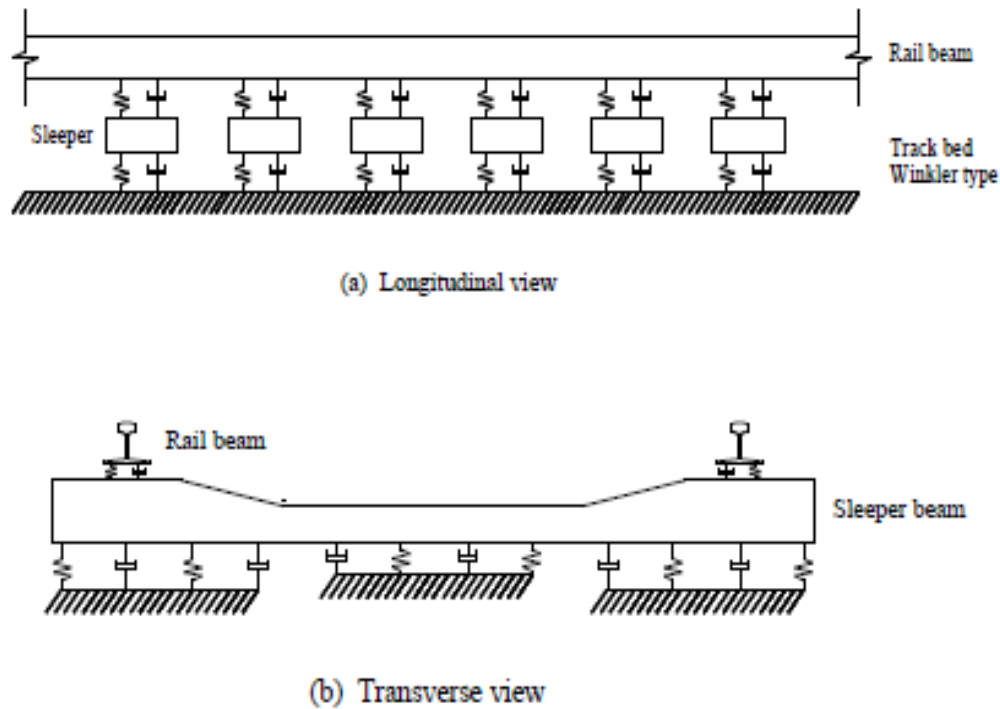


Figure 11 Rail on discrete support (Cai 1994)

The previous types of rail track models do not include the effect of real ballast behavior, such as ballast deformation. Ballast is formed by granular material, which cannot be modeled the same as sub grade soil. To fulfill this task, discrete ballast masses need to be used to mimic the mechanical responses of a granular material. Furthermore, when more and more details of the track such as geometry and/or material properties have to be considered, numerical solving techniques are employed. 3D Finite Element Model is one of the most widely used models as it is able to cover almost all geometry considerations and is commercially available. In order to investigate the train speed effect, XiTRACK Limited proposed a 3D FEM model incorporating all track components, including rail, sleepers, ballast, and subgrade. Train load is modeled as a sequence of constant load running at a constant speed over the rails. Detailed information about the model is given by Woodward et al. [Woodward, 2005].

### 3.1.3 2.5 Dimensional Track Models

Although 3D FEM can always serve as a benchmark program to calibrate different track models when field measured data is not available, it is time consuming and is usually utilized only for particular cases. Hence, realizing the limitations of two-dimensional models and the unfavorable time efficiency associated with three-dimensional models, researchers [Bian, 2008; Yang & Hung, 2001, 2003] proposed an innovative model called 2.5D FEM by assuming the track property remains uniform along the direction of train movement; only a profile of half-space vertical to the direction of load movement is considered. Also, the 2.5D approach is suitable for tunnels due to its assumption of uniform material properties along the movement direction. The accuracy of this approach is verified via comparison of results obtained from analytical solutions [Yang & Hung, 2009].

Bian (2008) modeled track-ground interaction by moving load using 2.5D FEM. The material properties and geometry are assumed consistent along the movement direction. The ground is modeled by isoparametric quadrilateral elements to condense the 3D issue to a plane strain problem. The 2.5 Dimensional BOEF is shown in Figure 12 below.

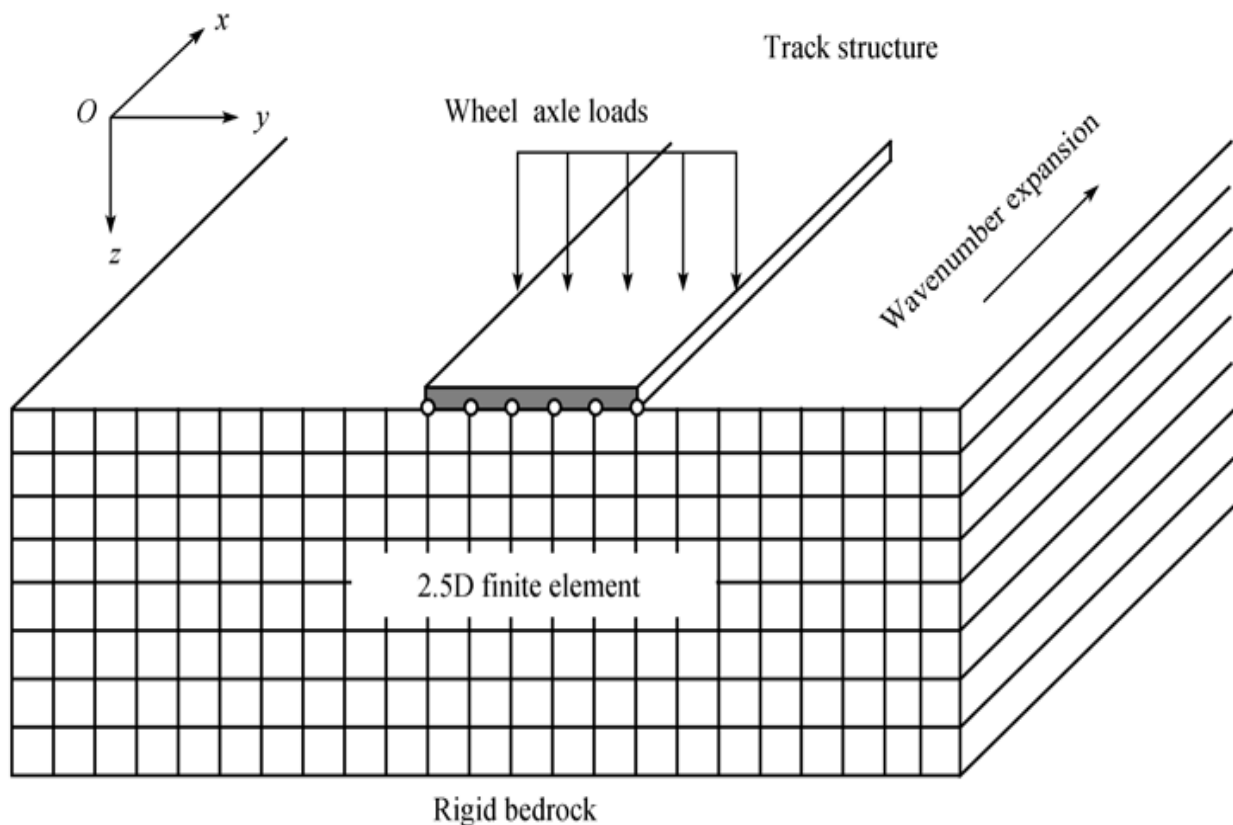


Figure 12 Track Ground interaction model with a moving load(Bian, 2008)

For 3D soil, the 2.5D FEM technique is employed. Fourier Transform was only performed in the direction of train movement, and the transverse and vertical directions were discretized by plane stress quadrilateral finite elements. A typical 2.5D element is illustrated in Figure 13 below.

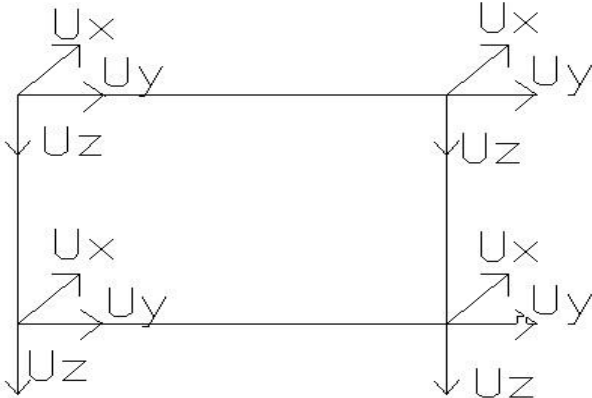


Figure 13 2.5 D element (2D element, 3D motion)

### 3.2 Train – Track Interaction Models

The vehicle-track dynamic has consistently not been valued in previous research. However, due to the track irregularities and vehicle dynamic properties, a train-track interaction model is essential. A 3D vehicle-track system allows dynamic loads to transmit to the track structure and sub grade through the wheel-rail contact. Over the past decade, research on train-track interaction models has been rapidly developing, with computer technology facilitating the possibility of analyzing large and sophisticated dynamic train-track coupled models (Sheng, 2004; Zhai, 2010).

In Sheng's model (2004), two rails are taken as a single Euler beam with mass per unit length and bending rigidity. The lower beam, also with a mass per unit length but no bending rigidity, represents sleepers. The continuous springs between two beams have a complex stiffness by considering spring stiffness and damping loss factor. The ballast has an infinite length and mass; vertical complex stiffness is modeled as a viscoelastic layer. The ground is simply modeled as a layered elastic medium of infinite extent. This model is shown in Figure 14 below.

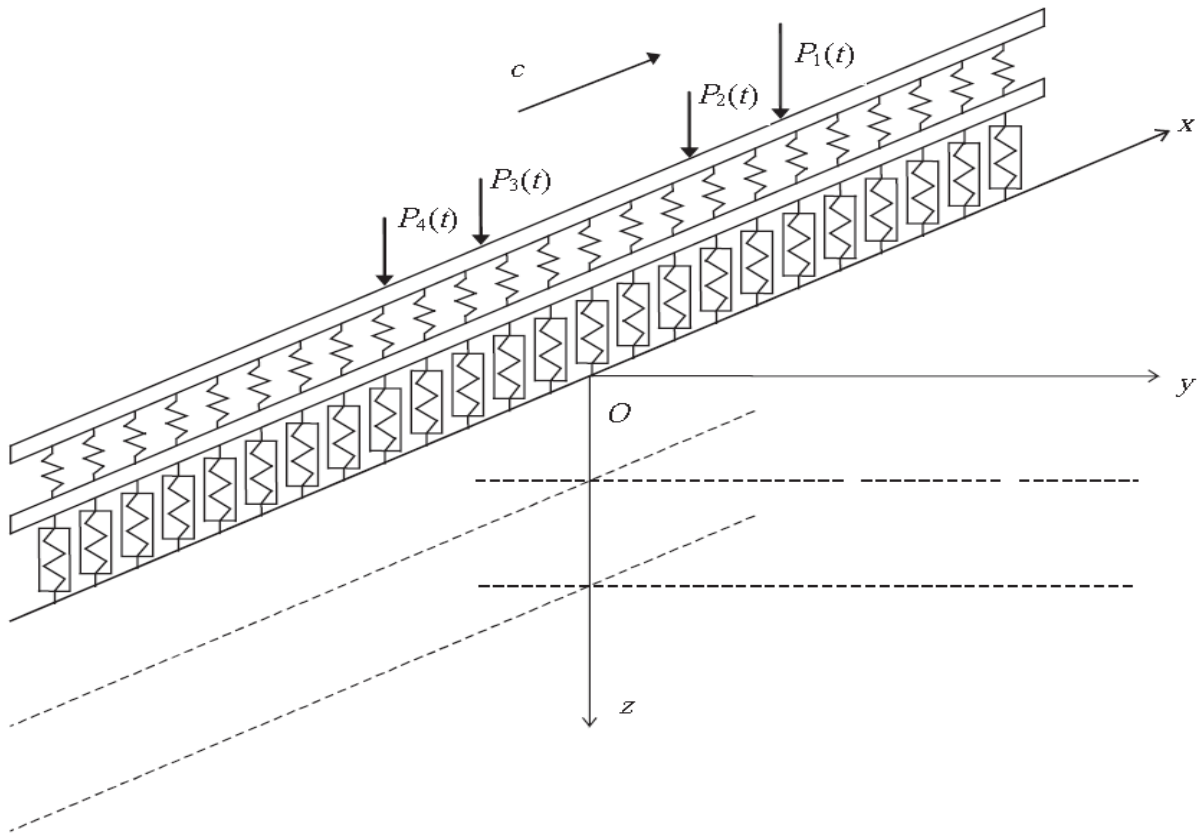


Figure 14 Sheng's Model (2004)



However, the validity of the assumption for the soil part (i.e. half infinite space) is still limited. Also, this model becomes insufficient in cases that tracks are laid on slopes, or inside tunnels, where cross-section geometry characteristics of the track and foundation are of great importance. In order to account for the continuity and coupling effect of the interlocking ballast, Zhai (2003) recommended shear stiffness and shear damping between adjacent ballast masses. The model is shown in Figure 15 below.

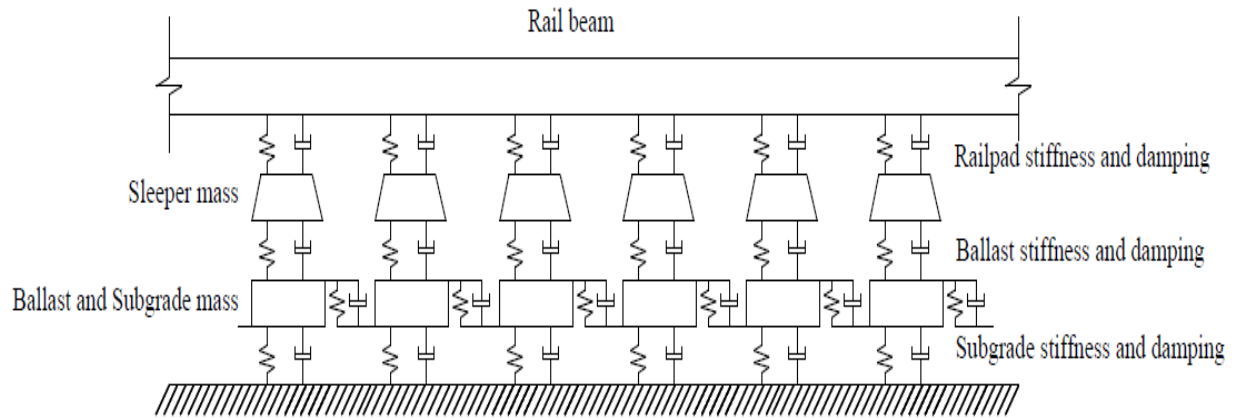


Figure 15 A complete model of discrete supports ballast mass, stiffness and damping(Zhai 2004)

### 3.3 Train Speed Effect

Vibration would not have been an unfamiliar occurrence in the past. Even during the age of horse-driven carriages on cobblestone streets, strong complaints of vibrations arose from occupants of buildings along the route. However, no convincing explanations were made in that period. Therefore, for a long period, train loads have been believed to be reasonably assumed as quasi-static moving loads. Krylov (1994) considered that a train will encounter the „sound barrier“ (critical speed) when reaching the velocity of Rayleigh surface waves propagating in the ground. The critical condition is explained as resonance between the moving train and the Rayleigh wave of the sub grade soil. This phenomenon can be compared with the Mach effect by supersonic jets and the Cherenkov radiation of light. Depending on whether the speed of a moving train is less than, great than or close to the velocity of Rayleigh wave, the train speed effect can be categorized as subsonic, supersonic and transonic. (Kaynia, 2000) When a train runs at a speed less than the Rayleigh wave velocity, the ground vibrations behave and represent a quasi-static condition. The increase in vibration magnitude is slow relative to increase in train speed. However, under transonic and supersonic cases, the dynamic effects of ground vibrations perform like the development of Mach lines and Mach surfaces. Vibration magnitude increases exponentially with train speed. Critical speed, the focus of this research, is defined as the speed at which moving trains resonate with waves traveling in the track and produce excessive ground vibrations. The effect of train speed is illustrated in Figure 16 and 17.

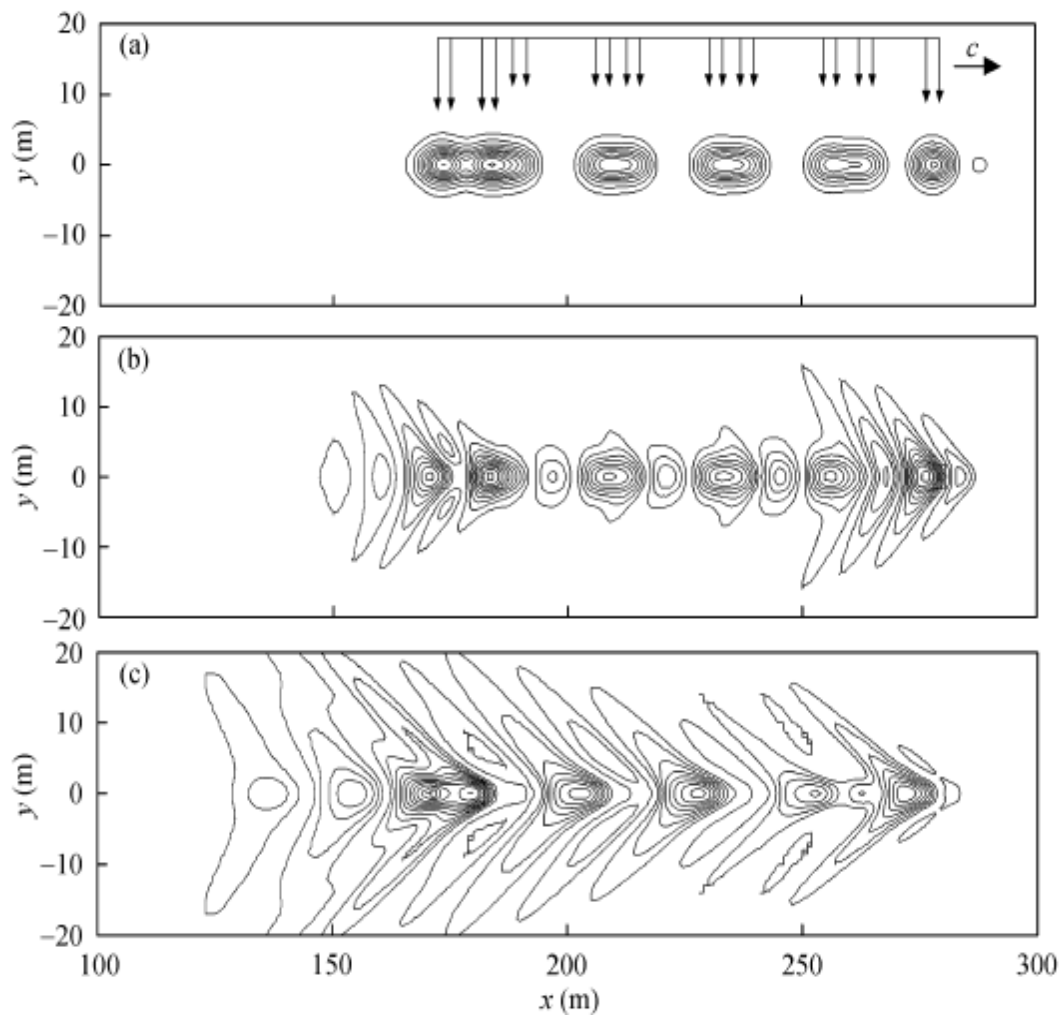


Figure 16 Ground surface deflection contour plots for trains running at different speeds: (a)  $c=100$  km/h; (b)  $c=200$  km/h; (c)  $c=300$  km/h [Bian, 2008]

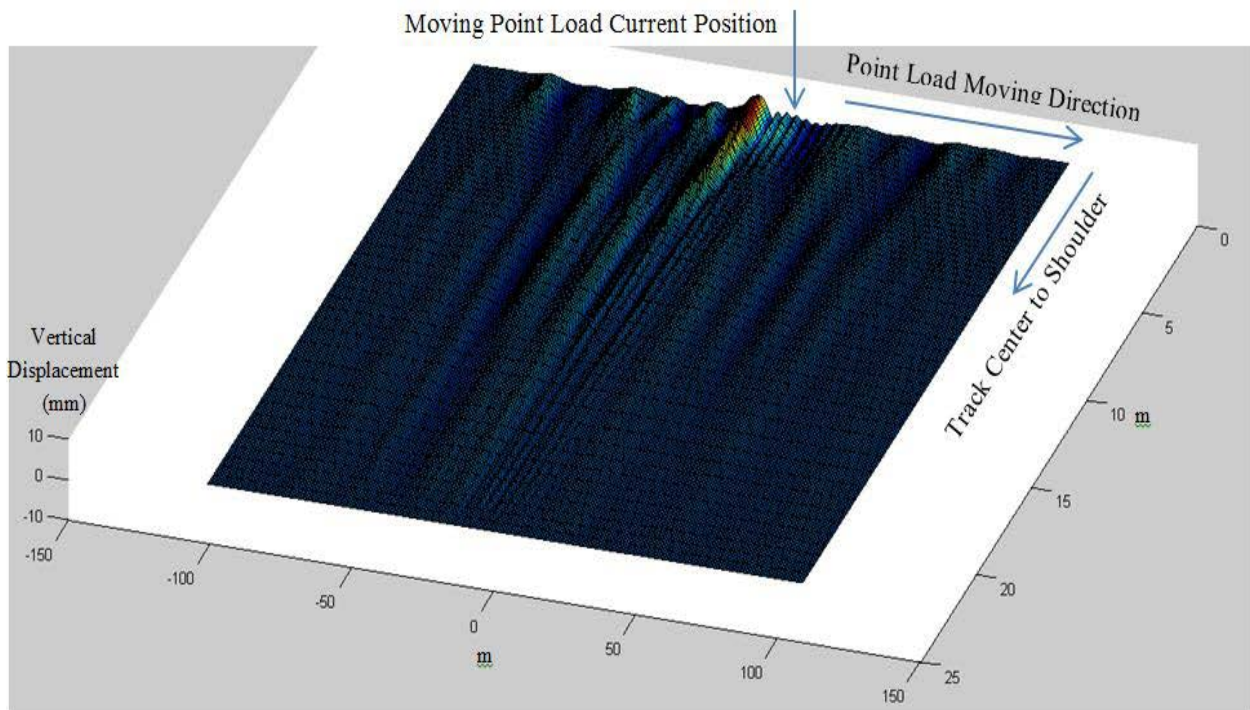
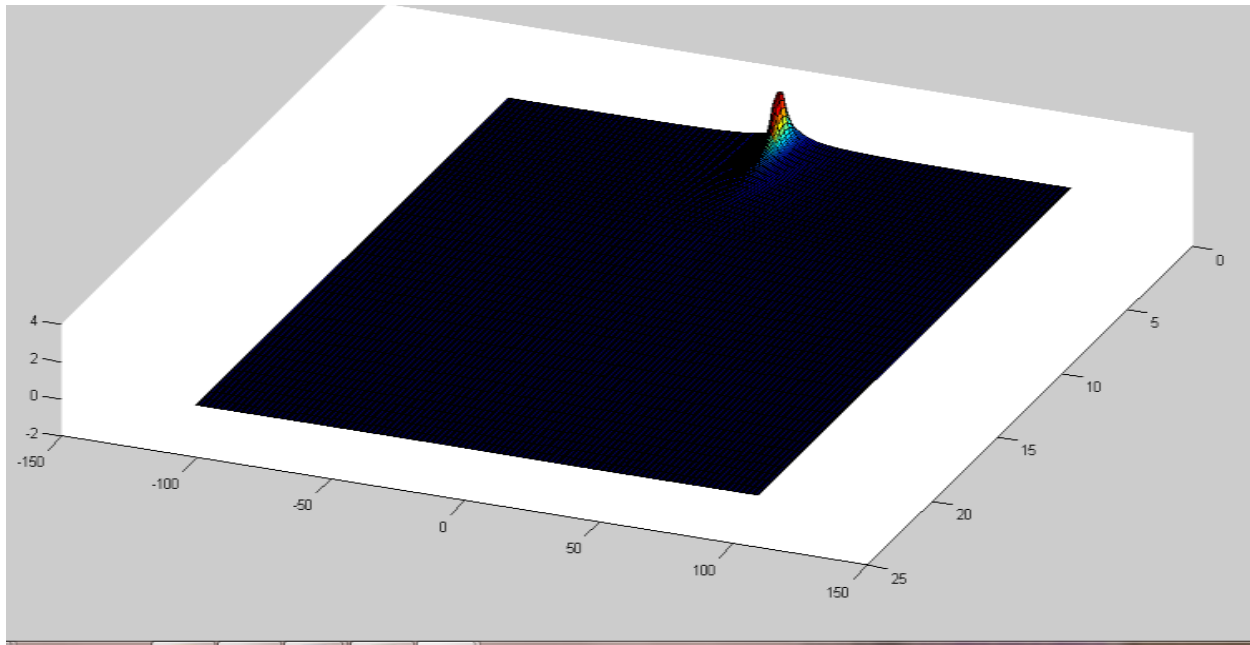


Figure 17 Ground surface deflection profiles under low-speed (top) and high-speed (bottom) moving objects [Huang, 2013]

As can be observed, both the displacement field and vibration level increase significantly when trains run at critical speed. The reason may be the surface irregularity of wheels and rails, and the rise and fall of axles over sleepers. In practice, the rails were found not to be straight, and the wheels were found not to be round. The vibrations can readily arise when the mass of the vehicle is being made to follow the alignment of the track. Eventually, this would result in dynamic forces. Similarly, the wheels would traverse the joints on the rails, leading to dynamic forces. In addition, it can be understood that the actual support stiffness under the rails is not uniform. Ties with intervals between each other are laid vertical to the movement direction, acting as periodic supports for the upper structure. The rails right under the ties offer higher support stiffness when the wheels are right over, but in between, the ties offer smaller stiffness, which makes the rail stiffness take over the role to support the train predominantly. Furthermore, even when trains are moving on smooth rails, vibrations can still be induced by the regular repetitive action of the moving train loads [Yang & Hung, 2009].

In rail engineering, the theory is similar. If train speeds reach critical velocity and the vibrations resonate with the natural frequency of the subgrade soil or the building nearby, it can cause increasing ground vibration, internal noise in building, and even structural damage in buildings up to 250m from the track. This critical velocity is relative to the speed of propagation of Rayleigh waves on the ground, which depends on geotechnical properties. Rayleigh waves are surface waves, commonly produced by earthquakes but also caused by heavy, fast-moving loads such as high speed trains. Rayleigh waves typically take two-thirds of the energy in a vibrating soil system, and also attenuate at a much slower rate than P- and S-waves. Consequently, Rayleigh waves are so important because they are able to cause considerable structural vibrations when trains reach critical speed. The critical speed effect is the combination of train suspension system, train speed, rail surface smoothness, track structure, and most importantly the ground soil or sub grade. It has been usually investigated case-by-case and there is a lack of general solutions. One possible solution is to develop a sophisticated computer model to identify any potentially critical train-track combinations. Through the computer model proposed in this research, the critical speed effect can be predicted which will greatly decrease the risk of the operation of railway traffic.

## Chapter 4 METHODOLOGY

### 4.1 Sandwich Dynamic Track Model

As previous models are insufficient to investigate the critical speed effect due to many reasons : 1) three dimensional numerical solutions are time consuming 2) two dimensional numerical solutions are effective in computation of vertical responses but do not consider track cross section. 3) simply using stiffness  $k$  is inadequate to study critical speed effect.

A three dimensional track model is used here in my thesis work which considers that vehicle and track are in contact to each other by Hertzian contact theory. In the sandwich dynamic track model rails considered as a beam that do not considers shear deformation. Rail pad, sleepers and ballast are modeled as a spring mass damper system with different spacing in between them.

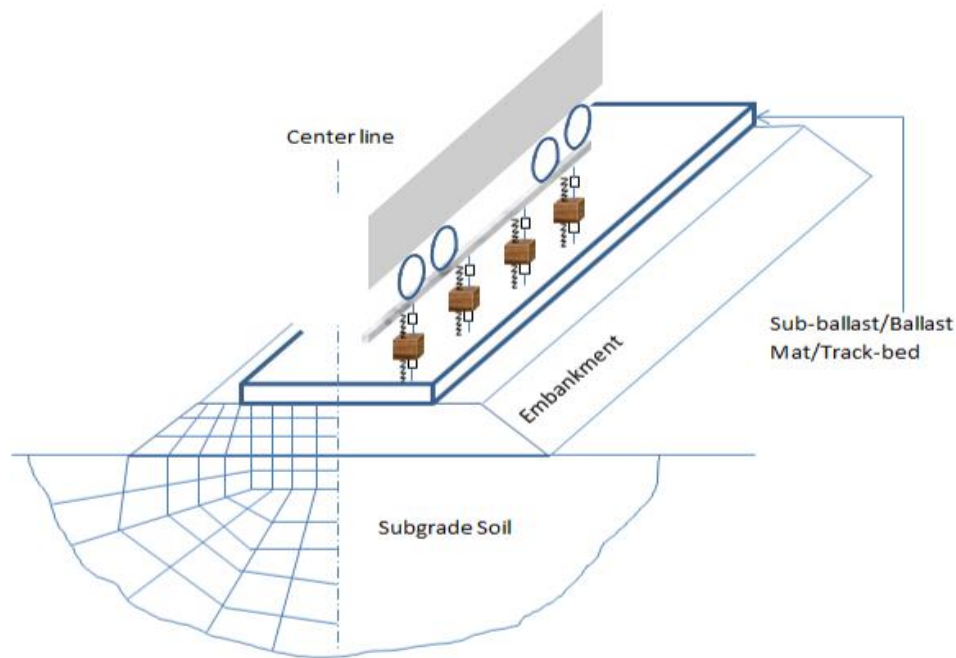


Figure 18 Sandwich Dynamic track Model

### 4.2 Mathematical Formulation

The physical train track ground system consists of vehicle track coupled subsystem and the sub grade system. The solution of the total system is divided into two parts, vehicle track subsystem and the sub grade system, which are coupled through the deformation compatibility and force equilibrium of nodes at the track subgrade interface. The reaction force of the track bottom is calculated by vehicle track coupled dynamics in the vehicle track subsystem. According to the compatibility condition of the force balance the reaction force will be loaded on the corresponding nodes of the subgrade. In the sub grade system the displacement of sub grade is

calculated by FEM, which in reverse excites the deformation of track structure. So two systems are essentially coupled to each other. In order to establish the coupling relationship between the track and the sub grade, the combined slab track element is adopted. In the combined beam rail beam, sleeper beam and foundation beam are connected together by a spring damper system. The combined element consists of two elastic beams, which involve four nodes:  $i, j, k, l$ . Each combined element has eight degrees of freedom (DOFs):  $z_{ri}, \theta_{ri}, z_{rj}, \theta_{rj}, z_{sk}, \theta_{sk}, z_{sl}, \theta_{sl}$ , where  $Z$  and  $\theta$  are the vertical displacement and the rotation angle of the node. The displacement of the arbitrary point of the combined beam can be expressed as:  $Z = N q$  where  $N$  is the shape function matrix of order 3 Hermite Interpolation functions and  $q$  are defined above. By means of the variation principle the stiffness matrix of combined element  $K$ , damping matrix of combined element  $C$  and mass matrix of the combined element  $M$  can be deduced by first variation of system potential energy.

$$K = \sum K_1 + \sum K_2 + \sum K_3$$

$$C = \sum C_1 + \sum C_2$$

$$M = \sum M_1$$

Where  $K_1$  is the element stiffness matrix of the rail and sleeper and  $K_2$  and  $K_3$  are element stiffness matrices of the rail pad and ballast;  $C_1$  and  $C_2$  are the damping matrices of the rail pad and the ballast; and  $M_1$  is the element mass matrix of the rail and slab. Based on the principle of total potential energy final mathematical formulation becomes as

$$M \ddot{q} + C \dot{q} + K q = F(t).$$

Here  $M$  is the mass matrix;  $C$  and  $K$  are the stiffness and damping matrices of track system.  $\ddot{q}$ ,  $\dot{q}$  and  $q$  are vectors of accelerations, velocity and displacements.

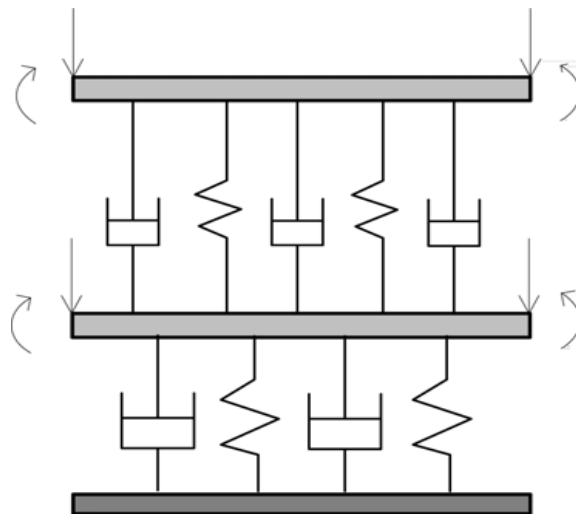


Figure 19 Sketch of Track Element

## 4.3 Modeling Procedures in ABAQUS/CAE

The ABAQUS/CAE environment is divided into different modulus, where each module defines a logical aspect of the modeling process; for instance, defining the geometry, defining the material properties, and generating a mesh. The GUI interface generates an input file with all information of the model, to be submitted to the solver, using ABAQUS/Standard or ABAQUS/Explicit routines. The solver performs the analysis and sends the information back to ABAQUS/CAE for evaluation of the results.

### 4.3.1 Modules

Most models created in ABAQUS/CAE are assembled from different parts. It always starts with creating different parts separately in the parts module. Different parts may need different material properties, which are defined in the property module. A full range of material properties are available in ABAQUS, such as elastic and plastic behavior, as well as thermal and acoustic behavior. The model then is assembled in the assembly model, by combing the different instances originates from different parts. In the step module the analysis is divided in different analysis step, such as static and dynamic analyses. These can be combined in a way to resemble the physical problem that is to be analyzed. The instances in the model will not interact with each other until they have been connected in the interaction module. Connector elements can be defined, to simulate for example spring or dashpot behavior. The loads acting on the model are defined in the load module, as well as boundary conditions. The load and boundary conditions can be defined to vary over time as well as over different steps. The whole model is then meshed in the mesh module. The meshing techniques vary with the element type and the geometry of the model.

### 4.3.2 Elements

Abaqus has an extensive element library to provide a powerful set of tools for solving many different problems. All elements used in ABAQUS are divided into different categories, depending on the modeling space. The element shapes available are beam elements, shell elements and solid elements and the modeling space is divided into 3D space, 2D planar space and axisymmetric space. Some examples of types of finite elements are shown in Figure 20 below.

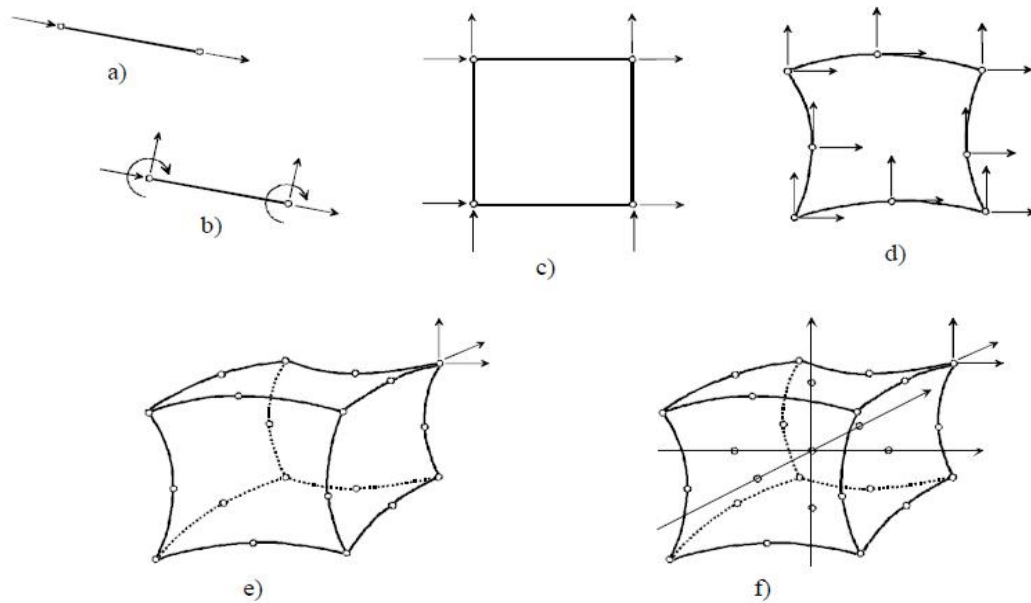


Figure 20 Examples of different types of finite elements (nodes and dofs indicated by small circles and small arrows, respectively): a) Two-noded linear truss element (for axial loads only), b) Beam element that allows axial, transversal and moment loads, c) Four-noded 2D element, d) Eight-noded 2D element, e) 20-noded 3D element and f) 27-noded 3D element (with local coordinate system indicated through the centre node). (Elements e) and f) with only three of the dofs displayed

#### 4.3.2.1 Beam elements

Beam elements have been used for the rails and sleepers modeling. A beam element is an element in which assumption are made so that the problems reduced to one dimension mathematically. The primary solution variable is then functions of the length direction of the beam. For this solution to be valid, the length of the element must be large compared to its cross-section. There are two main types of beam elements formulations, the Euler-Bernoulli theory and the Timoshenko theory, which have been discussed. The Euler-Bernoulli theory assumes that the plan cross-sections, initially normal to the beam axis, remain plane, normal to the beam axis, and undistorted. All beam elements in ABAQUS that use linear or quadratic interpolation are based on this theory. The Timoshenko beam theory allows the elements to have transverse shear strain, so that the cross-sections don't have to remain normal to the beam axis. This is generally more useful for thicker beams.

#### 4.3.2.2 Solid element

Solid elements in two and three dimensions are available in ABAQUS. The two-dimensional solid element allows modeling of plane and axisymmetric problems. In three dimensions the isoparametric hexahedron element is the most common, but in some cases complex geometry may acquires tetrahedron elements. Those elements are generally only recommended to fill in awkward parts of mesh. ABAQUS provides both first-order linear and second-order quadratic



interpolation of the solid elements. The first-order elements are essentially constant strain elements, while the second-order elements are capable of representing all possible linear strain fields and are more accurate when dealing with more complicated problems

#### 4.3.2.3 Rigid elements

For the discretely supported track including ballast mass model, the ballast is modeled as rigid element since only its mass is concerned. A rigid part represents a part that is so much stiffer than the rest of the model that its deformation can be considered negligible. Computational efficiency is the principal advantage of rigid parts over deformable parts. During the analysis element-level calculations are not performed for rigid parts. Although some computational effort is required to update the motion of the rigid body and to assemble concentrated and distributed loads, the motion of the rigid body is determined completely by the reference point. There are two kinds of rigid parts: discrete rigid part and analytical rigid part. When describing a rigid part an analytical rigid part will have the priority because it is computationally less expensive than a discrete rigid part.

#### 4.3.2.4 Spring and Dashpot elements

Spring and dashpot elements are widely used in this model. For instance, the railpads between the rail and the sleeper, connectors for bounding adjacent ballasts in each direction, and the boundary conditions for constraining the sleepers. SPRING1 and SPRING2 elements are available only in Abaqus/Standard. SPRING1 is between a node and ground, acting in a fixed direction. SPRING2 is between two nodes, acting in a fixed direction. The SPRINGA element is available in both Abaqus/Standard and Abaqus/Explicit. SPRINGA acts between two nodes, with its line of action being the line joining the two nodes, so that this line of action can rotate in large-displacement analysis. The spring behavior can be linear or nonlinear in any of the spring elements in Abaqus.

### 4.3.3 Analysis Type

#### 4.3.3.1 Linear Eigen value Analysis

Linear eigen value analysis is used to perform an eigen value extraction to calculate the natural frequencies and corresponding mode shapes of the model. The analysis can be performed using two different eigen solver algorithms, Lanczos or subspace. The Lanczos eigen solver is faster when a large number of eigen modes are required while the subspace eigen solver can be faster for smaller systems. When using the Lanczos eigen solver, one can choose the range of the eigen values of interested while the subspace eigen solver is limited to the maximum eigen value of interest.

#### 4.3.3.2 Steady-state Dynamic Analysis

One way to investigate the dynamic properties of a railway track is to load the track with a sinusoidal force. At frequencies up to about 200Hz, this can be done by using hydraulic cylinders. If one wants to investigate the track response at higher frequencies, the track may be excited by an impact load, for example from a sledge-hammer. In ABAQUS steady-state dynamic analysis provides the steady-state amplitude and phase of the response of a system due

to harmonic excitation at a given frequency. Usually such analysis is done as a frequency sweep by applying the loading at a series of different frequencies and recording the response; in Abaqus/Standard the direct-solution steady-state dynamic procedure conducts this frequency sweep. In a direct-solution steady-state analysis the steady-state harmonic response is calculated directly in terms of the physical degrees of freedom of the model using the mass, damping, and stiffness matrices of the system. By editing the keyword after create the model, the amplitude of the rail vibration in steady-state dynamic analysis could be obtained.

#### 4.3.3.3 General Static Analysis

The general static analysis can involve both linear and nonlinear effects and is performed to analyse static behavior such as deflection due to a static load. A criterion for the analysis to be possible is that it is stable. A static step uses time increments, not in a manner of dynamic steps but rather as a fraction of the applied load. The default time period is 1.0 units of time, representing 100% of the applied load. The nonlinear effects are expected, such as large displacements, material nonlinearities, boundary nonlinearities, contact or friction, the NLGEOM command should be used. When dealing with an unstable problem, such as in buckling or collapse, the modified Risk method can be used. It uses the load magnitude as an additional unknown, and solves simulations for loads and displacements. This method provides a solution even if the problem is nonlinear.

#### 4.3.3.4 Dynamic Implicit Analysis

The dynamic implicit analysis method is used to calculate the transient dynamic response of a structure. A direct-integration dynamic analysis in Abaqus/Standard must be used when nonlinear dynamic response is being studied. The general direct-integration method provided in Abaqus/Standard, called the Hilber-Hughes-Taylor operator, is an extension of the trapezoidal rule. The half-step residual is the equilibrium residual error halfway through a time increment,  $t + \Delta t/2$  and once the solution at  $t + \Delta t$  has been obtained, the accuracy of the solution can be accessed and the time step adjusted appropriately. The principal advantage of the Hilber-Hughes-Taylor operator is that it is unconditionally stable for linear systems; there is no mathematical limit on the size of the time increment that can be used to integrate a linear system. This nonlinear equation solving process is expensive; and if the equations are very nonlinear, it may be difficult to obtain a solution. However, nonlinearities are usually more simply accounted for in dynamic situations than in static situations because the inertia terms provide mathematical stability to the system; thus, the method is successful in all but the most extreme cases. The choice of the time increment depends on the type of analysis performed. In dynamic problems, a smaller time increment than the stable one might be used, to get an accurate result depending on the variations in the structure. There are two ways of defining the time increment: automatic or fixed incrementation. The automatic incrementation scheme is provided for use with the general implicit dynamic integration method. The scheme uses a *half-step residual* control to ensure an accurate dynamic solution. By defining initial, minimum, and maximum increment sizes the automatic time increments can be chosen. If no convergence is achieved, a smaller one is used until convergence is achieved, down to the minimum increment defined.

# CHAPTER 5

## ANALYSIS RESULTS

### 5.1 Verification

The purpose of verification is to improve and test the accuracy of proposed three dimensional sandwich dynamic track model by comparing with the results of FEM software ABAQUS. It would be time consuming and uneconomical to verify the model using field tests and also it is observed from previous researches that results are approximately same in both the cases. The results from dynamic loading will not be simulated exactly. So for dynamic analysis field test should be conducted to simulate and verify the model.

### 5.2 High Speed Track Modeling

High Speed Trains track consist of sub grade, frost protection layer, hydraulically bonded layer, concrete bearing layer, sleeper and rails. On concrete bearing layer are based for the sleepers, such that horizontal faces are carried by concrete anchors. Concrete bearing layer lies over the hydraulically bonded layer. A hydraulically bonded layer is a layer of soil mixed with water and cement for increase its strength and stability. Hydraulically bonded layer lies over the Frost protection layer. Frost protection layer is usually made up of compacted permeable soil. Frost protection layer lies over the subsoil.

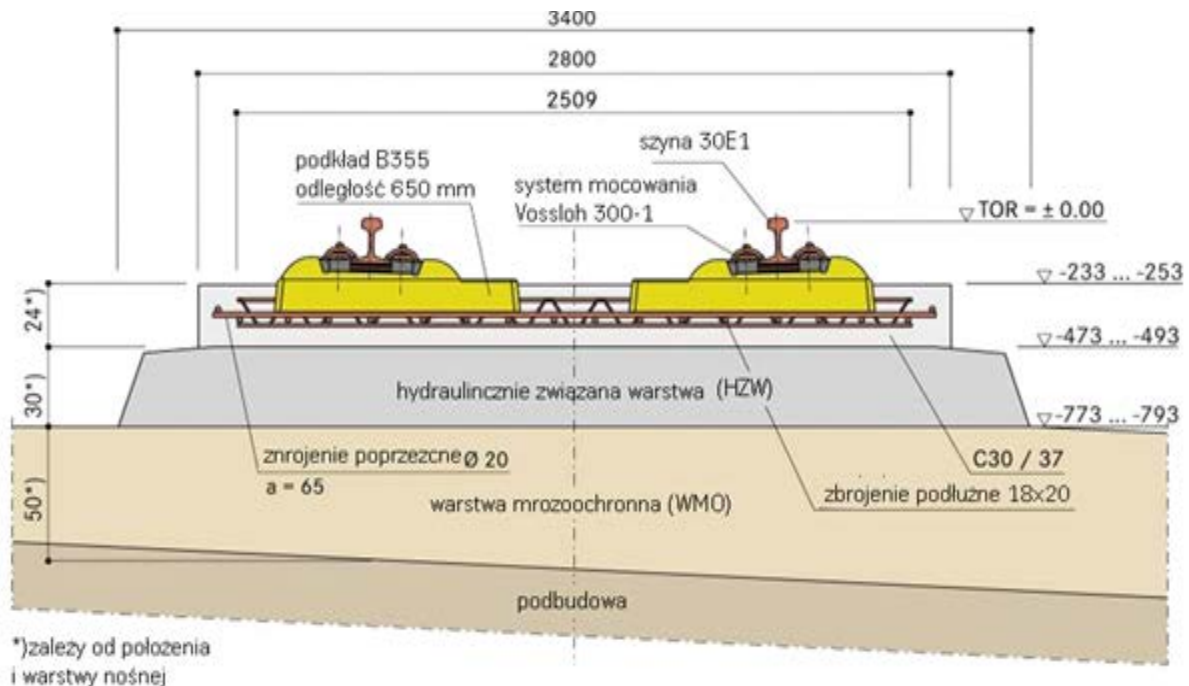


Figure 21 High Speed Track Layers

Values of different parameters of High Speed Train Track have been listed below:

PART	MASS DENSITY(kg/m <sup>3</sup> )	MODULUS OF ELASTICITY(GPa)	POISSON'S RATIO
RAIL	7850	207	0.28
SLEEPER	2400	70	0.2
CONCRETE BEARING LAYER(CBL)	2400	34	0.2
HYDAULICALLY BONDED LAYER (HBL)	2400	5	0.2
FROST PROTECTION LAYER (FPL)	2400	0.12	0.2
SUBSOIL	2000	0.01	0.4

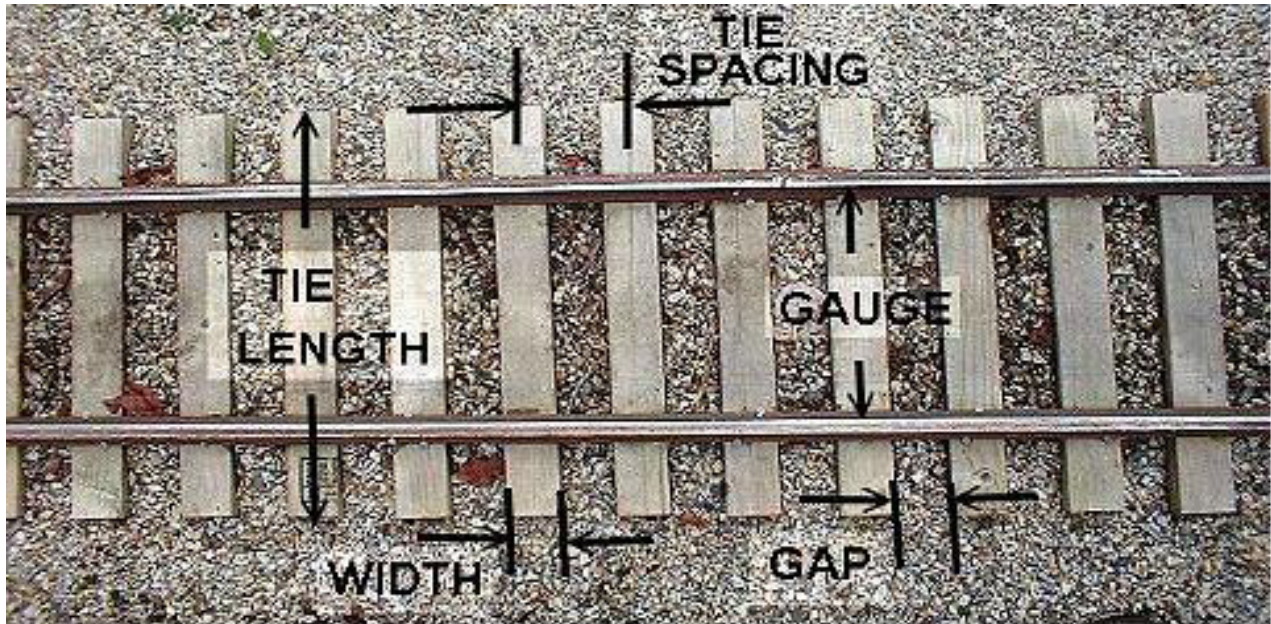
Table 2 values of poisson's ratio, modulus of elasticity and mass density

As in India the majority of soil is sandy and clayey so soil modulus is different for different type of soils. In the table given below values soil modulus of different soils is listed. My ABAQUS model is tested for these values and further effect is studied further.

SANDY SOILS	SOIL MODULUS (MPa)
DENSE SAND AND GRAVEL	108 to 215
DENSE SAND	27 to 108
LOOSE SAND	11 to 27
CLAYEY SOILS	SOIL MODULUS (MPa)
STIFF CLAY AND SILTY CLAY	54 to 108
MEDIUM CLAY	22 to 54
SOFT CLAY	5 to 22

Table 3 Soil Modulus of different soils

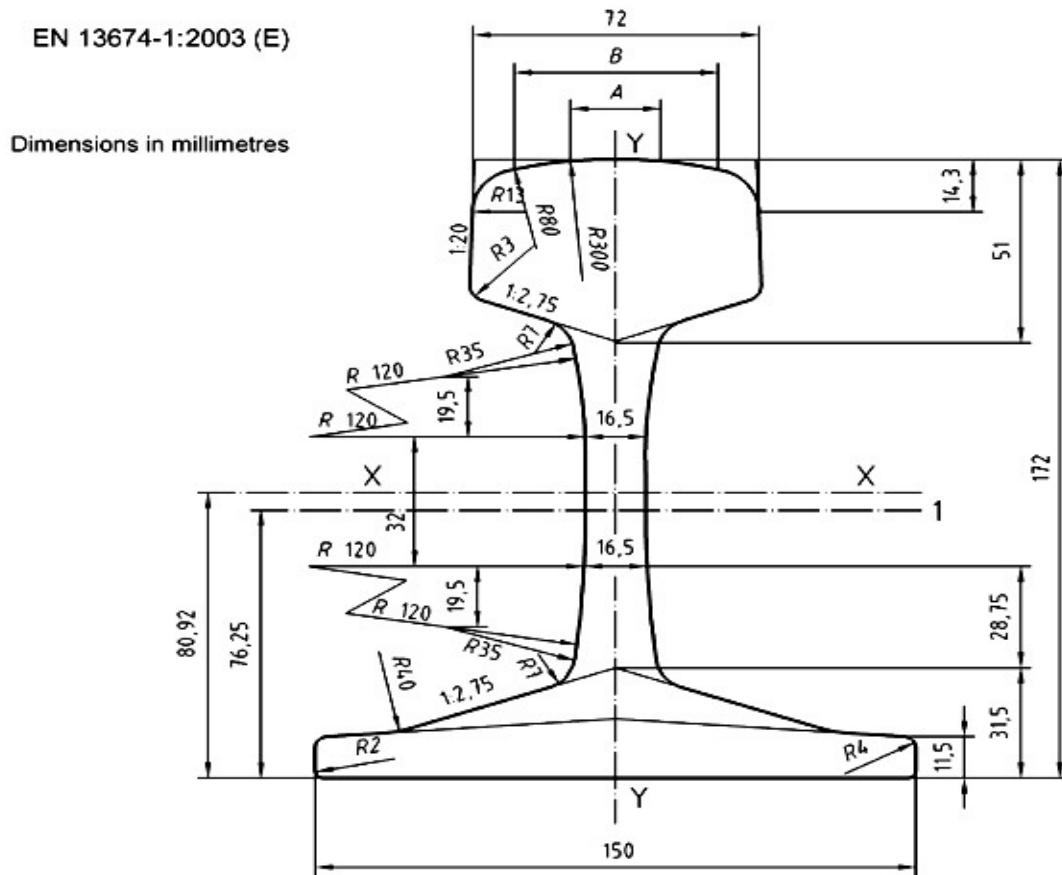
Sleepers to be used are pre-stressed concrete sleepers. Parameters considered for sleepers are listed below.



SLEEPER	FULL SCALE (meters)
SLEEPER LENGTH	2.6
SLEEPER WIDTH	0.25
SLEEPER HEIGHT	0.15
SLEEPER SPACING	0.65
SLEEPER GAP	0.30

Table 4 Sleeper Parameters

Rails to be used in modeling work are UIC 60 rail. Length of this rail section varies from 12 meters to 25 meters. Cross section of UIC 60 rail is shown below:



**Key**

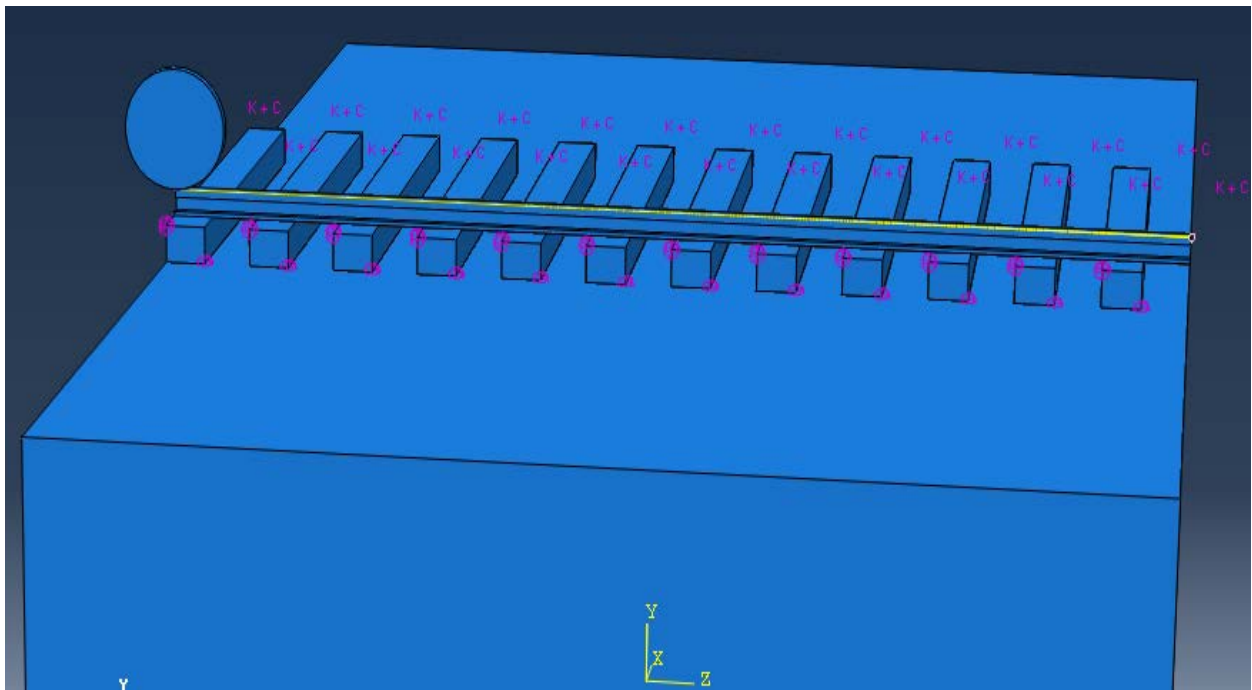
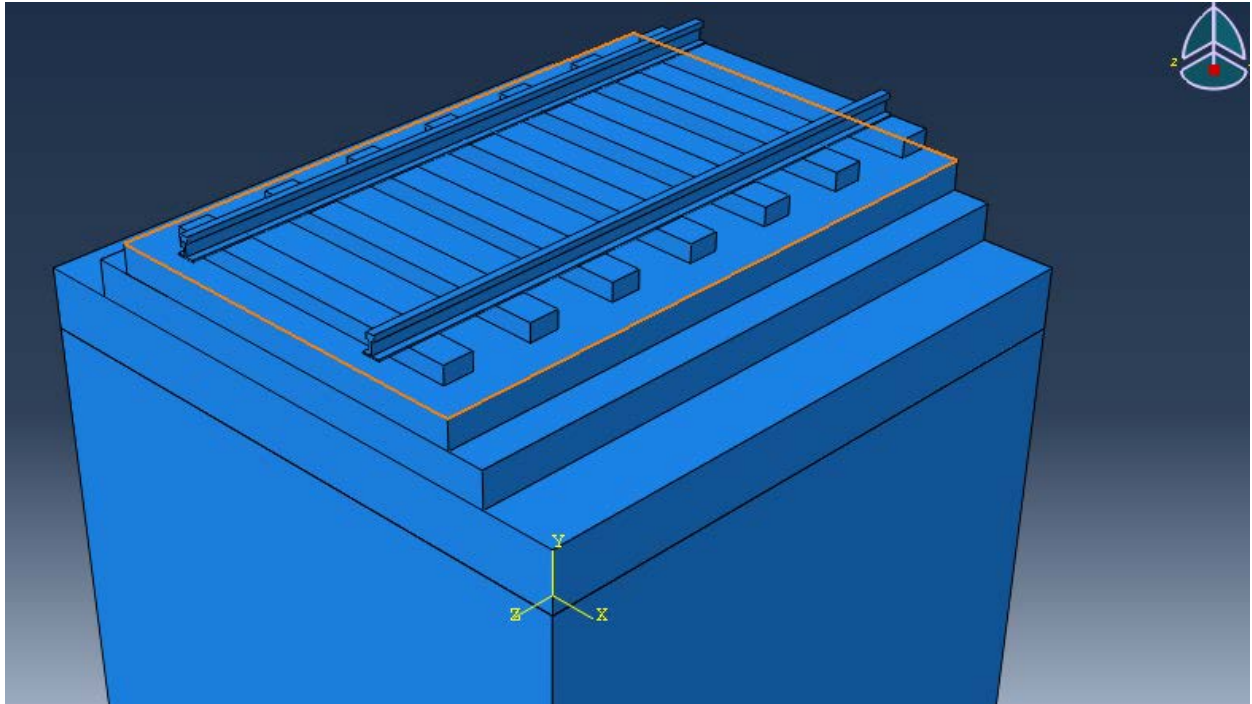
1 Centre line of branding

Cross-sectional area	: 76,70	cm <sup>2</sup>
Mass per metre	: 60,21	kg/m
Moment of inertia x-x axis	: 3038,3	cm <sup>4</sup>
Section modulus - Head	: 333,6	cm <sup>3</sup>
Section modulus - Base	: 375,5	cm <sup>3</sup>
Moment of inertia y-y axis	: 512,3	cm <sup>4</sup>
Section modulus y-y axis	: 68,3	cm <sup>3</sup>

Indicative dimensions : A = 20,456 mm  
B = 52,053 mm

Figure A.21 — Rail profile 60 E 1

Figure 22(a) Cross-Section of UIC 60 rails







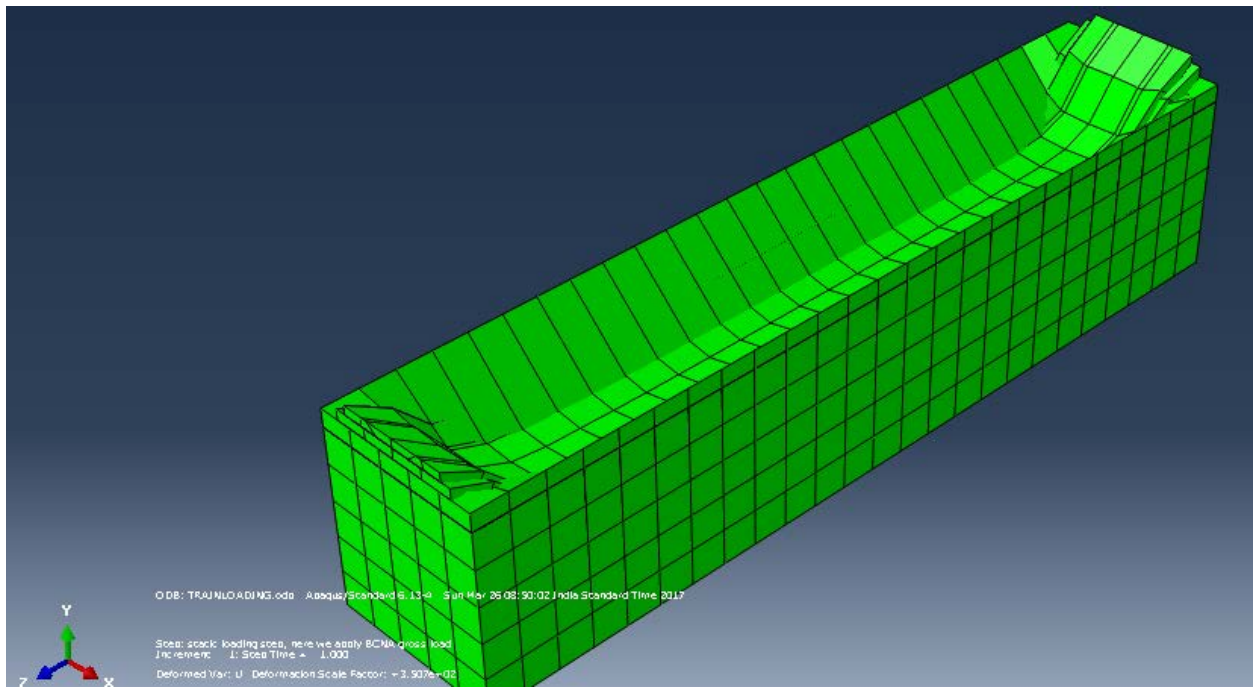
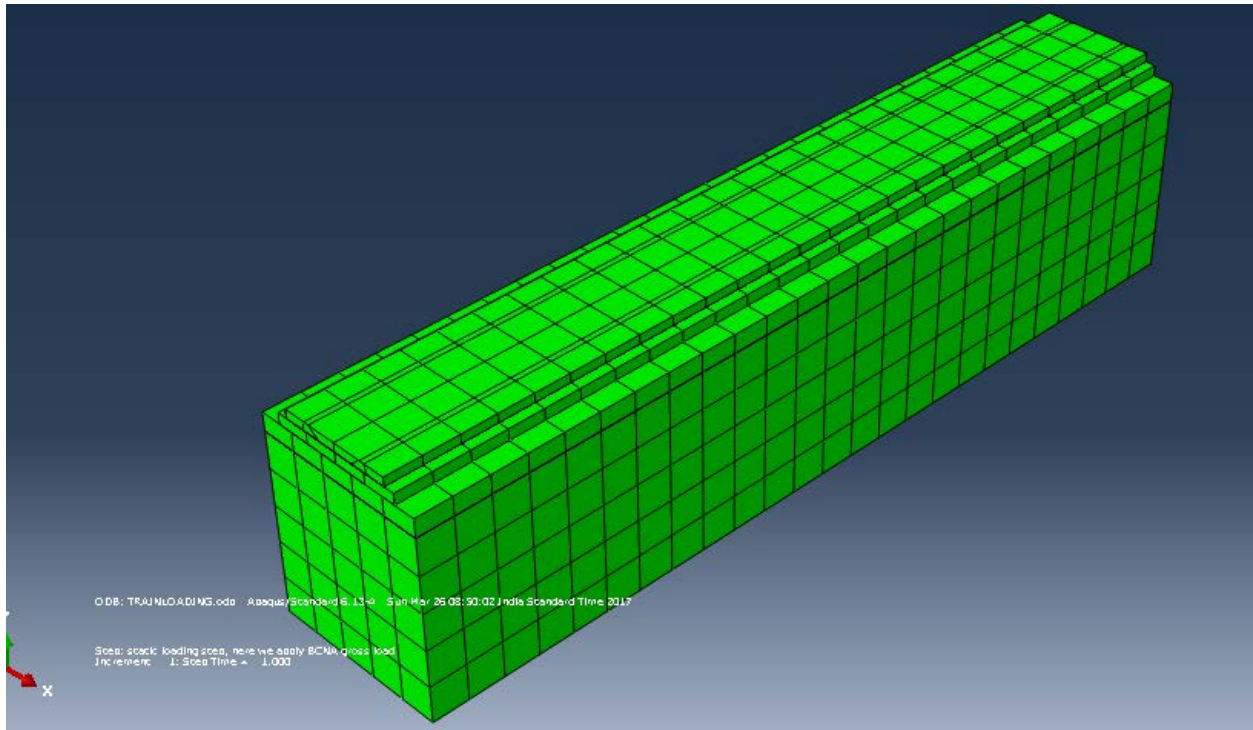


Figure 22(b) Three dimensional track Model and Deformed Shapes

### 5.3 Train Speed Effect

In order to study the ground induced vibrations generated by the high speed train, the velocity of train plays an important role in it to study the effect of vibrations and mode shapes. The railway track geometry and material remains the same. In order to study how velocity of train has effect on vibrations we find displacements under various speeds. The results of various displacements have been plotted below in the figure. When the high speed trains are running at very low velocities the displacements of the rail surface and soil change very little. For example the speeds 10m/s, 20 m/s, 30 m/s, 40 m/s, 50 m/s have displacements 15.5 mm, 15.6 mm, 15.68 mm, 16.00 mm and 16.20 mm. However as the speed of train increases there is frequent rise in vibration level or displacements of rail surface and soil. For example the speeds 60 m/s, 70 m/s, 80 m/s, 90m/s and 100m/s have displacements 16.4 mm, 17.0 mm, 19 mm, 22.3 mm and 25.5 mm. The results explain the previous researches well and show that there is significant increase in vibration level when train runs at critical speed.

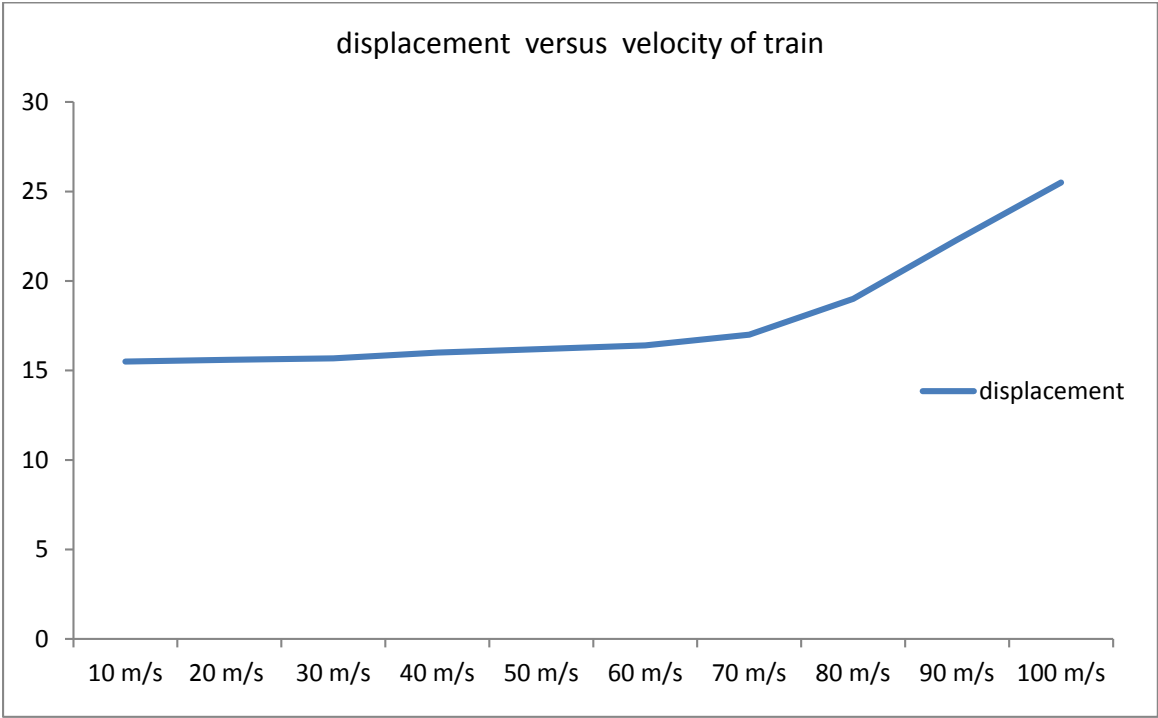
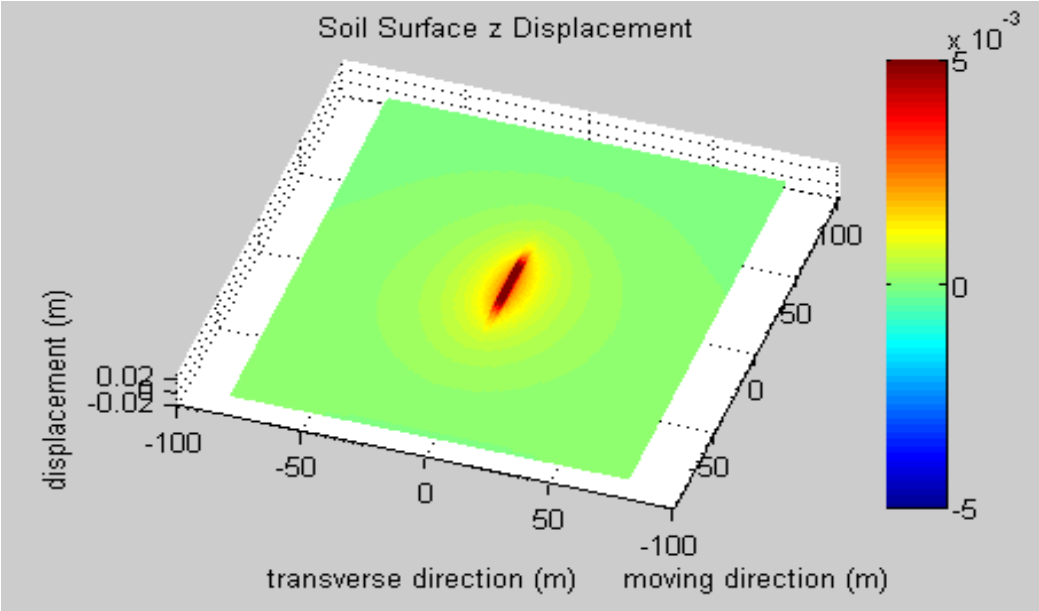
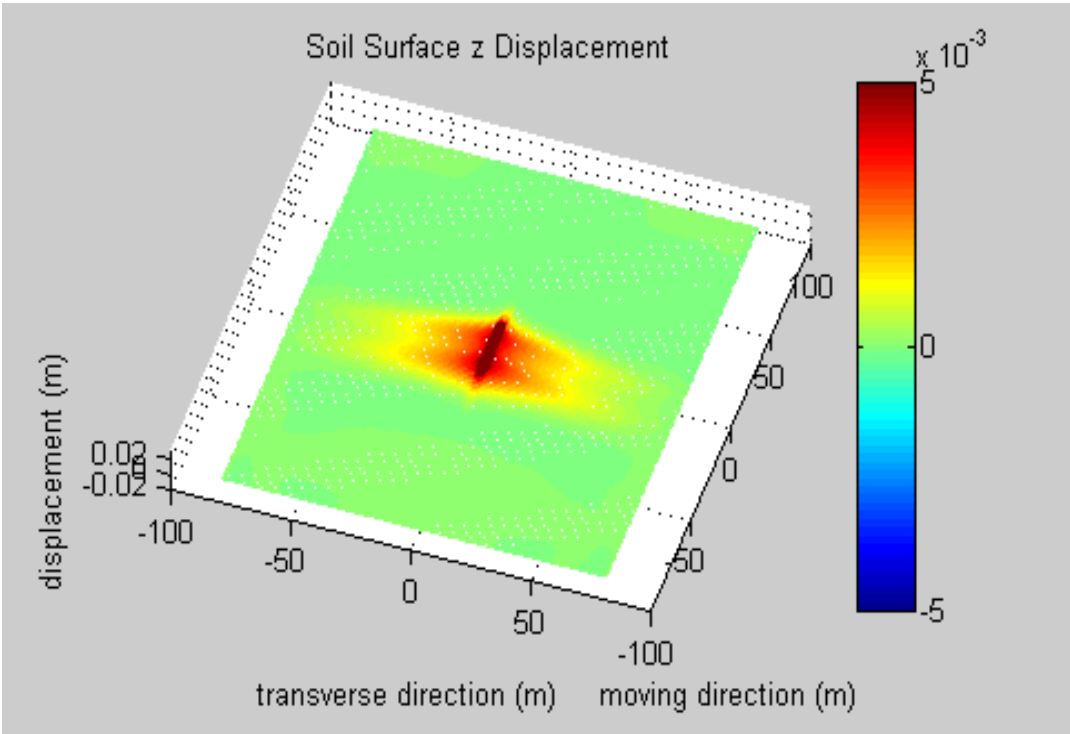


Figure 23 Maximum soil and rail surface displacement changed with velocity of train.

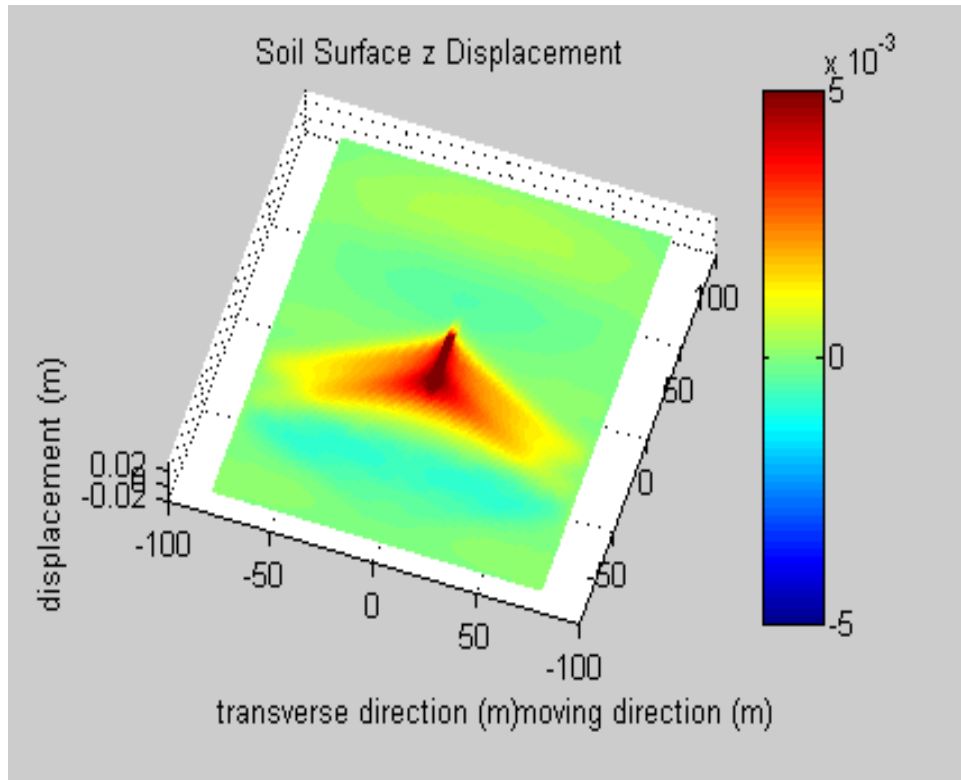
The soil surface vertical displacement contours are shown in below :



V= 10 m/s



V= 90 m/s



V= 100 m/s

Figure 24 Contour soil surface displacements for different speed of trains

From above contour displacements it can be seen that at low speeds displacement pattern is symmetric. The loading type is similar to static loading and displacement field is moving with the train. At low speeds of train displacement pattern resembles the static loading displacements of train. As the speed increase to 90 m /s the displacement area tends to increase and start moving with the train. However it is seen from previous researches that as speed reaches to critical speed the displacement increase to large area and full dynamic response is attained. At high speeds response is not similar to static loading but is similar to cone shaped displacement field.

#### 5.4 Material Property Effect (Soil Poisson's ratio)

In this result analysis subsoil of the track model was tested for different values of Poisson's ratio and further vertical rail surface displacements were seen on the model. The tested values and rail surface vertical displacement are provide in the table below. A decrease in Poisson's ratio results in increase in displacement because lower Poisson's ratio yields lower impact of radial stresses on vertical deformation.

Poisson's ratio	Vertical rail surface displacement
0.20	17.6 mm
0.30	16.8 mm
0.40	15.4 mm

Table 5 Poisson's ratio and vertical rail surface displacement

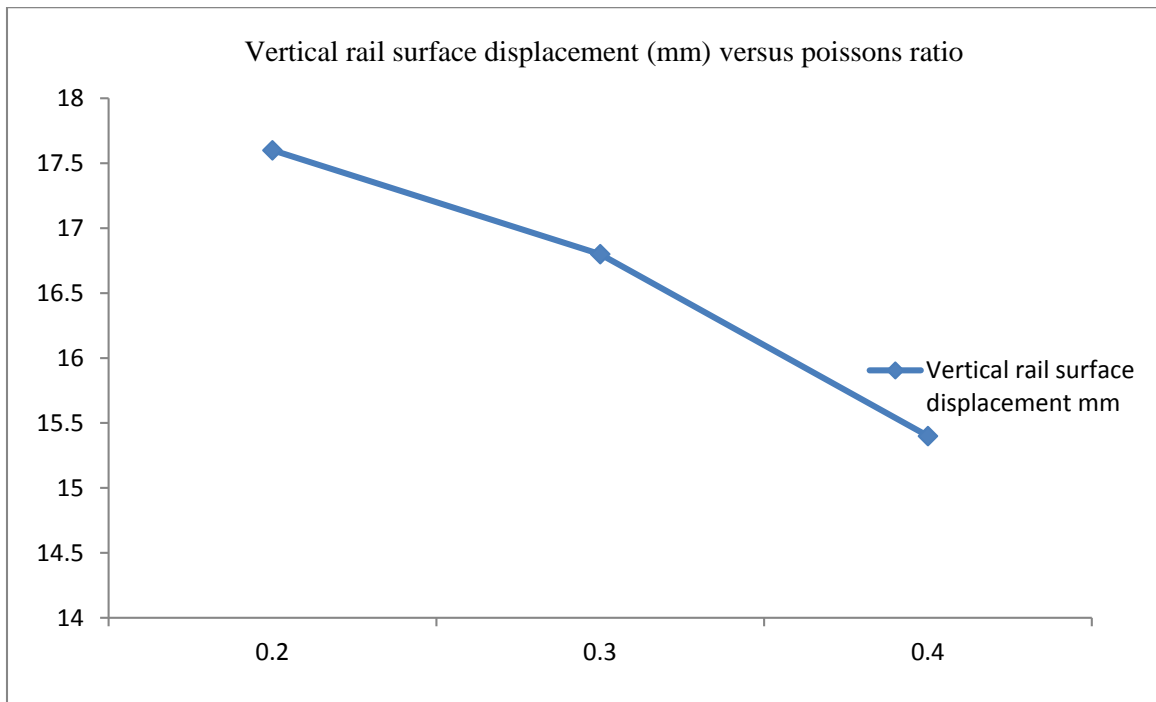


Figure 25 Effect of Poisson's ratio on rail surface displacement at low speed

## 5.5 Material Property Effect (Soil Modulus)

The Soil Track Interaction model was tested for different values of soil modulus and the displacements were observed specifically on sandy soils and clayey soils found in India. Various tested values are listed below.

Sandy soils	Soil Modulus(Mpa)	Rail Surface Vertical Displacement(mm)
Dense Sand and Gravel	120	15
Dense Sand	50	16.4
Loose Sand	20	18.2

Table 6 Soil Modulus of sandy soils and rail surface vertical displacement

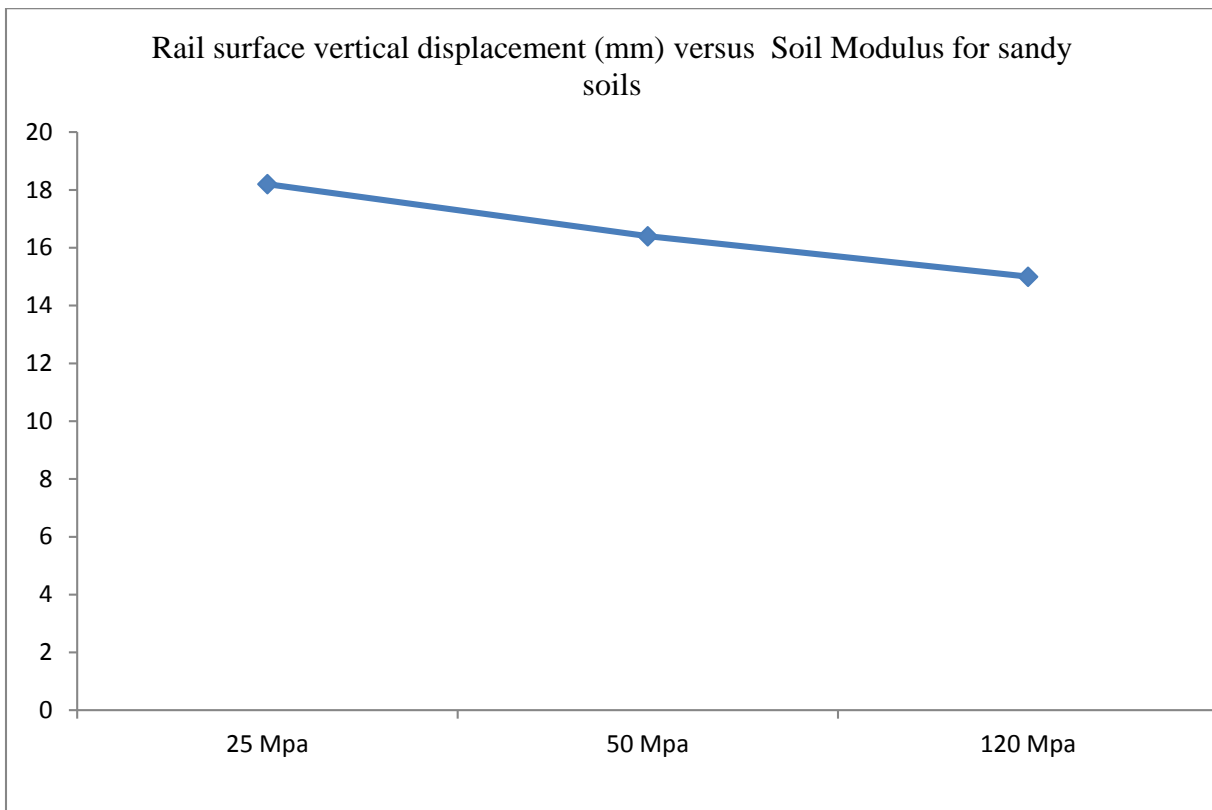


Figure 26 Effect of Soil Modulus of Sandy Soils on Rail surface displacement at low speed

Clayey Soil	Soil Modulus(MPa)	Rail Surface Vertical Displacement(mm)
Stiff Clay and Silty Clay	80	15.4
Medium Clay	40	17.5
Soft Clay	15	25.2

Table 7 Soil Modulus of Clayey Soil and Rail surface displacement (mm)

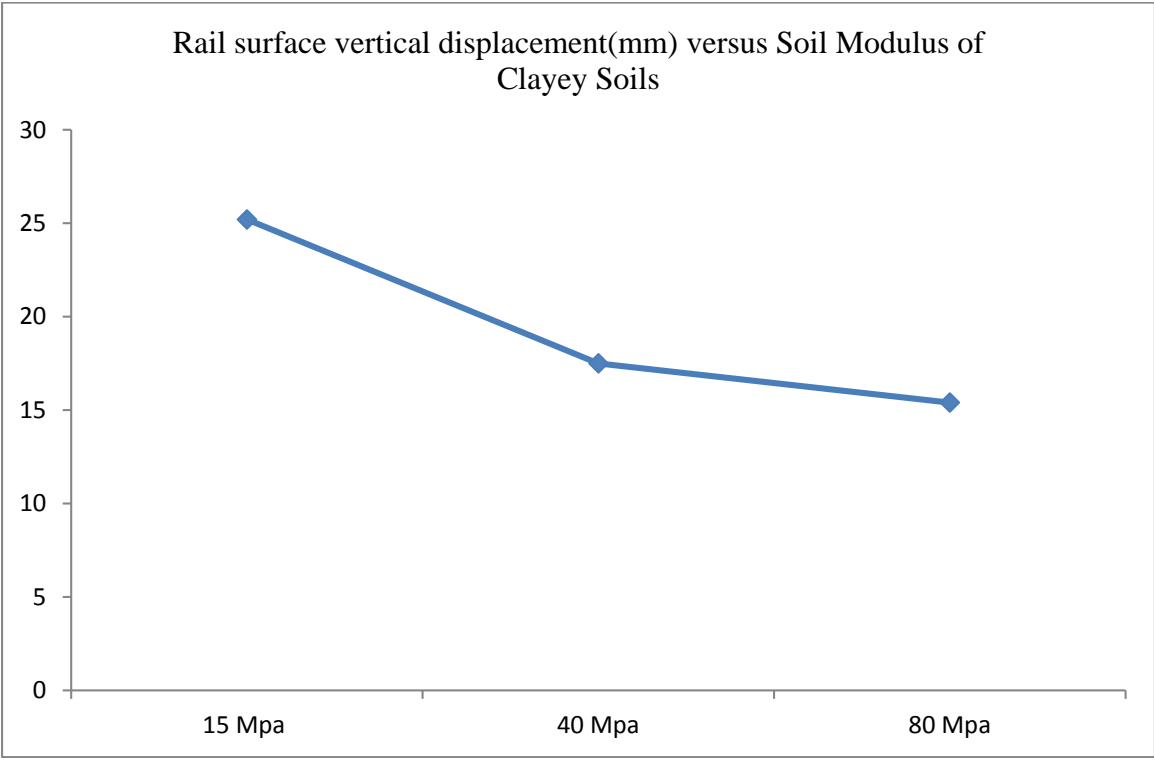


Figure 27 Effect of Soil Modulus of Clayey Soil on Rail surface Displacement at low speed

From the data plotted Table 6, Table 7, Figure 25 and Figure 26 it can be observed that increase in soil modulus can reduce the rail surface displacement or reduce the vibration level. Also it can be explained that stiffer the material and smaller is the displacement.



## 5.5 Track Component Properties

Soil Track interaction Model was tested for different values of Rail pad stiffness, Rail pad damping, Ballast stiffness and Ballast damping. Original values of above are listed below and further displacement was observed on the model.

Tested Parameters	Tested Values
Rail pad stiffness	90 MN/m
Rail pad damping	1 MNsec/m
Ballast stiffness	160 MN/m
Ballast damping	30 MNsec/m

Table 8 Stiffness and Damping

For static loading springs and dashpot do not contribute to the system performance. For the values listed in Table 8 and Table 2 the three dimensional track model rail surface displacement was 15.6 mm. For other trial values it was observed that by reducing stiffness and damping values of rail pad and ballast the rail surface displacement value increase.

# CHAPTER 6

## CONCLUSIONS

### 6.1 Conclusions

Based on the results of a series of analyses using the proposed model, trains running at critical speed will induce a significant increase in vibration level on both rail track and sub grade soil. The vibration level depends on train speed and soil properties. The track components also have important effects on vibrations. Verification using benchmark FEM software ABAQUS indicates that the proposed model is reasonable under low speed conditions. However, it still needs to be further verified by field tests and computer modeling, particularly for high speed conditions.

With regards to the train speed effect, it was observed that the rail surface and soil surface vertical displacements increased with the train speed. Also when a train is operating at a speed less than the critical speed, the vibration level increases slowly with increasing train speed. However, the vibration level increases significantly when the train speed approaches or exceed the critical speed.

With respect to the soil properties, by increasing the soil elastic modulus, the initial critical speed is increased. By reducing the Poisson's ratio, the initial critical speed is also increased. Very soft clay could generate the critical speed effect easily, because the speed of Rayleigh wave in soft clay is much lower than that in harder clay.

### 6.2 Scope of Future Work

The current goal is to obtain overall track dynamic responses. Therefore, ballast is only modeled as spring/dashpot system in this research. The effect of ballast mass should be further studied in future work. In the current sensitivity analysis, only one property is changed for each run. However, the combined effects of changing two or more properties should be investigated. How these parameters act together, and the relative effect in changing system responses for each parameter, should be further explored.

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