REALIZATION OF A 2-DIMENSIONAL LINEAR PHASE FIR FILTER USING ORTHOGONAL POLYNOMIALS

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CERTIFICATE

This is to certify that the work entitled, "Realization of 2 Dimensional FIR Linear Phase Filter Using Orthogonal Polynomials" submitted by A. Raghavendra Kumar ID 031074, S. Bharat Kumar ID 031066, P.Varun Choudary ID 031004 in partial fulfillment for the award of degree of Bachelor of Technology in Electronics And Communications of Jaypee University of Information Technology has been carried out under my supervision. This work has not been submitted partially or wholly to any other University or Institute for the award of this or any other degree or diploma.

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Acknowledgment

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Abstract

Image Processing has grown by leaps and bounds in the last few years. Modern digital technology has made it possible to manipulate multi-dimensional signals with systems that range from simple digital circuits to advanced parallel computers. The goal of this manipulation can be divided into three categories:

* Image Processing i.e. *Image in -> image out*

* Image Analysis i.e. *Image in -> measurements out*

* Image Understanding i.e. *Image in -> high-level description out*

As part of our project we propose to design a two dimensional linear phase fir filter which performs the first action as mentioned above. We also provide the various results and analysis associated with the image processing of respective filters.

1. INTRODUCTION:

1.1 What is an Image?

An image as defined in the "real world" is considered to be a function of two real variables, for example, a(x,y) with a as the amplitude (e.g. brightness) of the image at the real coordinate position (x,y).

Further, an image may be considered to contain sub-images sometimes referred to as regions-of-interest, ROIs, or simply regions. This concept reflects the fact that images frequently contain collections of objects each of which can be the basis for a region.

1.2 What is Image Processing?

In the broadest sense, image processing is any form of information processing for which both the input and output are images, such as photographs or frames of video.

In a sophisticated image processing system it should be possible to apply specific image processing operations to selected regions. Thus one part of an image (region) might be processed to suppress motion blur while another part might be processed to improve color rendition.

The following picture shows what exactly an image processing does:

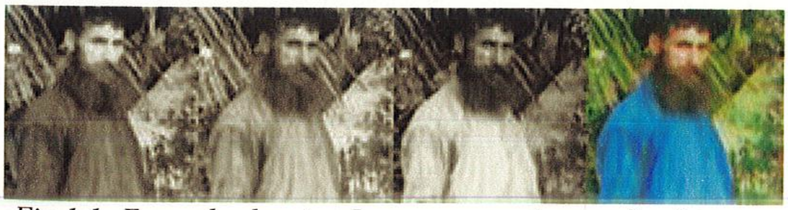

Fig 1.1: Example showing Image processing application

The first three pictures show red, green, and blue color channels of a photograph. The fourth image is a composite

1.3 What can be done by Image Processing?

- Geometric transformations such as enlargement, reduction, and rotation
- Color corrections such as brightness and contrast adjustments, quantization, or conversion to a different color space
- Registration (or alignment) of two or more images
- Combination of two or more images, e.g. into an average, blend, difference, or image composite
- Interpolation, and recovery of a full image from a RAW image format
- Segmentation of the image into regions
- Image editing and digital retouching
- Extending dynamic range by combining differently exposed images

1.4 Applications:

Image Processing finds applications in the following areas:

- Photography and printing
- Satellite image processing
- Medical image processing
- Face detection, feature detection, face identification
- Microscope image processing

2 FILTERS AND THEIR CLASSIFICATION:

2.1 What is a filter?

A filter is a device that discriminates according to one or more attributes at its input, what passes through it.

One example is the color filter which absorbs light at certain wavelengths.

We shall describe frequency-selective filters in this discussion. It is called frequency-selective because it discriminates among the various frequency components of its input. By filter design we can create filters that pass signals with frequency components in some bands, and attenuate signals with content in other frequency bands.

2.2 Classification of Filters:

1-> Low Pass Filter: A low-pass filter is a filter that passes low frequencies but attenuates frequencies higher than the cutoff frequency.

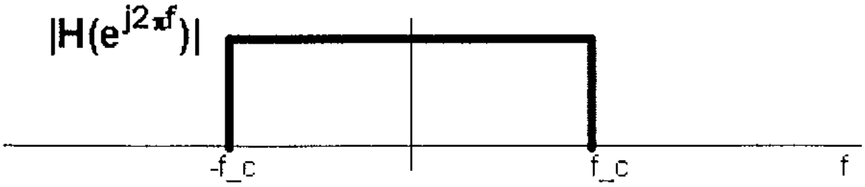

Fig 2.1 Ideal Low Pass Filter

2-> High Pass Filter: A high-pass filter is a filter that passes high frequencies well, but attenuates frequencies lower than the cutoff frequency.

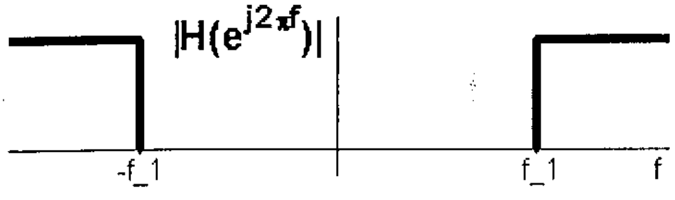

Fig 2.2 Ideal High Pass Filter

3->Band Pass Filter: A band-pass filter is a device that passes frequencies within a certain range and rejects frequencies outside that range. It is a combination of a high-pass and a low-pass filter

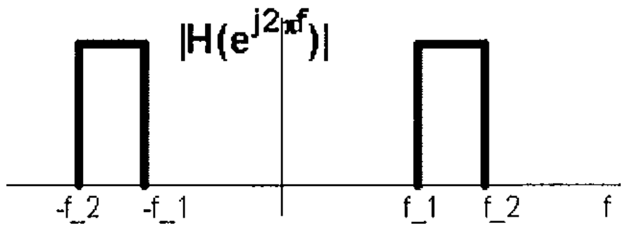

Fig 2.3 Ideal Band Pass Filter

4->Band Reject Filter: A band reject filter is a device that passes most frequencies unaltered, but attenuates those in a specific range to very low levels. It is the opposite of a band-pass filter.

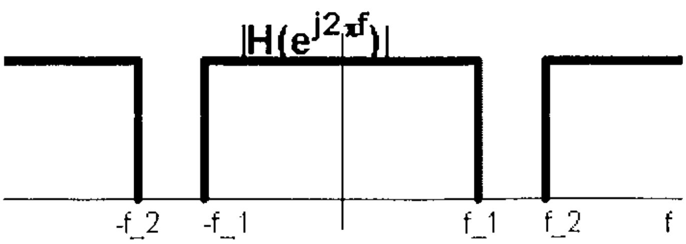

Fig 2.4 IdealBand Reject Filter

3 PROJECT DESCRIPTION:

3.1 Design of Filter:

The prima facie of our project involves designing of a two dimensional filter i.e. a filter whose characteristics vary in both X and Y coordinates.

A two dimensional filter is realized using cylindrical co ordinates (P, O). O denotes surface area which is a constant for any given picture and is equal to 2*pie. So, we vary the co ordinates with respect to P (where P = (X^2+Y^2) ^0.5)

The process we employed in realization of filter is to fit in a curve along the ideal response of the filter output by using the concept that **any given curve can be represented as a linear combination of orthogonal polynomials.**

Step 1:

The first and foremost step in the designing of filter is calculating the required orthogonal functions. Now a question immediately crosses our mind.

3.2 What is an orthogonal function?

The orthogonal function F(P) over the interval 0 to 1 is defined as follows:

Let A= 2*pi*[integral(Fi(P)*Fj(P)dP)][P,0,1]

Then, A = 1 if i=j

A = 0 if i! =j

If the above conditions are satisfied, then it is called an orthogonal function.

3.3 Polynomials:

For the realization of required filter, we have calculated orthogonal functions up to 32^{nd} degree. The polynomials that are calculated are as follows:

P0 = 0.56418

P1 = 1.953*P^2 - 0.976

P2 = 7.565*P^4 - 7.565*P^2 + 1.26

P3 = 118.422*P^8 - 236.844*P^6 + 152.257*P^4 - 33.834*P^2 + 1.691

P4 = 118.422P^8 – 236.844P^6 + 152.257P^4 – 33.83P^2 + 1.691

P5 = 471.314*P^10 - 1178.285*P^8 + 1047.364*P^6 - 392.761*P^4 + 56.108*P^2 - 1.87

P6 = 1878.699*P^12 - 5636.097*P^10 + 6404.655*P^8 - 3415.816*P^6 + 853.954*P^4 - 85.395*P^2 + 2.033

P7 = 7495.601*P^14 - 26234.604*P^12 + 36324.836*P^10 - 25225.5808*P^8 + 9172.938*P^6 - 1651.128*P^4 + 122.305*P^2 - 2.184

P8 = 29923.787*P^16 - 119695.144*P^14 + 195502.055*P^12 - 167573.177*P^10 + 80564.02*P^8 - 21483.736*P^6 + 2929.6001*P^4 - 167.405*P^2 + 2.325

P9 = 119510.294*P^18 - 537796.713*P^16 + 1.01232400147*10^6*P^14 - 1.033414*10^6*P^12 + 620049.474*P^10 - 221446.433*P^8 + 45424.948*P^6 - 4866.962*P^4 + 221.225*P^2 - 2.458

P10 = 477443.027*P^20 - 2.38709970529*10^6*P^18 +5.08801916066*10^6*P^16 - 6.02988550764*10^6*P^14 +4.344774053058*10^6*P^12 - 1.9550004599*10^6*P^10 +543008.76*P^8 - 88645.632*P^6 + 7670.356*P^4 - 284.047*P^2 + 2.581

P11 = 1.907780396227*10^6*ρ^22 - 1.049151346616*10^7*P^20 + 2.497602507944*10^7*P^18 - 3.3711311666308*10^7*P^16 + 2.8381822123294*10^7*P^14 - 1.544776962137*10^7*P^12 +5.4501124446702*10^6*P^10 - 1.21595762415*10^6*P^8 +162025.957*P^6 - 11563.632*P^4 +355.40005*P^2 - 2.688

P12 = 7.62137164031*10^6*P^24 - 4.5711892173738*10^7*P^22 + 1.2020796553161*10^8*P^20 - 1.820988665514*10^8*P^18 + 1.7558380914733*10^8*P^16 - 1.1238400471054*10^8*P^14 + 4.8319518223*10^7*P^12 - 1.38130489747*10^7*P^10 + 2.541305480923*10^6*P^8 - 282704.0301*P^6 +16988.562*P^4 - 442.057*P^2 + 2.838

P13 = 5.1396268963*10^6*P^26 - 2.607415638393*10^7*P^24 + 5.26188724181*10^7*P^22 - 4.81992029409103*10^7*P^20 + 5.7827014040557*10^6*P^18 + 3.233339436228*10^7*P^16 - 3.6233891082256*10^7*P^14 + 2.0063500606953*10^7*P^12 - 6.60823686072*10^6*P^10 + 1.322484922619*10^6*P^8 -154310.817*P^6 + 9465.102*P^4 -245.44*P^2 + 1.532

P14 = 1.2304743927683*10^7*P^28 - 7.66638994425*10^7*P^26 + 2.08416204837905*10^8*P^24 - 3.24583000292682*10^8*P^22 + 3.20457682758143*10^8*P^20 - 2.11245167673457*10^8*P^18 + 9.7546722342869*10^7*P^16 - 3.458224945668*10^7*P^14 + 1.0916287383266*10^7*P^12 - 3.207179988*10^6*P^10 + 741225.638*P^8 - 109853.882*P^6 + 8779.331*P^4 - 298.826*P^2 + 2.471

P15 = 1.5756285072692*10^7*P^30 - 8.39226830795*10^7*P^28 + 1.77660692352168*10^8*P^26 - 1.75484985711878*10^8*P^24 + 5.45863851630001*10^7*P^22 + 2.87841105370209*10^7*P^20 + 1.31531013145809*10^7*P^18 - 8.1909996377724*10^7*P^16 + 8.510255979902*10^7*P^14 - 4.6000094228671*10^7*P^12 + 1.4901567623063*10^7*P^10 - 2.94921843782*10^6*P^8 + 342896.214*P^6 - 21175.013*P^4 + 560.419*P^2 - 3.636

P16 = 5.17578170865*10^6*P^32 - 1.2962865855147*10^7*P^30 - 1.834538884972*10^7*P^28 + 1.03581493603741*10^8*P^26 - 1.46640720787066*10^8*P^24 + 8.606596146965*10^7*P^22 - 2.56536669441*10^7*P^20 + 5.085647019095*10^7*P^18 - 9.4967337612885*10^7*P^16 + 8.5505081614414*10^7*P^14 - 4.4184067648872*10^7*P^12 + 1.4006043288873*10^7*P^10 - 2.73189516184*10^6*P^8 + 313772.997*P^6 - 19157.292*P^4 + 501.544*P^2 - 3.221

3.4 Mathematical Calculations:

The above functions were realized with the help of **mathematica**, a mathematical tool that helps in performing very complex calculations. As an example, we would like to show the calculations involved in determining the function F3 (P)

Let F3 (P) = (a*P^6 + b*P^4 + c*P^2 + d)

The following are the steps involved in calculating coefficients a, b, c, d

Integrate [(a*P^6 + b*P^4 + c*P^2 + d)*P, {P, 0, 1}]

Ans-> (a/8 + b/6 + c/4 + d/2)

Integrate [(a*P^6 + b*P^4 + c*P^2 + d)*(1.953P^2 - 0.976)*P, {P, 0, 1}]

Ans-> 0.0733 a + 0.0814583 b + 0.0815 c + 0.00025 d

Integrate [(a*P^6 + b*P^4 + c*P^2 + d)^2*P, {P, 0, 1}]

Ans-> (a^2/14 + (ab)/6 + 1/10(b^2 + 2ac) + (cd)/2 + d^2/2 + 1/4(bc + ad) + 1/6(c^2 + 2bd))

Solve [{(a/8 + b/6 + c/4 + d/2) == 0, 0.0733 a + 0.0814583 b + 0.0815 c + 0.00025 d = 0, (a^2/14 + (ab)/6 + 1/10(b^2 + 2ac) + (cd)/2 + d^2/2 + 1/4(bc + ad) + 1/6(c^2 + 2bd)) == 0.159, {a, b, c, d}]

Ans-> {{a -> -29.1792, b -> 45.3517, c -> -19.0904, d -> 1.72277},

{a -> 29.1792, b -> -45.3517, c -> 19.0904, d -> -1.72277}}

The above two values are conjugates of each other, so we select one of them and proceed with the calculations.

Note: In all our calculations, we have selected set of values whose first coefficient is positive.

3.5 Fitting Of Curve:

Step 2:

After finding the orthogonal functions, the second step involves in **fitting an appropriate curve into the required filter response**. We do it because it is not possible to actually realize a filter with ideal filter characteristics i.e. a filter with exact step response output. So, for practical purposes we fit a curve along these step responses.

Fitting in of curve is obtained as a linear combination of all the orthogonal functions.

That is,

F = a0*P0 + a1*P1 + a2*P2 + a3*P3 + a4*P4 + a5*P5 + a6*P6 + a7*P7 + a8*P8 + a9*P9 + a10*P10 + a11*P11 + a12*P12 + a13*P13 + a14*P14 + a15*P15 + a16*P16

Now, each of the coefficients is calculated using the following concept

Suppose we want to calculate a0, then multiply both sides of equation by P0 and then integrate on both sides over the entire surface.

From the concept of orthogonality we know that except for the first term, rest all of them are equal to zero and the term under integral in first one is equal to 1

i.e. we have, a0=Integral [F*P0*PdP] {P, 0, 1}

From this we get a0, repeating the same process, we get all other coefficients.

4 FILTERS DESIGNED:

As a part of our project, we have designed the following 3 filters:

1-→ Low Pass Filter

2-→ High Pass Filter

3-→ Band Pass Filter

4.1 Coefficients:

The various coefficients for each of the filter are as follows:

Coefficients	*Low Pass Filter*	*High Pass Filter*	*Band Pass Filter*
A0	0.102493	0.236015	0.169254
A1	-0.110621	0.0656916	-0.0923009
A2	0.0373995	-0.0553064	-0.062176
A3	0.0237323	0.0293055	0.0475122
A4	-0.0265923	-0.00284119	0.00913599
A5	-0.0012922	-0.0128686	0.0173793
A6	0.0145348	-0.0148882	-0.0324866
A7	-0.0051587	-0.00790506	0.000257429
A8	-0.00519307	-0.000459967	0.00485416
A9	0.00428304	0.00480324	0.00904902
A10	0.000481852	-0.00430179	0.00183674
A11	-0.00127414	0.00151933	-0.0117739
A12	0.000289132	0.000422947	0.00384538
A13	-0.000132157	0.000885961	0.000872882
A14	0.00102109	-0.00237585	0.0111637
A15	-0.000868193	-0.000508211	-0.00656217
A16	-0.000702642	-0.000508876	-0.00452921

4.2 Transformation Used:

For the analysis of our filter, we require to convert all the above obtained polynomials into frequency domain. The transformation used for this purpose is explained below:

P= p0*cos(w/2)   ----(1)

Where,

P →orthogonal polynomial in p

p0 →constant(we take it as 1)

w →frequency

To convert our polynomial into frequency domain, we assume w to be sqrt(u^2+v^2) (u and v are frequency domain variables).This gives an entire circular domain of frequencies in the two dimensional plot. Thus, we have the final transformation as

P=cos (sqrt (u^2+v^2)/2)

We resort to this transformation because it results in an output characteristic curve which has a sharp transition in the pass band and low side lobes in the stopband.

Note:

On using this transformation, all the lower order values in the original polynomial are transformed to higher components in the frequency domain for the simple reason that substituting P=0 in (1) gives w= pie and P=1 in (1) gives w=0.

Thus, our low pass filter has high pass characteristics in frequency domain and high pass filter has low pass characteristics.

5 FILTER CHARACTERISTICS:

5.1 Results:

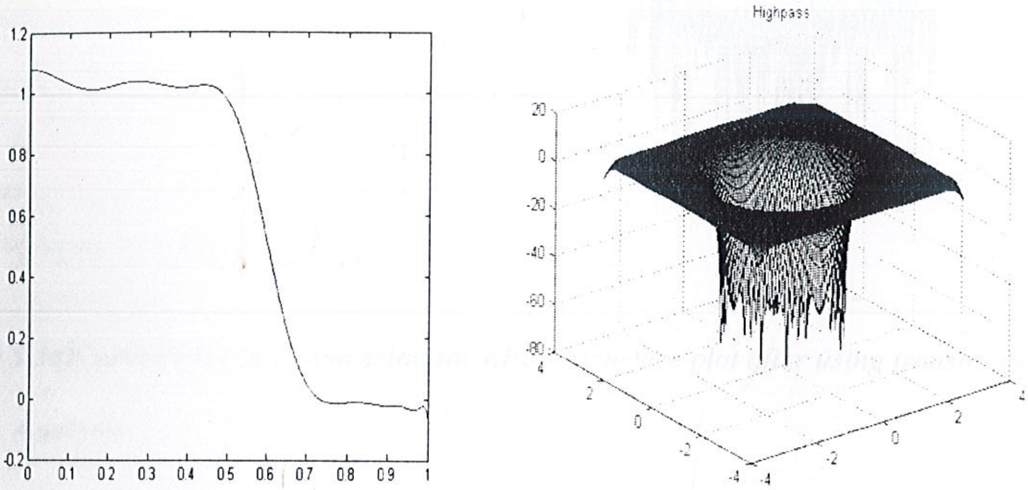

Fig 5.1a)Curve fitting in 2-dimension for LPF b)Surface plot after using transformation

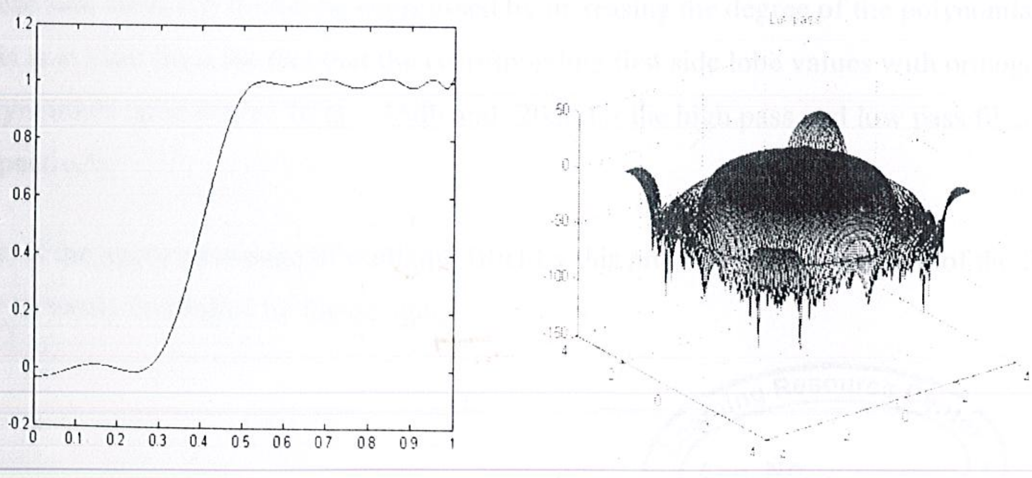

Fig 5.2a)Curve fitting in 2-dimension for HPF b)Surface plot after using transformation

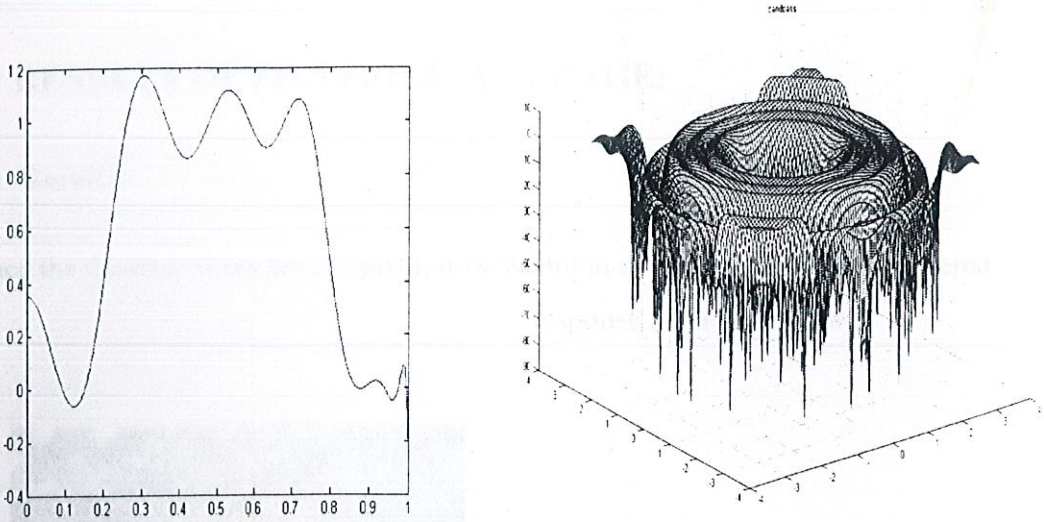

Fig 5.3a)Curve fitting in 2-dimension for BPF   b)Surface plot after using transformation

5.2 Analysis:

As shown in the figures above, the resultant filter outputs are quite close to the ideal filter characteristics. Even the side lobes are quite low. The first side lobes for the high pass and low pass filters are at -40db and -30db respectively

These side lobes can further be suppressed by increasing the degree of the polynomial. This is evident from the fact that the corresponding first side lobe values with orthogonal polynomials up to degree 16 is -35db and -20db for the high pass and low pass filters respectively.

This is the major advantage of realizing filter by this process. The parameters of the filter can be easily controlled by the designer.

6 RESULTS OF FILTERING AN IMAGE:

6.1 Results:

Once the desired filters are designed, now we input a image and obtain its filtered response as shown below:

Fig 6.1 Original Image

Fig 6.2 Low Pass filtered image

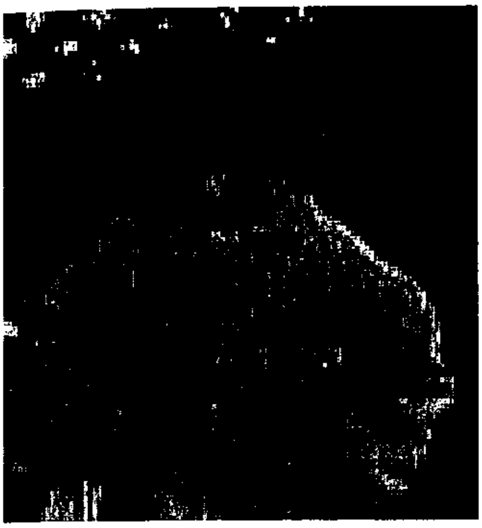

Fig 6.3 High Pass Filtered image

Fig 6.4 Band Pass Filtered image

6.2 Analysis:

Low frequencies in the frequency domain are responsible for the general gray-level appearance of an image over smooth areas, while high frequencies are responsible for detail such as edges and noise.

A filter that attenuates high frequencies and passes lower components is called a low pass filter whereas a filter that has opposite characteristics is called a high pass filter. A band pass filter on the other hand, passes only select frequency components as specified by the pass band.

Thus, a low pass filtered image has less sharp detail in the image than the original one because all the higher frequencies are attenuated.

A high pass filtered image has less gray level variations in smooth areas and emphasized transitional gray level detail. Such an image will appear sharper.

The band pass filter has an output image that contains characteristics which are a mixture of both the high pass and low pass filters.

7 ADDING NOISE TO THE IMAGE:

7.1 Noise:

Our next step involves in adding noise externally to the image at the desired frequency components. This is accomplished in the same manner as we have designed our filter i.e. by assuming it to be a linear combination of orthogonal polynomials. and finding the corresponding coefficients.

The noise we have added and the images we obtained after their filtration are as follows:

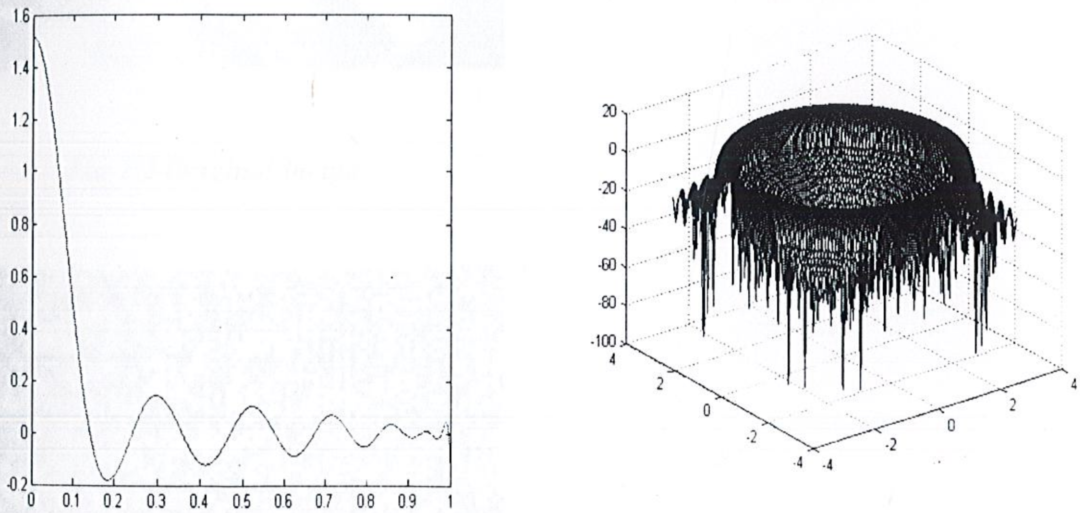

Fig 7.1a)Curve fitting in 2-dimension for Noise b)Surface plot after using transformation

We have added a high frequency noise to our image. The corresponding outputs we got on adding noise to the image and the filtering of noised image are as follows:

7.2 Results:

Fig 7.2 Original Image

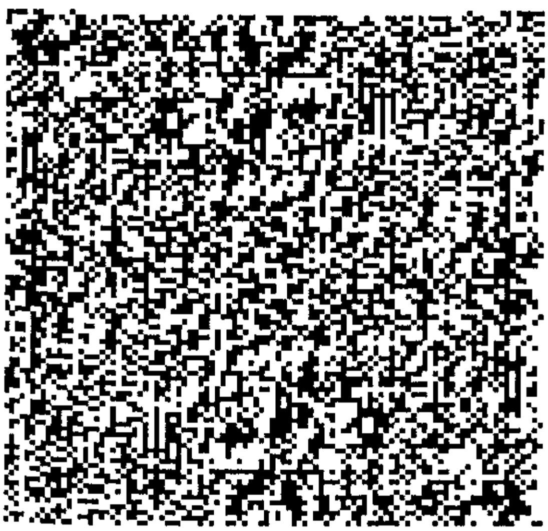

Fig 7.3 Noisy Image

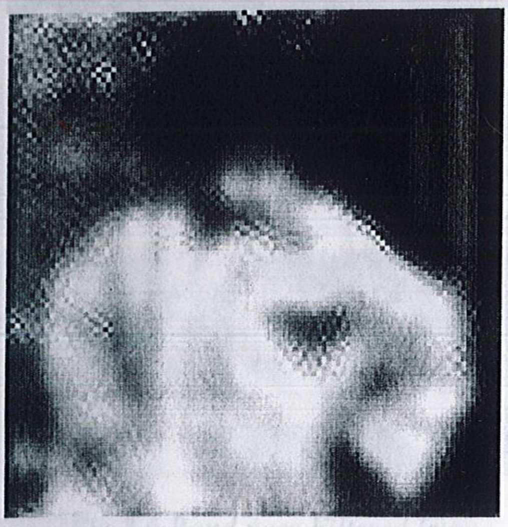

Fig 7.4 Low pass Filtered Image

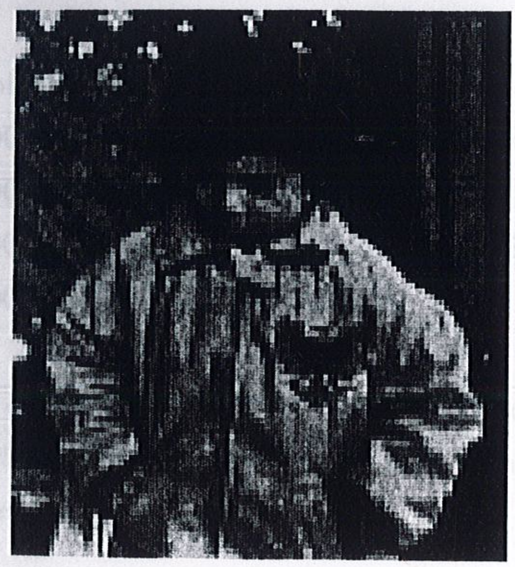

Fig 7.5 High pass Filtered Image

Fig 7.6 Band pass Filtered Image

The noise that we have added in this case is given by the following polynomial:

fnoise=2*pi*(0.0028209*p0-0.00483481*p1+0.00611688*p2-0.00701972*p3+0.00763793*p4-0.00801291*p5+0.00816935*p6-0.00812649*p7+0.00790245*p8-0.00751622*p9+0.00698069*p10-0.00629207*p11+0.00564231*p12+0.00292156*p13+0.00621799*p14-0.00729859*p15-0.00639602*p16)

As evident from the output images, the noise that has been added to the image has been perfectly filtered by the low pass filter, whereas the high pass and band pass filters have some traces of noise left.

This step of adding noise to an image and then filtering it out has been done so as to show the response of filter designed to noisy images.

8 CONCLUSION:

There are numerous filters with the same specifications as the filter we have designed and further the filters we have designed are some of the very common filters that find vast applications and have numerous design techniques. So what makes our project special and different from other design methods?

The basic concept underlying in our filter design is that any given curve can be obtained as a linear combination of orthogonal polynomials and further these orthogonal polynomials once realized remain the same for all applications. Only the linear combination of these polynomials varies according to the curve that has to be fit into.

Further, the curve fitting is obtained almost as desired by the designer. Also various other filter characteristics like the 3-db cut off, side lobe height are completely under the control of designer. The accuracy in the design of filter can be enhanced by increasing the degree of orthogonal polynomial. As a part of our project we have calculated these polynomials upto 32nd degree.

Currently, design of filters with such high precision and such high degree of control vested with the designer are not available. We hope that our project is going to find applications in the near future.

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