Jaypee University of Information Technology, Waknaghat
Test-I Examination, February 2019
B.Tech (ECE/CSE/IT)

Course Title: Probability Theory and Random Processes
Course Code: 10B11MA411
Semester: IV

Max. Marks: 15 marks

Max. Time: 1 hour

Note: Answer all the questions. ALL questions carry equal marks. Use of calculators is allowed.

1. A tube manufacturing factory has three machines E_1, E_2, E_3 that produce 50%, 25% and 25%, respectively, of the total daily output of electric tubes. It is known that 4% of the tubes produced one each of machines E_1 and E_2 are defective, and that 5% of those produced on E_3 are defective.

   (a) If one tube is picked up at random from a day's production, calculate the probability that it is defective.

   (b) Calculate the probability that the defective tube was produced on machine E_1.

2. The length of time X needed by students in a particular course to complete a 1 hour exam is a random variable with PDF given by

$$f(x) = \begin{cases} \frac{6}{5}(x^2 + x) & , \quad 0 \le x \le 1 \\ 0 & , \quad \text{else} \end{cases}$$

   (a) Find the *cumulative distribution function* of X.

   (b) Use the CDF to find (i) $\mathbb{P}(X \le -0.5)$ (ii) $\mathbb{P}(X \le +0.5)$.

3. Consider the density function $f(x) = 1$, where $0 < x < 1$.

   (a) Find the *moment generating function* of X.

   (b) Use *moment generating function* to find $E(X)$ and $E(X^2)$.

4. Consider the joint density of X and Y:

$$f(x, y) = \begin{cases} \frac{1}{y} & , \quad 0 \le x \le y, \ 0 < y < 1 \\ 0 & , \quad \text{else} \end{cases}$$

   (a) Find the *marginal* density $f_X(x)$ of X.

   (b) Write down the *conditional* density $f_{Y|X}(y|x)$ of Y given $X = x$.

5. Suppose that $E(X) = 3$, $\sigma_X^2 = 2$ and $Y = -6X + 22$.

   (a) Find the *covariance* $\text{Cov}(X, Y)$ between X and Y.

   (b) Are X and Y correlated random variables? Justify your answer.

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