## Joint Optimization of Spectrum and Energy **Efficiency in Cognitive Radio Networks**

Dissertation submitted in partial fulfillment of the requirement for the degree of

## **BACHELOR OF TECHNOLOGY**

IN

## **ELECTRONICS AND COMMUNICATION ENGINEERING**

By

SUMIT KUMAR(131021) SUBHAM RANA(131022) ANMOL WADHWA (131103)

UNDER THE GUIDANCE OF

PROF. DR. GHANSHYAM SINGH



JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT MAY 2017

## **DECLARATION BY THE SCHOLAR**

I hereby declare that the work reported in the B-Tech thesis entitled "Joint Optimization of Spectrum and Energy Efficiency in Cognitive Radio Networks" submitted at Jaypee University of Information Technology, Waknaghat India, is an authentic record of my work carried out under the supervision of Prof. Ghanshyam Singh

I have not submitted this work elsewhere for any other degree or diploma.

Sumit Kumar (131021)

Subham Rana (131022)

Anmol Wadhwa (131103)

# DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

Jaypee University of Information Technology, Waknaghat, India

May,2017

## SUPERVISOR'S CERTIFICATE

This is to certify that the work reported in the B-Tech. thesis entitled "Joint Optimization of Spectrum and Energy Efficiency in Cognitive Radio Networks" submitted by SUMIT KUMAR(131021), SUBHAM RANA(131022) AND ANMOL WADHWA(131103) at Jaypee University of Information Technology, Waknaghat, India, is a bonafide record of his / her original work carried out under my supervision. This work has not been submitted elsewhere for any other degree or diploma.

( Signature of Supervisor ) Prof. Dr. Ghanshyam Singh May,2017

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## LIST OF ACRONYM

AP	Access Point
CR	Cognitive Radio
DC	difference of two convex function
EE	Energy Efficiency
ICI	Inter Carrier Interference
MIMO	Multi Input Multi Output
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
PSK	Phase Shift Keying
QPSK	Quadrature Phase Shift Keying
SE	Spectral Efficiency
SFT	Space Frequency Time
SHF	Super High Frequncy
UHF	Ultra High Frequency

## **CHAPTER 1**

#### INTRODUCTION

#### **1.1 The Cognitive Radio Network**

Cognitive Radio is a type of wireless communication in which transceiver can detect channels which are currently in use and not in use and easily move between vacant channels while avoiding used ones. For identifying which channels are in use and which are not different functionalities are required in cognitive radio networks. Cognitive engine is capable of configuring transmitter's parameters. These parameters include "waveform, protocol, operating frequency, and networking". This unit plays as an autonomous unit in communication environment, exchanging information about the parameters with the networks it accesses and other cognitive radios (CRs). The performance and output is continuously read by CR monitor which then uses this information to determine the RF environment, channel conditions, link performance, etc. and adjusts the radio's parameter's to deliver the required quality of service subject to an appropriate combination of user requirements, operational limitations, and regulatory constraints.

## **1.2 Cycle of Cognitive Radio**

#### 1.2.1 Spectrum Sensing

CR system is designed so that it can well aware of the small and sensitive changes in its surrounding thus the main task of the cognitive radio is to sense the environment, detect which part of the spectrum is available for the transmission of other signal what are the parameters of the spectrum available for e.g. bandwidth, at what transmission power it should transmit

#### **1.2.2 Spectrum Decision**

Spectrum decision is based on the characteristics of the spectrum available for e.g. bandwidth. Furthermore, spectrum decision can also be affected by the presence of primary and other cognitive users in surroundings. It involves two steps: the first is that, each spectrum is characterized according to their bandwidth, it is not only based on local observation but also on local decision of primary user. Then, taking that into consideration proper spectrum is selected.

#### **1.2.3 Spectrum Sharing**

It requires cooperation with the other cognitive users. The existing work in spectrum sharing can be classified by four aspects: the architecture, spectrum allocation behavior, spectrum access technique, and scope. As the characteristics of available spectrum holes can vary over time, each hole can be shown as both the time-varying radio environment and the spectrum parameters, like operating frequency and bandwidth. The classification on the basis of architecture can be centralized or distributed.

**Centralized Spectrum:** In this a central user controls the spectrum allocation and spectrum features and construct a new map.

**Distributed spectrum sharing**: In this each node distributively access the network. Between different networks we usually use distributed solutions such that a base station (BS) competes with its interferer BSs according to the QoS requirements of its users to allocate a portion of the spectrum. Distributed solutions also follows centralized solution by sending messages between nodes as revealed by recent studies.

**Spectrum mobility**: When a CR user gets in contact of primary user who wants to use the spectrum he was he using, he immediately has to shift to the new spectrum.

**Spectrum access**: After decision is made spectrum is accessed by cognitive media access control protocols which intends to avoid collisions with the primary user and other user.

## 1.3 Why Do We Need Cognitive Radio

- i. In the present world the wireless devices are exponentially increasing so does the requirement of the spectrum requirement is also increasing and Spectrum is getting scarce in this situation becomes one of the useful device. Cognitive radio provides one of the promising solutions for this problem.[13-16]
- ii. Due to its ability it will cause no interference to the primary user.
- iii. Moreover as it has cross layer design it is adaptable and flexible to adjust itself according to the spectrum sensing results.
- iv. It uses OFDMA which provides high flexibility in providing in adaptation of resources and can provide air interference for the cognitive radio.
- v. MIMO system has become the requirement of present technology and it supports MIMO system.

## **CHAPTER 2**

#### **MATHEMATICAL TOOLS**

#### 2.1 Mathematical Optimization

A optimization problem has the form

Minimize f(x)

Subject to  $g_i(\mathbf{x}) \leq b_i$ ,  $i = 1, \ldots, m$ .

where,

vextor  $x = (x_1, ..., x_n)$  is the optimization variable,

 $f(\mathbf{x}): \mathbf{R}^{n} \rightarrow \mathbf{R}$  is th *objective* function,

 $g_i(\mathbf{x}): \mathbf{R}^n \rightarrow \mathbf{R}$  are the *inequality constraints*,

*b<sub>i</sub>* are *bounds* or *limits* for constraints.

A vector  $x^*$  is optimal if it, among all vectors, satisfies the constraints with the smallest objective value: for any  $z f_1(z) \le b_1, \ldots, f_m(z) \le b_m$ , we have  $f_0(z) \ge f_0(x^*)$ .

#### 2.2 Convex sets

Suppose  $x_1 \neq x_2$  are two points in  $\mathbf{R}_n$ . Points of the form

$$y = \vartheta x_1 + (1 - \vartheta) x_2,$$

where  $\vartheta \in \mathbf{R}$ , form the line passing through  $x_1$  and  $x_2$ . The parameter value  $\vartheta=0$  corresponds to  $y = x_2$ , and the parameter value  $\vartheta = 1$  corresponds to  $y = x_1$ . Values of the parameter  $\vartheta$  between 0 and 1 correspond to the (closed) line segment between  $x_1$  and  $x_2$ .

A set **C** is convex if the line segment between any two points in **C** lies in **C**, *i.e.*, if for any  $x_1, x_2 \in C$ and any  $\theta$  with  $0 \le \vartheta \le 1$ , we have

$$\vartheta x_1 + (1 - \vartheta) x_2 \in \boldsymbol{C}.$$

#### **2.3 Convex function**

A function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is convex if **domf** is a convex set and if for all x,  $y \in \text{domf}$ , and  $\theta$  with  $0 \le \vartheta \le 1$ , we have

$$f(\vartheta x + (1 - \vartheta)y) \leq \vartheta f(x) + (1 - \vartheta)f(y),$$

Geometrically, this inequality means that the line segment between (x, f(x)) and (y, f(y)), which is the chord from x to y, lies above the graph of f. A function f is strictly convex if strict inequality holds in above equation whenever  $x \neq y$  and  $0 < \theta < 1$ . We say f is concave if -f is convex, and strictly concave if -f is strictly convex as shown in fig.2.1.

A function is convex if and only if it is convex when restricted to any line that intersects its

domain.



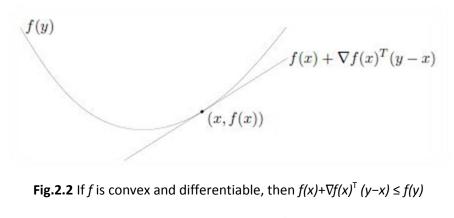
**Fig.2.1** Graph of a convex function. The chord (i.e., line segment) between any two points on the graph lies above the graph.

#### **2.3.1** First order condition

Suppose f is differentiable (*i.e.*, its gradient  $\nabla f$  exists at each point in **domf**, which is open). Then f is convex if and only if **domf** is convex and

$$f(y) \ge f(x) + \nabla f(x)T(y-x)$$

holds for all  $x, y \in$  **domf**. This inequality is illustrated in fig.2.2.



for all  $x, y \in$ **domf**.

Above inequality states that for a convex function, the first-order Taylor approximation is in fact a *global underestimator* of the function. Conversely, if the first-order Taylor approximation of a function is always a *global underestimator* of the function, then the function is convex.

#### 2.3.2 Second order condition

We now assume that f is twice differentiable, that is, its Hessian or second derivative  $\nabla^2 f$  exists at each point in **domf**, which is open. Then f is convex if and only if **domf** is convex and its Hessian is positive semidefinite: for all  $x \in \text{domf}$ ,

$$\nabla^2 f(x) \geq 0.$$

For a function on R, this reduces to the simple condition  $f''(x) \ge 0$  (and **domf** convex, *i.e.*, an interval), which means that the derivative is nondecreasing. The condition  $\nabla^2 f(x) \ge 0$  can be

interpreted geometrically as the requirement that the graph of the function have positive (upward) curvature at x.

#### 2.3.3 Properties

1. A convex function *f*, is continuous on *C* and differentiable at all or at most, countable many points. If *C* is closed, then *f* may fail to be continuous at the end points of *C*.

2. If a function's derivative is monotonically non-decreasing on that interval then that function is called convex

3. If the function lies above all of its tangents:  $f(b) \ge f(a) + f'(a)(b-a)$  for all a and b in the interval then a continuously differentiable function of one variable is convex.

A strictly convexfunction will have only one minimum which is also the global minimum of that function.

Examples:

- The second derivative of  $x^2$  is 2; it shows that  $x^2$  is a convex function of x.
- The absolute value function |x| is convex, even though it does not have a derivative at x =
   0.

#### 2.4 Concave function

If a function's derivative f' is decreasing on an interval or has decreasing slope then that function f is **concave**. A convex function is also called as concave upwards function, and a concave function is also called concave downwards function. If the second derivative of a twice differentiable function f''(x), is positive then the graph is convex (or concave upward); if the second derivative is negative, then the graph is concave (or concave downward). Points, at which concavity changes, are called **inflection points**.

Any point at the "bottom" of a convex (*i.e.*, concave upward) is called minimal extremum. The "apex" of a concave (*i.e.*, concave downward) function is called maximal extremum. A function f(x) is said to be **concave** on an interval if, for all a and b in that interval it follows,

For every 
$$t \in [0, 1]$$
,  $f(ta + (1 - t) b) \ge tf(a) + (1 - t) f(b)$ 

Additionally, f(x) is **strictly concave** if

For every 
$$t \in [0, 1]$$
,  $f(ta + (1 - t)b) > tf(a) + (1 - t)f(b)$ 

These relationships are illustrated in Fig. 2.3

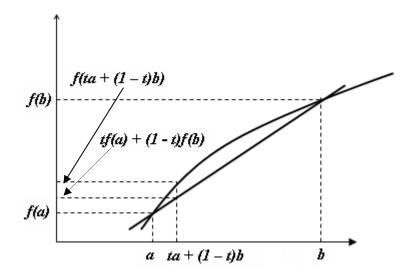


Fig. 2.3 Concave function

## 2.4.1 Testing for concavity of a single variable function

A function has non increasing slope or  $\partial^2 f / \partial x^2 \leq 0$  then that function is called concave. That function will be is strictly concave if its slope is continually decreasing or  $\partial^2 f / \partial x^2 < 0$  throughout the function.

### 2.4.2 Properties of a concave functions

A continuous function on domain C is concave if and only if

$$f\left(\frac{a+b}{2}\right) \ge \frac{f(a)+f(b)}{2}$$

for any x and y in C.

If an  $x_0$  point of a function such that for all  $x < x_0$ , f(x) is non-decreasing and for all  $x > x_0$  it is non-increasing function then that function is called **quasiconcave**.  $x_0$  can also be  $\pm \infty$ , making the function non-decreasing (non-increasing) for all x. **quasiconvex** can be called as the opposite of quasiconcave.

#### 2.5 Quasi-convexity and Quasi-concavity

A function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is called *quasi-convex* (or *unimodal*) if its domain and all its sublevel sets

$$S\alpha = \{x \in domf \mid f(x) \le \alpha\},\$$

for  $\alpha \in \mathbf{R}$ , are convex. A function is *quasi-concave* if -f is *quasi-convex*, *i.e.*, every superlevel set  $\{x \mid x \in \mathbf{R}\}$ 

 $f(x) \ge \alpha$  is convex. A function that is both quasiconvex and quasiconcave is called *quasilinear*. If a function *f* is *quasilinear*, then its domain, and every level set

 $\{x \mid f(x) = \alpha\}$  is convex.

An Examples of Quasi-concave Functions:

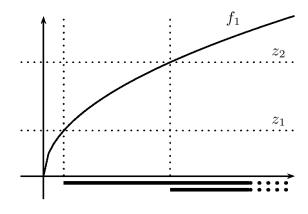


Fig.2.4 Graph of Quasi-concave function

#### 2.6 Newton's method

Newton's method sometimes called *damped* Newton method or *guarded* Newton method which is different from *pure* Newton method which uses constant step size=1. In this method instead of minimizing function f, we minimize its second order Taylor's approximation around x',

$$f(x) \approx g(x) = f(x') + \nabla f(x')'(x - x') + \frac{1}{2}(x - x')' \nabla^2 f(x')(x - x')$$

#### 2.6.1 Newton step

For  $x \in$ **domf**, the vector,

$$\Delta \mathbf{x}_{\rm nt} = -\nabla^2 f(\mathbf{x})^{-1} \nabla f(\mathbf{x})$$

is called the Newton step (for *f*, at *x*). Positive definiteness of  $\nabla^2 f(x)$  implies that

$$\nabla f(x)^{\mathsf{T}} \Delta x_{\mathsf{nt}} = -\nabla f(x)^{\mathsf{T}} \nabla^2 f(x)^{-1} \nabla f(x) < 0$$

unless  $\nabla f(x) = 0$ , so the Newton step is a descent direction (unless x is optimal). It gives steepest descent direction for the quadratic norm defined by the Hessian  $\nabla^2 f(x)$ , *i.e.*,

$$||u||_{\nabla_{2f(x)}} = (u^{T} \nabla^{2} f(x) u)^{1/2}$$

It gives good estimate of search direction if x is near  $x^*$ , *i.e.* minimizer of function f, hence a very good minimum search algorithm.

#### 2.6.2 Algorithm

**given** a starting point  $x \in \text{domf}$ , tolerance  $\in >0$ .

#### repeat

1. Compute the Newton step and decrement.

$$\Delta \mathbf{x}_{nt} := -\nabla^2 f(\mathbf{x})^{-1} \nabla f(\mathbf{x}); \quad \lambda^2 := \nabla f(\mathbf{x})^T \nabla^2 f(\mathbf{x})^{-1} \nabla f(\mathbf{x})$$

- 2. Stopping criterion. quit if  $\lambda^2/2 \le \epsilon$ .
- 3. Line search. Choose step size t by backtracking line search.
- 4. Update.  $x := x + t\Delta x_{nt}$ .

#### 2.7 Barrier method

The basic idea of barrier method is to approximate the constrained problem into unconstrained problem by using different methods introducing some barriers into the objective problem.One of the easy to implement barrier functions is logarithmic function.

In barrier method we solve a sequence of unconstrained minimization problems, using previous point found as the starting point for the next unconstrained minimization problem.

#### 2.7.1 Logarithmic barrier

Indicator function

$$\hat{l}_{-}(u) = -(1/t) \log(-u), \text{ dom } \hat{l}_{-} = -\mathbf{R++},$$

where t > 0 is a parameter that sets the accuracy of the approximation. Like *I*., the function  $\hat{I}_{-}$  is convex and nondecreasing, and (by our convention) takes on the value  $\infty$  for u > 0. Unlike *I*., however,  $\hat{I}_{-}$  is differentiable and closed: it increases to  $\infty$  as u increases to 0.

Figure 11.1 shows the function  $I_{-}$ , and the approximation  $\hat{I}_{-}$ , for several values of t. As t increases, the approximation becomes more accurate.

Substituting  $\hat{l}_{\perp}$  for  $l_{\perp}$  in (11.3) gives the approximation

Minimize 
$$f_0(x) + \sum_{i=1}^{m} -(1/t) \log(-f_i(x))$$

Subject to Ax = b.

The objective here is convex, since  $-(1/t) \log(-u)$  is convex and increasing in u, and differentiable. Assuming an appropriate closedness condition holds, Newton's method can be used to solve it.

The function

$$\Phi(x) = -\sum_{i=1}^{m} \log(-f_i(x)),$$

with **dom**  $\mathcal{P} = \{x \in \mathbf{R}_n \mid f_i(x) < 0, i = 1, ..., m\}$ , is called the logarithmic barrier or log barrier for the above problem. Its domain is the set of points that satisfy the inequality constraints strictly. No matter what value the positive parameter *t* has, the logarithmic barrier grows without bound if  $f_i(x) \rightarrow 0$ , for any *i*.

#### 2.7.2 Central path

The problem becomes

minimize  $t f_0(x) + \Phi(x)$ 

Now the problem can be solved by Newton's method and for each t > 0, it has a unique

solution. The set of points  $x^{*}(t)$ , t > 0, which is called the central points. Points on the central path are characterized by the following necessary and sufficient conditions:  $x^{*}(t)$  is strictly feasible, *i.e.*, satisfies

A 
$$x^{*}(t) = b$$
  $f_{i}(x^{*}(t)) < 0$ , i= 1, ..., m,

and there exists  $v \in \boldsymbol{R}_n$  such that

$$0=t \nabla f_0(x^*(t)) + \nabla \Phi (x^*(t)) + A^T v$$

#### 2.7.3 Algorithm

**Given** *x*,  $t=t^{(0)}>0, \mu>1$ , tolerance  $\epsilon>0$ .

#### Repeat

1. Centering step

*Calculate*  $x^*(t)$  *by minimization*  $t f_0 + \Phi$ , *subject to* Ax=b, *starting at* x.

- 2. *Update*  $x=x^{*}(t)$ .
- 3. Break if  $m/t < \epsilon$ .
- 4. Increase t.  $t=\mu t$ .

At every iteration we compute  $x^*(t)$  and then increase t by factor  $\mu > 1$ . Centering step (since a central point is being computed) is known as process executed after step 1.  $x^*(t^{(0)})$ as the initial centering step taking t=0. The inner iterations are the Newton iterations or steps executed during the centering step. After execution of each inner step, we have a primal feasible point; and after each outer step we will have dual feasible point.

#### 2.7.4 Accuracy of centering

Since the central path has no significance beyond the fact that it leads to a solution of the original problem as  $t \rightarrow \infty$ , computing  $x^*(t)$  exactly is not necessary. Inexact centering will still lead a sequence of points  $x^{(k)}$  that converges to an optimal point. By adding a correction term this can be corrected provided the computed x is near the central path, *i.e.*,  $x^*(t)$ .

On the other hand, the cost of computing an extremely accurate minimizer of  $t f_0 + \Phi$ , as compared to the cost of computing a good minimizer of  $t f_0 + \Phi$ , is only marginally more, *i.e.*, a few Newton steps at most. For this reason it is notunreasonable to assume exact centering.

#### 2.7.5 Choice of $\mu$

The trade-off in the number of inner and outer iterations requires choice of the parameter  $\mu$ . If  $\mu$  is small (*i.e.*, near 1) then at each outer iteration t increasesby a small factor. Hence the initial point for the Newton process, *i.e.*, theprevious iterate x, is a very good starting point, and the number of Newton stepsneeded to compute the next iterate is small. Thus for small  $\mu$  we expect a smallnumber of Newton steps per outer iteration, but of course a large number of outeriterations since each outer iteration reduces the gap by only a small amount. Inthis case the iterates closely follow the central path. This explains the alternate name path-following method.On the other hand if  $\mu$  is large we have the opposite situation. After eachouter iteration t increases a large amount, so the current iterate is probably not a very good approximation of the next iterate. Thus we expect many more inneriterations. This 'aggressive' updating of t results in fewer outer iterations, since theduality gap is reduced by the large factor  $\mu$  at each outer iteration, but more inneriterations. With  $\mu$  large, the iterates are widely separated on the central path; theinner iterates veer way off the central path.

#### 2.7.6 Choice of t<sup>(0)</sup>

This can be reffered as initial value of t. If  $t^{(0)}$  is chosen too large, the first outer iteration will require too many iterations. And extra outer iterations will require if  $t^{(0)}$  is chosen too small, and possibly too many inner iterations in the first centering step. Since  $m/t^{(0)}$  is the duality gap that will result from the first centering step, onereasonable choice is to choose  $t^{(0)}$  so that m/t(0) is approximately of the same order as  $f_0(x^{(0)}) \cdot p^*$  or  $\mu$  times this amount. Thus, in the first outer iteration we simply compute a pair with the same dualitygap as the initial primal and dual feasible points.

Another possibility is suggested by the central path condition We can interpret

$$\inf_{v} || t \nabla f_{0}(x^{(0)}) + \nabla \phi(x^{(0)}) + A^{T}v||_{2}$$

as a measure for the deviation of  $x^{(0)}$  from the point  $x^*(t)$ , and choose for  $t^{(0)}$  thevalue that minimize.

#### 2.7.7 Hessian matrix

Square matrix of second order partial derivative of scalar valued function. Specifically, suppose f:  $\mathbf{R}^n \rightarrow \mathbf{R}$  is a function taking as input a vector  $\mathbf{x} \in \mathbf{R}^n$  and outputting a scalar  $f(\mathbf{x}) \in \mathbf{R}$ ; if all second partial derivatives exist and are continuous over the domain of the function, then the Hessian matrix  $\mathbf{H}$  of f is a square n x n matrix, usually defined and arranged as follows:

$$|D| = \begin{vmatrix} 0 & f_1 & \cdots & f_k \\ f_1 & f_{11} & \cdots & f_{1k} \\ \vdots & \vdots & \ddots & \vdots \\ f_k & f_{k1} & \cdots & f_{kk} \end{vmatrix}$$

or, component-wise:

$$H_{i,j}=d^2f/dx_idx_j$$

The determinant of the above matrix is also sometimes referred as hessian.

## Chapter 3 CHANNEL MODELS

#### **3.1 Wireless Channel**

Frequencies of Interest: In the UHFwe have frequency in which we can transmit is from (.3GHz –3GHz) and in SHF (3GHz – 30 GHz) bands;

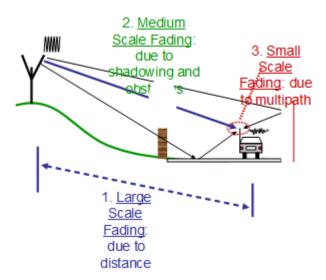


Fig.3.1 Types of channel fading

Due to following reasons path loss occurs:

- As the wave moves over distance path loss due to dissipation of energy occur.
- Obstacle such as buildings, trees, walls causes Shadowing that causes absorption, reflection, scattering etc.
- Multiple waves interfere with each other .

Multipath fading in wireless communication systems is commonly modeled by Rayleigh and Riciandistributions. A close approximation of attenuation due to multipath fading in wireless channels can be done by Rayleigh fading (for the case where no line of sight component present) and Rician fading (for the case where line of sight component present)

Rayleigh- No line of sight component present Rician-Line of sight component present

#### 3.2 Rayleigh Channel Fading

It is a statistical model showing effect of propogation of radio signal. That are used in wireless devices. As the signal propogates its magnitude will fade according to Rayleigh distribution. It can be highly used for communication where tropospher and ionosphere layers are considred. It can be highly considred where density of obstacle are much more and Radio signal will likely to scatter before reaching receiver. Just like highly build up city where ther is no line of sight between transmitter and receiver. The probability distribution function is given as

$$P_r(r)=2r/\Omega e^{-r^{2/\Omega}}, r>=0$$

where  $\Omega = E(R^2)$ .

Channel fading also on how fast the receiver or transmitter is moving *i.e.* Doppler shift also causes fading

#### **3.3 OFDMA**

ODFM is very beneficial as it removes several multichannel defects such as multichannelfading ,shadowing effect and narrow band interference without using channel equalisation filters. In this channel equalisation also becomes very simple as it contains many slowly modulating narrow band signals rather than rapidly varying modulating wide band signals. The slowly modulating signal can also make use of guard band which will help in avoiding intersymbol interference. OFDM can also be used as SFT's it can be assumed as

several transmitters transmits the same signal [17].

### **3.3.1 Principle of operation of OFDM:**

In OFDM the sub carrier frequencies are choosen so that the subcarrier are perpendicular to each other meaning that there is no guard band required. This greatly simplifies the design of the receiver and the transmitter system. The ororthogonality should satisfy the spacing condition between the subcarrier[19-21]

$$f = f_n - f_{(n-1)} = k/T_u$$

where  $T_u$  is symbol duration and k is the positive integer typically set to 1. Therefore if there are N sub-carrier the total passband bandwidth will be

$$B = Nf.$$

high spectral efficiency with a total efficiency of the signal equal to nyquist rate for the equivalent baseband signal can be provided using the OFDM and almost the whole frequency band can beutilized. OFDM has a nearly white spectrum.

OFDM requires synchronization frequency between the transmitter and receiver as we transmit the signal if the receiver or transmitter are moving the doppler shift occurs and the frequency deviation will affect the orthogonality of the signal and causing intersymbol interference between the signal. While this gets more worsen if the multipath effect is added to it and this causes the reflections at various offsets this further worsen when there is high speed movement of the vehicles. To mitigate it we can use ICI in such scenario one can change the shape of the subcarrier signal in order to minimize the signal interference resulting in non-orthogonal sub-carrier overlapping.

# Chapter 4

### Abstract

In this project we are jointly improving energy efficiency and spectral efficiency in OFDM (Orthogonal Frequency Division Multiplexing) based cognitive radio networks. A multiobjective resource allocation process is used to optimize energy efficiency and spectral efficiency.

- We will first find the Relation between energy efficiency and spectrum efficiency and show that EE is quasiconcave function of spectrum efficiency.
- Based on the relation between energy efficiency and spectral efficiency we will find Pareto-optimal set (most feasible set of points) of multi objective problem and will characterize the problem.
- We will transform this multiobjective problem into single objective problem and will find that this problem has a difference convex function structure and gain a standard optimization problem.
- We will find a global optimal solution for the single-objective problem by proposing a unified energy and spectral efficiency theory.
- Simulation results will validate the algorithm.

## **Chapter 5**

## SYSTEM MODEL AND PROBLEM FORMULATION

#### 5.1 System Model

Consider a downlink CR system which is based on OFDM system, consists L active primary user and  $K=\{1,23...K\}$  active secondary user. The secondary can access the primary user licensed spectrum via an access point. The total bandwidth is W allocated to primary user and this bandwidth is divided into N no. of OFDM subchannels. In our project we assume that real channel state information is present at the transreceiver of secondary user, primary user.

Let M denote the set of subchannelscorresesponding to sub-band corresponding to subband licensed to the lth primary user.Now CR will do periodic spectrum to find the frequency band which are not active and selects a subsets form  $\{1,2,3,4...N\}$  to transmits information.The bandwidth of subchannel is B and each has a span from  $f_s+(n-1)B$  to  $f_s+nB$ , where fs is starting frequency. When information is transmitted by CR over the nth subchannel the interference caused to the jthsubchannel sub-band of the lth primary user can be shown as[2]:-

$$I_{k,l}^{n} = \oint_{(j-1)B - (n-1/2)B}^{(j)B - (n-1/2)B} g_{n,l} \Phi(f) df$$

Where  $g_{n,l}$  is the power gain from access point to receiver of lth primary user in the nth

$$\phi(f) = T(\frac{\sin(\pi ft)}{\pi ft})^2$$

sub-band  $\Phi(f)$  denotes power spectral density of the OFDM channel and is given by:

where T is the symbol duration.

When the CR system is sensing the empty spectrum two kinds of errors[3] usually occurs first is false alarm which occurs when CR system detects that the primary user signal is present but actually spectrum is empty, second is misdetection which occurs when CR detects that spectrum is empty but actually primary user is present in the spectrum. Generally access point collects all the sensing information and then allocates the subchannel for the secondary user then the set of available subchannel are found in the subband of the lth primary user  $M_v^l$  and  $M_o^l$  set of unavailable subchannels are found.Now probability of misdetection and false alarm are  $q_n^m$ ,  $q_n^f$  respectively.

The values of these probability can be found out by local spectrum sensing by the transceiver or by mining the spectrum usage data to analyze the statistics of primary traffic pattern[4]. Misdetection causes in co-channel interference and false alarm results lowers the usage of the spectrum.

S no.	Real state	Resultof sensing	Information of heprobality						
1	Active	Occupied	$P\{O2 O1\}=1-q_n^m$						
	{01}	{O2}							
2	Active	Vacant	$P\{H2 O1\}=q_n^m$						
	{01}	{H2}							
3	Idle	Vacant	$P\{H2 H1\}=1-q_n^m$						
	{H1}	{H2}							
4	Idle	Occupied	$P{O2 H1}=q_n^f$						
	(H1)	{O2}							
	Table 5.1 Probability information from imperfect spectrum sensing								

There are four posible scenarios:

Where H1 and O1 are the hypothesis of present and absence of certain primary user signal of certain primary user on the nth sub-channel and H2, O2 denotes the hypothesis that the nth sub-channel is available or not based on information available from the sensing

- $P_{(1,n)}$  is the probability that nth sub-channel is used by the primary user.
- $P_{(2,j)}$ As the probability that jth sub-channel is used when the CR system finds it empty.

$$\mathsf{P}_{1,n} = \mathsf{P}\{\{\mathcal{O}_n | \tilde{\mathcal{O}}_n\}\} = \frac{\mathsf{P}\{\tilde{\mathcal{O}}_n | \mathcal{O}_n\} \mathsf{P}\{\mathcal{O}_n\}}{\mathsf{P}\{\tilde{\mathcal{O}}_n | \mathcal{O}_n\} \mathsf{P}\{\mathcal{O}_n\} + \mathsf{P}\{\tilde{\mathcal{O}}_n | \mathcal{H}_n\} \mathsf{P}\{\mathcal{H}_n\}}$$

$$=\frac{(1-q_n^m)q_n^L}{(1-q_n^m)q_n^L+q_n^f(1-q_n^L)},$$

where  $q_n^L$  the probability of sub band of the nth user is used by primary user.

Then the interference introduced to the *l*th PU by the access of the SU transmission power equal to unity

$$I_{n,l}^{SP} = \sum_{j \in \mathcal{M}_o^l} P_{1,j} I_{j,l}^n + \sum_{j \in \mathcal{M}_v^l} P_{2,j} I_{j,l}^n.$$

Define the signal-to-noise ratio(SNR) of the kth SU on the nth subchannel as

$$H_{k,n} = \frac{g_{k,n}^{ss}}{\Gamma(N_0 B + \sum_{l=1}^{L} I_{k,n,l}^{PS})},$$

where  $\Gamma$  is SNR gap and given by for uncoded MQAM[5]

$$\Gamma = -\frac{\ln(5BER)}{1.5}$$

with the BER specified in []. The trasnmission rate of nth subchannel used by kth SU is

$$r_{k,n} = \rho_{k,n} B \log(1 + p_{k,n} H_{k,n}),$$

where  $p_{k,n}$  is the kth SU's transmitted power on the nth subchannel,  $\rho_{k,n}$  can take only 1 or 0 indicating whether the nth channel is used by kth SU or not.

Now we defined spectrum spectrum efficiency as system throughput per unit bandwidth

$$\eta_{SE} = \sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} \log(1 + p_{k,n} H_{k,n}).$$

We also define energy efficiency as

$$\eta_{EE} = \frac{\eta_{SE}}{\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} \mathbf{p}_{k,n} + \mathbf{P}_{c}},$$

where  $P_c$  is the circuit power consumption [6]

#### 5.2 Problem Based on Given Constraints

Our optimization problem can be formulated as:

$$\max_{p_{k,n},\rho_{k,n}} \{\eta_{SE}, \eta_{EE}\}$$
  
s.t. C1 :  $p_{k,n} \ge 0, \forall k, n$   
C2 :  $\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} p_{k,n} \le P_t$   
C3 :  $\sum_{n=1}^{N} \sum_{k=1}^{K} \rho_{k,n} p_{k,n} I_{n,l}^{SP} \le I_l^{th}, \forall l$   
C4 :  $\rho_{k,n} \in \{0, 1\}, \forall k, n$   
C5 :  $\sum_{k=1}^{K} \rho_{k,n} = 1, \forall n.$ 

where  $P_t$  is the maximum power CR system can transmit and  $I_l^{th}$  is the interference power threshold of the *l*th user[7]. C2 and C3 maximum power that can be used by CR and interference constraints. C4 and C5 indicate that all subchannel are not shared among SUs.

#### 5.3 Heuristic allocation of the subchannel

Above equations defined the mixed integer problem that involves binary variable and real varaiable. To optimize EE and SE we are are considering two step procedure. We will first optimize subchannel allocation then we will optimize distribution of the power. We will use *heuristic allocation of the subchannel* to figure out what should be the value of  $\rho$ . Now, total no. of interference constraints should be L because there are L primary user. If the nth channel is interference limited then total power alloctedt the nth channel should be interference threshold constraints max power that can be allocated to the nth channel is

$$p_{k,n}^{max} = min(P_t, min(I_l^t / I_{n,l}^{SP}))$$

To solve above equation let m denote the subchannel used by the kth SU and N is the set of the subchannels. We allocate N subchannel one by one. While allocating the (n+1)th channel we find the value of m which is defined as follows

$$\eta_{\delta}^{k} = \left(\frac{\eta_{SE}^{k} - \eta_{SE}^{n}}{\eta_{SE}^{n}}\right) \left(\frac{\eta_{EE}^{k} - \eta_{EE}^{n}}{\eta_{SE}^{n}}\right),$$

The subchannel will be allocated to the SU with maximal  $\eta^k_{\delta}$ . Subchannel allocation scheme is as follows:

```
1. Initialization:
2. \mathcal{N} = \{1, 2, ..., N\}, \ \Omega_k = \emptyset, \ \forall k, \ k^* = 0, \ \eta_{SE}^0 = 0, \ \eta_{EE}^0 = 0
3. For n from 1 to N
Set tmp=0
5. For k from 1 to K
         Calculate \eta_{SE}^{k} and \eta_{EE}^{k} when subchannel n is added
6.
to \Omega_k,
7.
          Calculate \eta_{\delta}^{k}.
8.
         If \eta_{\delta}^{k} > tmp
             Set k^* = k
9.
           Endif
10.
           Update tmp = \eta_{\delta}^{k}
11.
12. Endfor
13. Update \Omega_{k^*} = \Omega_{k^*} \cup n
14. Calculate \eta_{SE}^n and \eta_{EE}^n.
15. Endfor
```

#### 5.4 EE-SE tradeoff metric

We transform our multiobjective problem into single objective problem using scalarization methods to facilitate our system design. Define EE-SE tradeoff metric as[9]:

$$U(P) = [\eta_{SE}(P)]^{\omega} \times [\eta_{EE}(P)]^{1-\omega},$$

where  $\omega \in [0,1]$ . ( $\omega$ ,1- $\omega$ ) is a given preference configuration for SE and EE. U(P) is referred

as the utility function.

#### **5.4 DC programming**

Now we do following transformation,

$$V(P) = \log U(P)$$
  
=  $\omega \log \eta_{SE}(P) + (1 - \omega) \log \eta_{EE}(P)$   
=  $\log \eta_{SE}(P) - (1 - \omega) \log(P_{sum} + P_c)$ ,

where,

$$P_{sum} = \sum_{n=1}^{N} p_n$$

Above equation is equivalent to,

$$\max_{p_n} f(P) - g(P)$$
  
s.t. C1 :  $p_n \ge 0, \forall n$   
C2 :  $\sum_{n=1}^{N} p_n \le P_t,$   
C3 :  $\sum_{n=1}^{N} p_n l_{n,l}^{SP} \le l_l^{th}, \quad l = 1, ..., L,$ 

where,

$$f(P) = \log \eta_{SE}(P)$$
 and  $g(P) = (1 - \omega)\log(P_{sum} + P_c)$ .

The f(P)-g(P) is d.c. function since both are concave function. The gradient of g(P) is  $\nabla g(P) = \left(\frac{\partial g}{\partial p_1}, \frac{\partial g}{\partial p_2}, \dots, \frac{\partial g}{\partial p_n}\right)_{,}$ 

where,

$$\frac{\partial g}{\partial p_n} = \frac{1-\omega}{\sum_{n=1}^N p_n + P_c}.$$

We expand g(P) with the taylor series[10-12] approximation at fairly large neighbourhood of  $P^{(t)}$  to get first order approximation, as g(P) is slowly sensitive to change in variable P, as

$$\begin{aligned} \max_{p_n} f(P) - g(P^{(t)}) - &< \nabla g(P^{(t)}), P - P^{(t)} > \\ \text{s.t.} \quad C1 : p_n \ge 0, \, \forall n \\ C2 : \sum_{n=1}^{N} p_n \le P_t, \\ C3 : \sum_{n=1}^{N} p_n l_{n,l}^{SP} \le l_l^{th}, \quad l = 1, ..., L, \end{aligned}$$

As function g(P) is concave, its gradient is also its super-gradient, so

$$g(P) \le g(P^{(t)}) + \langle \nabla g(P^{(t)}), P - P^{(t)} \rangle$$

Thus a non-convex optimization problem is converted into convex optimization problem. Generally, barrier method is a standard technique to solve convex optimization problems. For the barrier method, original problem is converted into a sequence of unconstrained minimization problems by defining alogarithmic barrier function with parameter t which decides the accuracy of the approximation. The solution to each minimization problem is called a centralpoint in the central path related to the original problem. As t increases, the central point will be more and more close to the optimal solution of the original problem. For searching the center point with a given t, Newton method is generally employed. Therefore, the barrier method is always carried out via two essential steps, namely centering step and Newton step. The former is the outer iteration which is executed to compute the central point starting from the previously computed one. And the latter is the inner iteration implemented during each centering step [34].

## CHAPTER 6 SIMULATION RESULT

#### Channel allocation matrix for N=16

omeg	a =															
						1										
	1	0	1	1	0	0	0	1	1	1	1	0	0	1	0	0

#### Results for different tradeoff preference $\boldsymbol{\omega}$

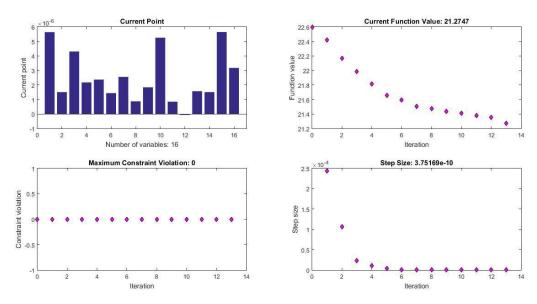
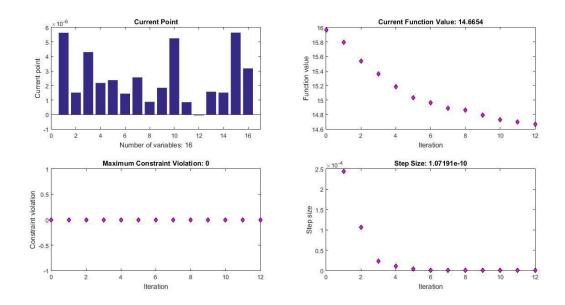


Fig 6.1 For  $\omega$ =0.2



**Fig 6.2** For ω=0.5

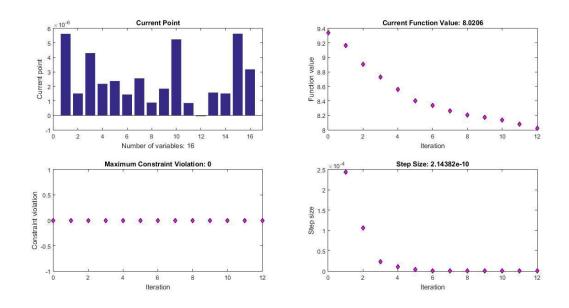
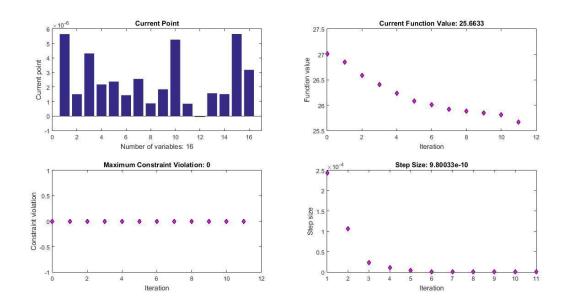
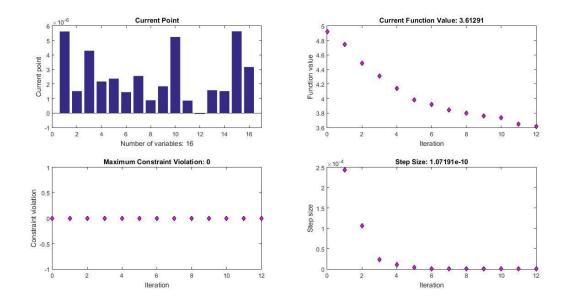


Fig 6.3 For  $\omega$ =0.8



**Fig 6.4** For ω=0



**Fig 6.5** For ω=1

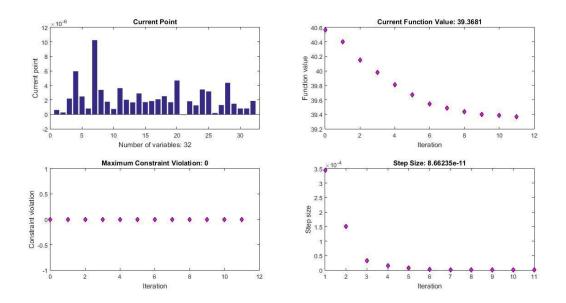
Channel allocation matrix for N=32

#### omega =

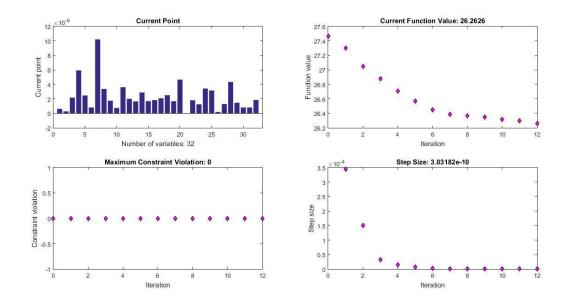
```
Columns 1 through 20
```

1	0	0	0	1	1	1	0	1	1	1	0	1	0	1	1	1	0	1	1
0	1	1	1	0	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0
Columns 21 through 32																			

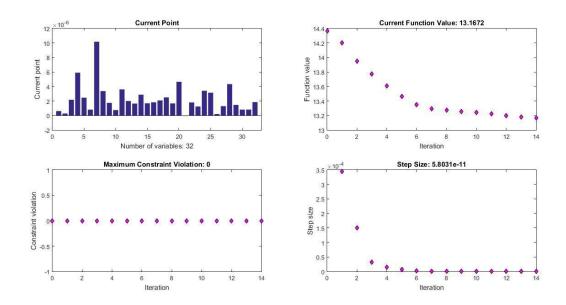




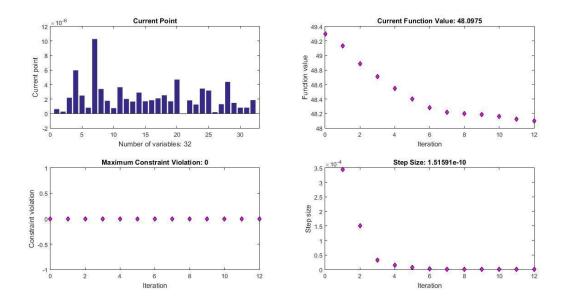




**Fig 6.7** For ω=0.5



**Fig 6.8** For ω=0.8



**Fig 6.9** For ω=0

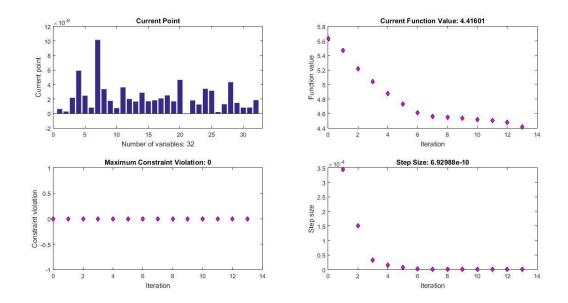


Fig 6.10 For  $\omega$ =1

# CONCLUSION

- We have formed the problem of the joint optimisation of energy aand spectral efficiency in OFDM based Cognitive radio networks.[22]
- We have found out globally optimum method to allocate the subcarrier to the secondary user.
- Then we converted the multiobjective problem into single objective one, we formed pareto sets of optimal solution.
- From the pareto set of solution we formed a globally optimised solution.
- Simulation results were presented.

### **APPENDIX** A

### **Fundamentals for EE-SE relation**

As the CR system is power limited and inteference-limited, assume there is greatest value of  $\eta_{se}^{max}$  and SE region is  $[0, \eta_{SE}^{max}]$ ,

**Theorem 1:** For a SE achieved by the power allocation matix P that satisfy all the conditions, the maximal EE,[8]  $\eta_{EE}^m(\eta_{SE}) = \max_P \eta_{EE}(\eta_{SE})$ , strictly quasiconcave in  $\eta_{SE}$ . Moreover, in the SE region  $[0, \eta_{SE}^{max}]$  the  $EE \eta_{EE}^m(\eta_{SE})$ 

- i. strictly increases with  $\eta_{SE}$  and is maximized at  $\eta_{SE} = \eta_{SE}^{max}$  if  $\frac{d\eta_{EE}^m}{d\eta_{SE}} |\eta_{SE} = \eta_{SE}^{max} \ge 0$
- ii. first strictly increases and then strictly decreases with  $\eta_{SE}$  and is maximized at  $\eta_{SE} = \eta_{SE}^*$  if  $\frac{d\eta_{EE}^m}{d\eta_{SE}} | \eta_{SE} = \eta_{SE}^{max} \ge 0$  where  $\eta_{SE}^* = argmax \eta_{EE}^m (\eta_{SE})$ .

The EE is quasi concave in SE in the CR-system model and there are two possible SE-EE curves. The SE-EE curves can be obtained by solving the following optimization problem:

$$\max_{p_n} \eta_{EE}$$
  
s.t. C1 :  $\eta_{SE} = C$ ,  
C2 :  $\sum_{n=1}^{N} p_n l_{n,l}^{SP} \le l_l^{th}$ ,  $l = 1, ..., L$   
C3 :  $p_n \ge 0, \forall n$ .

The above equation is calculated by pareto solution. Feasible solution  $P_1$  selected todominate solution  $P_2$ . If  $\eta_{SE}(P_1) \ge \eta_{SE}(P_2)$  and  $\eta_{EE}(P_1) \ge \eta_{EE}(P_2)$ . A Pareto solution will be called as Pareto optimal solution  $P_0$  if no solution feasible solution dominating it. There may be multiple Pareto optimal solutions for the multi-object optimization problem. The pareto optimal set is the set of all pareto optimal points.

**Theorem 2**: The Pareto optimal set of problem is

$$P^{POS} = \begin{cases} \{P | \eta_{SE}^* \le \eta_{SE} \le \eta_{SE}^{max}\} & \text{if } \eta_{SE}^* < \eta_{SE}^{max} \\ P | \eta_{SE} = \eta_{SE}^{max} \end{cases} \text{if } \eta_{SE}^* \ge \eta_{SE}^{max}.$$

## **APPENDIX B**

## Matlab Code

clc							
close all;							
clear all;							
	%k=no. of secondary	vuser=2					
	%n=no. of subchann	el=16					
	%l=no.of primary use	er=2					
N=16;	%no. of subcarrier						
omega=zeros(2,N);							
pta=rand(1,N);							
ptau=round(pta);							
gnl=10^-12.*exprnd(C receiver	).06,N,2);	%channel gain between AP and primary user					
T=110*10^-6;		%OFDM symbol duration					
B=16*10^3;	%Band	width of subchannel					
in=zeros(N,N,2);							
pc=40*10^-3;		%Circuit power consumption					
for n=1:N	%interference ir	ntroduced					
for j=1:N	%to the jth subcl	nannel					

```
for I=1:2
                          %in the sub-band of lth PU
      syms f;
      in(n,j,l)=integral(@(f)fun(T,gnl(n,l),f),((j-1)*B-((n-1)/2)*B),j*B-((n-1)/2)*B);
    end
  end
end
p1j=zeros(N,2);
p2j=zeros(N,2);
for j=1:N
  if ptau(j)==1
         p1j=p1j+0.98*squeeze(in(:,j,:));%probability that the jth subchannel is truly occupied
  end
                                                    %by PU while CR system makes a correct
judgement
  if ptau(j)==0
                                            %probability that the jth subchannel is truly
      p2j=p2j+0.01*squeeze(in(:,j,:));
occupied
  end
                                                    %when CR system deems it as vacant
end
isp=p1j+p2j
                               %interference introduced to the lth PU by the access of n
                          %SU on the nth subchannel with unit transmission
gkn=exprnd(0.06,2,N);
                                     %channel gain between AP and secondary user receiver
gam=-log(5*10^-6)/1.5;
                                     %SNR gap
n0=10^-13;
                                %noise PSD
l=1;
```

```
for n=1:N
  ispf(n,l)=isp(n,l)+isp(n,l+1);
end
ispf';
for k=1:2
for n=1:N
h(k,n)=gkn(k,n)/(gam*(n0*B+ispf(n,1)));
                                             %SNR of the kth SU on the nth subchannel
end
end
ispf;
ith=5*10^-12;
                                                   %interference threshold
inte=(5*10^-12)./isp;
Pt=0.8;
                                                   %total transmitted power
Pk=Pt/N;
  for n=1:N
    if Pt>min(inte(n,:))
       p(n)=min(inte(n,:));
    else
       p(n)=Pk;
    end
  end
nsen=0;
neen=0;
ktmp=0;
```

```
for n=1:N
```

tmp=0;

omegatmp1=omega;

#### for k=1:2

if k==1

omegatmp1(1,n)=1;

#### else

omegatmp1(2,n)=1;

omegatmp1(1,n)=0;

### end

nsek=sum(log2(1+p.\*sum(omegatmp1.\*h)));

neek=nsek/(sum(p));

if n==1

ndel=nsek\*neek; %normalized change index

#### else

ndel=(nsek-nsen)\*(neek-neen)/(nsen\*neen);

end

if ndel>tmp

ktmp=k;

end

tmp=ndel;

#### end

```
omega(ktmp,n)=1;
```

nsen=sum(log2(1+p.\*sum(omega.\*h)));

neen=nsen/(sum(p));

end

omega

x0=zeros(1,N);

w=0.5;

tpkn=sym('pkn%d%d',[2,N],'real');

pkn=(sum(omega.\*tpkn));

t=0;

z=0;

tol=10^-6;

```
pkn0=zeros(1,N);
```

```
pkn1=10^-4.*ones(1,N);
```

t=t+1;

```
con=@(pkn)[-sum(pkn)+Pt:-sum(sum(pkn*isp)+ith):sum(pkn)]; %constraints matrix
```

```
V=@(pkn)log(sum(log2(1+pkn.*sum(omega.*h))))-sum((1-w)*log10(sum(pkn1)+pc)*(1-pkn+pkn1));
```

%objective function

optimtool;

%optimization toolbox

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