DESIGN OF SFRC MIXES AND ANALYSIS OF PAVEMENT USING MATLAB

BY

MADHUR KHANDELWAL - 041611

VIKAS SINGH - 041615

विद्या तत्व ज्योतिसमः

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY

MAY - 2008

Submitted in Partial fulfillment of the Degree of Bachelor of Techonology

to

DEPARTMENT OF CIVIL ENGINEERING

JAYPEE UNIVERSITY O INFORMATION TECHNOLOGY

WAKNAGHAT

CANDIDATE'S DECLARATION

We hereby certify that the work which is being presented in this report , **"Design of SFRC mixes and Analysis of Pavement using MATLAB"** in fulfillment of the requirement for the award of B.TECH degree, submitted in the department of Civil Engineering, Jaypee University of Information Technology, Waknaghat, Solan, is an authentic record of our own work carried out from July '07 to May '08 under the guidance of **Prof. Dr. R.M Vasan**, HOD, Department of Civil Engineering, Jaypee University of Information Technology, Waknaghat, Solan (H.P).

We have not submitted the matter embodied in this report for the award of any other degree.

Dated: 29.5.2008

Madhur

(Madhur Khandelwal)

041611

Vikas

(Vikas Singh)

041615

CERTIFICATE

This is to certify that the work entitled, **"Design of SFRC mixes and Analysis of Pavement using MATLAB"**, submitted by **Madhur Khandelwal** (041611) and **Vikas Singh** (041615) in fulfillment of the award of degree of Bachelor of Technology in Civil Engineering of Jaypee University of Information Technology, Waknaghat, Solan, has been carried out under my supervision. This work has not being submitted partially or wholly to any other University or Institute for the award of this or any other degree or diploma.

(Dr. R.M. VASAN)
Professor and Dean
Jaypee University of Information Technology
Waknaghat, Solan (H.P)

ACKNOWLEDGEMENT

The success of any project depends largely on the encouragement and guidelines of many others. Therefore we take this opportunity to express my sincere gratitude to the people who have been instrumental in the successful completion of the project. We would like to express my sincere appreciation and gratitude to my guide **Professor Dr. R.M. Vasan**, HOD, Civil Engineering Department, Jaypee University of Information Technology, Waknaghat without whose able guidance, tremendous support and continuous motivation, the project would not have been carried to perfection. We sincerely thank him for spending all his valuable time and energies during the execution of project. Constant encouragement, generous help and guidance given by Senior Lecturer Ramakant Dwivedi and Mr. Anil Dhiman throughout this study is gratefully acknowledged.

We wish to gratefully acknowledge the support provided by the Faculty of Civil Engineering, Waknaghat, throughout the project schemes.

We wish to gratefully acknowledge the library staff and several authors of various text books and research papers which have been referred in this study but have remained unmentioned in the list of references.

ABSTRACT

The conventional plain cement concrete has been widely used but it has a certain limitations like low brittleness, tensile and flexural strength. Therefore, to overcome these limitations fibers are incorporated in the concrete mix. Fiber reinforced concrete consists of hydraulic cement, water, fine or fine and coarse aggregate and discontinuous discrete fibres. It may also contain pozzolans, superplasticisers and other admixtures commonly used with concrete to increase its workability. There are various types of fibers such as steel, carbon, nylon, polypropylene, asbestos and alkali resistant glass but steel is the principal type of fibre and is most commonly used while others are used for specific applications only. The incorporation of steel fibres in concrete increases its resistance to tensile stresses and modifies its ductility depending upon the fibre concentration and its distribution. The physical characteristics of the fibres affecting the concrete behavior are the aspect ratio, shape and volume fraction.

Formation of voids and cracks reduces the strength of plain cement concrete in the hardened state as the increase of stress causes the cracks to merge and propagate and thus the number and size of cracks increases. This initiation and growth of new cracks reduces the load carrying capacity of plain concrete and hence the specimen fails. The incorporation of fibres in concrete enhances the crack arrest mechanism, exhibiting superior post cracking ductility and significantly higher load carrying capacity with substantially reduced cracking in terms of size and number of cracks. So fibres appear as a good tool for improving the strength characteristics of cement concrete matrix.

As steel fiber has high ductility and resistance to tensile stress, incorporation of steel fibre in the mix results into the increase of flexural strength and also the compressive strength of the matrix. In the present study we are going to design a matrix with varying percentage of cement, fly-ash, steel fibres, water and aggregate; so as to obtain a steel fibre reinforced concrete which is effective, efficient and economical. For obtaining this result blocks of size 150 x 150 x 150mm for various proportions of mixes have been tested and their results have been compared.

The method of analysis taken was the Finite Element and the soft language used is the MATLAB. The code was to be developed in MATLAB involving steps of analysis of structures by Finite Element Method.

TABLE OF CONTENT

LIST OF FIGURES

LIST OF TABLES

CHAPTER-1

INTRODUCTION

1.1 GENERAL

From a rail dominant economy in the 1950s, India has become a decidedly road dominant economy in the 1990s. The roadways have grown rapidly in independent India. Ranging from the cross-country link of the national highways to the roads in the deepest interiors, the country has a road network of 3.3 million km, making it the second largest road network in the world (19). US lead the world with 6.43 m km of roadways. China has 1.87 million km's and Brazil has 1.75 million km's and Japan at 1.18 million km's.

With such a vast network of roads what really does matter is the condition of these roads. No doubt the condition of roads has improved in the recent years with technological and economical advancement. But outside the major cities, main roads and other roads are poorly maintained and congested. The main factors affecting a highway are vehicular loads, variation in temperature and moisture. The combined affect of these are responsible for setting up tensile stresses in concrete. Now concrete which has a weaker tensile strength than compressive strength tends to develop cracks.

Micro cracks start to generate in the matrix of the concrete. The magnitude of the combined stresses of concrete should always be below the flexural strength in order to safeguard against the formation and prorogation of cracks, as also to ensure the structural stability and desired serviceability under the worst traffic and environmental conditions. To improve the condition of roads so as to improve the tensile strength, flexural strength, toughness and impact resistance of concrete various researches has been conducted. These researches have provided information that all these parameters can be improved upon by reinforcing the concrete by various types of fibre (7).

It has been observed that by varying the size and volume of fibre, the steel fibre reinforced concrete mix do not show a substantial improvement in strength as compared to mixes without fibres. But using fibres in concrete increases the post cracking tensile behavior and ductility. Incorporation of fibre in concrete can be used to reduce the thickness of pavements and hence improves the strength characteristics and load bearing capacity of the pavement. The fibre tends to increase the initial capital cost but reduces the subsequent maintenance cost. Various fibre types are currently being specified in bridge decks, ultra-thin white topping pavements, thin unbounded overlays, and concrete bus pads.

Steel Fibre Reinforced Concrete (SFRC) is a composite material comprising:

a. hydraulic (Portland) cement
b. aggregate
c. sand
d. water
e. A dispersion of discontinuous steel fibres.

It may also contain supplementary cementitious material (such as fly ash, silica fume or ground granulated blast furnace slag) and/or additives (admixtures) commonly used in conventional concrete. The uniform distribution of fibres in a well proportioned and graded concrete is essential to achieving appropriate ductility which is fundamental to the performance of Steel Fiber Reinforced Concrete.

1.2 SCOPE OF STUDY

The properties of fibre reinforced concrete mixes and the effect of various mix design parameters such as aggregating grading, water cement ratio, fibre types, volume and aspect ratio of fibres on the strength characteristics has now been well established as a result of numerous research investigations carried out to date. Recent work preformed in U.S.A. by *Zheng and Chung* showed the approximate doubling of the flexural strength with only 0.3 ~01.8 carbon fibres - an improvement resulting from the use of chemical agents(9). Another research by M.Sargaphuti, S. P. Shah, and K. D. Vinson showed how introduction of cellulose fibre reduces the adverse effects of shrinkage cracking. However, after being subjected to repeated wet/dry cycling, a 40 percent drop in toughness of both cellulose and polypropylene fibre composite was also observed.

In order to utilize the present study for rigid pavements, it would be necessary to examine the experimental behavior of plain cement concrete, plain cement concrete with fly ash and steel fibre reinforced concrete mixes in comparison to each other and then determine the cost and strength associated with the use of each. Due to superior crack arrest properties and improved flexural strength of the steel fiber composites, it could be a fair judgment that fibre reinforced concrete pavements would be able to bear loads up to the upper limit and may show an entirely different cracking behavior under increasing loads beyond the first cracking load.

The finite element method technique can be extended towards the solution of layered system problems by incorporating numerically integrated finite elements. Finite element software should have been used in this analysis but due to unavailability of the software the analysis is done using MATLAB to evaluate the stresses and deflection in pavements and subgrade. The main advantage of using software for the analysis is that it can save a lot of time of calculations.

1.3 EXPERIMENTAL PROGRAM

The present investigation aims at evaluating the experimental behavior of SFRC mixes. This is done by investigating the effectiveness and structural adequacy of SFRC composites. The experimental study has been divided into the following:

i. Preparing concrete mixes.
ii. Laboratory testing of plain cement concrete mixes for evaluation of strength characteristics i.e. compressive strength.
iii. Preparing PCC mixes by adding fly ash to the concrete.
iv. Laboratory testing of PCC mixes for evaluation of strength characteristics.
v. Preparing FRC mixes by adding various percentages of fibres in the mix.
vi. Laboratory testing of FRC mixes for evaluation of strength characteristic.
vii. Comparing the results of various mixes.
viii. Analysis of stresses, strains and deflections of subgrade due to application of load on pavement using MATLAB.

CHAPTER - 2

LITERATURE REVIEW

2.1 GENERAL

By adding steel fibre it is possible to improve the tensile strength and increase the ductility but fibre should be of appropriate size, shape and aspect ratio. Due to these properties the thickness of pavement can be reduced up to 40% (7). Fibre addition also improves the impact and fatigue resistance and thermal cracking is eliminated resulting in a smoother riding surface and higher stability of the pavement.

One of the important properties of steel fibre reinforced concrete is its superior resistance to cracking and crack propagation. As a result of this ability to arrest cracks, fibre composites possess increased extensibility and tensile strength, both at first crack and at ultimate, particular under flexural loading; and the fibres are able to hold the mix together even after extensive cracking. The net result of all these is to impart to the fibre composite pronounced post – cracking ductility which is not present in ordinary concrete. The addition of fibre transforms the mix from a brittle to a ductile type of material would increase substantially the energy absorption characteristics of the fibre composite and its ability to withstand repeatedly applied, shock or impact loading.

2.2 HISTORICAL REVIEW

2.2.1 Early Developments

The oldest mode of travel obviously was on foot-paths. Later on the discovery of wheel brought up the necessity of providing a hard surface for these wheeled vehicles to move. Such a hard surface is believed to have existed in Mesopotamia in the period about 3500 B.C. The first road on which there is some authentic record is that of Assyrian empire by about 1900 B.C (3).

2.2.2 Roman Roads

Only during the roman period the roads were constructed on vast scales which were mainly aimed for military operations. Hence they are considered to be pioneers in road construction. The Apian Way as built in 312 B.C. extending over 580 km still exists after 2000 years.

2.2.3 Treasaguet Construction

After the fall of Romans, their technique of road construction did not gain popularity in other countries. Until the eighteenth century there is no evidence of any new road construction method, expect of using the older concept of thick construction of road beads as roman did. Pierre Tresaguet (1716 - 1796) developed an improved construction in France by the year 1964 A.D. The main features of his proposals was that the thickness of construction need be only the order of 30 cm due to the consideration given by him to subgrade moisture condition and drainage of surface water.

2.2.4 Macadam Construction

John Macadam (1756-1836) put forward an entirely new method of road construction as compared to all the previous methods in 1815. He improved the subgrade drainage and compactions were recognized. He replaced the heavy foundation stones with smaller broken stones placed at the bottom, with advantage. This technique was having better load dispersion characteristics of compacted broken stone aggregate of smaller size. It was the first method based on scientific thinking, One of the most popular methods which is even now prevalent in many countries is the water bound macadam (WBM), known after Macadam's technique (3).

2.2.5 Further Developments

The WBM were considered to be one of the superior methods until the fast moving vehicles start using these roads causing problems of dust, mud in monsoon season etc...The next development was the penetration and bituminous macadam and other type of surface dressing methods using bituminous materials. For better performance superior mixes like the bituminous carpet and bituminous concrete were also developed in a scientific way. The uses of cement concrete for roads have been popular even prior to the use of bituminous mixes. The cement concrete roads could be designed to keep up the heaviest loads expected on the roads even in adverse soil and climatic condition and to last for a long service life (3).

2.3 HISTORY OF CONCRETE

In Serbia, remains of a hut dating from 5600 BC have been found, with a floor made of red lime, sand, and gravel. The pyramids of Shaanxi in China, built thousands of years ago, contain a mixture of lime and volcanic ash or clay.

The Assyrians and Babylonians used clay as cement in their concrete. The Egyptians used lime and gypsum cement. In the Roman Empire, concrete made from quicklime, pozzolanic ash/ pozzolana and an aggregate made from pumice was very similar to modern Portland cement concrete. The secret of concrete was lost for 13 centuries until in 1756, the British engineer John Smeaton pioneered the use of hydraulic lime in concrete, using pebbles and powdered brick as aggregate. Portland cement was first used in concrete in the early 1840s. In modern times the use of recycled materials as concrete ingredients is gaining popularity because of increasingly stringent environmental legislation. The most conspicuous of these is fly ash, a byproduct of coal fired power plants. This has a significant impact by reducing the amount of quarrying and landfill space required. The properties of concrete have been altered since Roman and Egyptian times, when it was discovered that adding volcanic ash to the mix allowed it to set under water. Similarly, the Romans knew that adding horse hair made concrete less liable to crack while it hardened, and adding blood made it more frost resistant. In modern times, researchers have added other materials to create concrete that is extremely strong, and even concrete that can conduct electricity. As of 2005 about six billion cubic meters of concrete are made each year, which equals one cubic meter for every person on Earth. Concrete is used more than any other man-made material on the planet (10, 19).

2.4 HISTORY OF STEEL FIBRE REINFORCED CONCRETE

The concept of using fibre as reinforcement is not new. Fibres have been used as reinforcement since ancient times. Historically, horsehair was used in mortar and straw in mud bricks. In the early 1900s, asbestos fibre were used in concrete, and in the 1950s the concept of composite materials came into being and fiber reinforced concrete was one of the topics of interest. There was a need to find a replacement for the asbestos used in concrete and other building materials once the health risks associated with the substance were discovered. By the 1960s, steel, glass (GFRC), and synthetic fibre such as polypropylene fibre were used in concrete, and research into new fiber reinforced concretes continues today.

The application of SFRC was first started in 1968 when a short length of fibres reinforced road leading to Machinery Division of the National Standard Company Plant at Niles, Michigan was constructed (7, 36). In 1973, an extremely interesting project was undertaken by Collin D. Johnson at the University of Calgary, Calgary, Alberta, Canada. Under this project plain concrete and SFRC were laid on gravel sub base under adverse conditions of temperature and weather. It was observed that SFRC pavement with fibre reinforced percentage 0.5-1.0 % offered the possibility of reducing the pavement thickness by 60-70% of that of PCC. Also the damage incurred by SFRC pavement slabs was appreciably lower than concrete pavements (7, 20).

SFRC has been recognized as an acceptable material for heavy duty pavements and numerous other uses. Progress in the applications of the material is also being made in Canada, Japan, Australia, England, Western Europe, New Zealand and India.

2.5 USE OF FLY ASH IN CONCRETE PAVEMENTS

2.5.1 Introduction

Coal fly ash is a coal combustion product that has numerous applications in highway construction. Since the first edition of Fly Ash Facts for Highway Engineers in 1986, the use of fly ash in highway construction has increased and new applications have been developed. Fly ash has been used in roadways and interstate highways since the early 1950s. In 1974, the Federal Highway Administration encouraged the use of fly ash in concrete pavement (21).

Fly ash is most commonly used as a pozzolan in PCC applications. Pozzolans are siliceous or siliceous and aluminous materials, which in a finely divided form and in the presence of water, react with calcium hydroxide at ordinary temperatures to produce cementitious compounds. The unique spherical shape and particle size distribution of fly ash make it good mineral filler in hot mix asphalt (HMA) applications and improves the fluidity of flowable fill and grout. The consistency and abundance of fly ash in many areas present unique opportunities for use in structural fills and other highway applications.

The use of fly ash in Portland cement concrete pavements has many benefits and improves concrete performance in both the fresh and hardened state. Fly ash use in concrete improves the workability of plastic concrete, and the strength and durability of hardened concrete. Fly ash use is also cost effective. When fly ash is added to concrete, the amount of Portland cement may be reduced. Benefits to concrete vary depending on the type of fly ash, proportion used, other mix ingredients, mixing procedure, field conditions and placement.

2.5.2 Benefits of Using Fly Ash

1. Benefits to Fresh Concrete: - Generally, fly ash benefits fresh concrete by reducing the mixing water requirement and improving the paste flow behavior. The resulting benefits are as follows:

a. **Improved workability:** The spherical shaped particles of fly ash act as miniature ball bearings within the concrete mix, thus providing a lubricant effect. This same effect also improves concrete pump ability by reducing frictional losses during the pumping process and flat work finish ability.

b. **Decreased water demand:** The replacement of cement by fly ash reduces the water demand for a given slump. When fly ash is used at about 20 percent of the total cementitious, water demand is reduced by approximately 10 percent. Higher fly ash contents will yield higher water reductions. The decreased water demand has little or no effect on drying shrinkage/cracking. Some percentage of fly ash is known to reduce drying shrinkage in certain situations.

c. **Reduced heat of hydration:** Replacing cement with the same amount of fly ash can reduce the heat of hydration of concrete. This reduction in the heat of hydration does not sacrifice long-term strength gain or durability. The reduced heat of hydration lessens heat rise problems in mass concrete placements.

2. Benefits to Hardened Concrete: One of the primary benefits of fly ash is its reaction with available lime and alkali in concrete, producing additional cementitious compounds (21). The following equations illustrate the pozzolanic reaction of fly ash with lime to produce additional calcium silicate hydrate (C-S-H) binder:

Cement Reaction: $C_3S + H \xrightarrow{\text{(hydration)}} C\text{-}S\text{-}H + CaOH$

Pozzolanic Reaction: $CaOH + \underset{\text{silica from ash constituents}}{S} \rightarrow C\text{-}S\text{-}H$

a. **Increased ultimate strength:** The additional binder produced by the fly ash reaction with available lime allows fly ash concrete to continue to gain strength over time. Mixtures designed to produce equivalent strength at early ages (less than 90 days) will ultimately exceed the strength of straight cement concrete mixes.

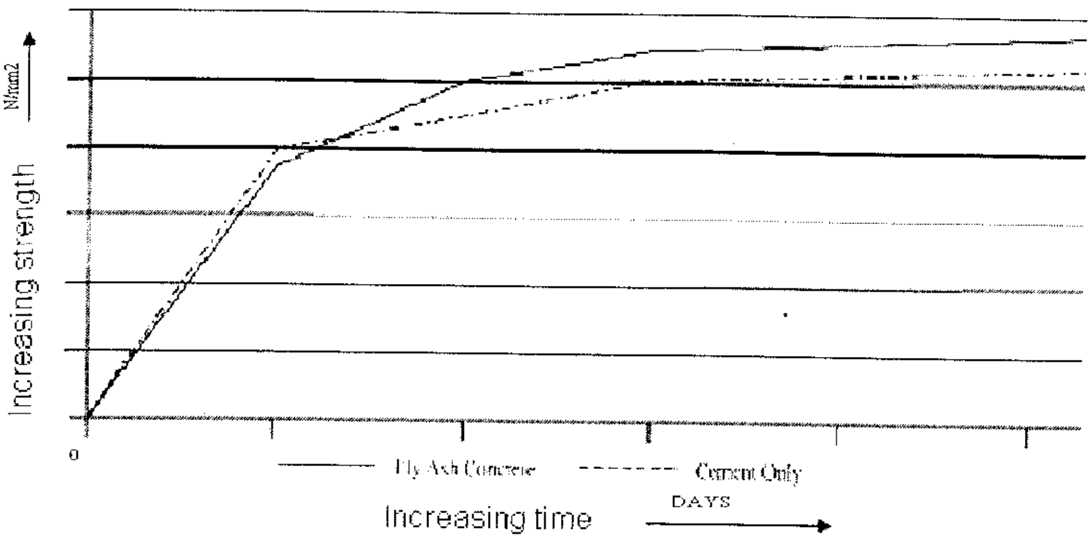


Figure 2.1: - **Typical strength gain of fly ash concrete (21).**

b. **Reduced permeability:** The decrease in water content combined with the production of additional cementitious compounds reduces the pore interconnectivity of concrete, thus decreasing permeability. The reduced permeability results in improved long-term durability and resistance to various forms of deterioration.

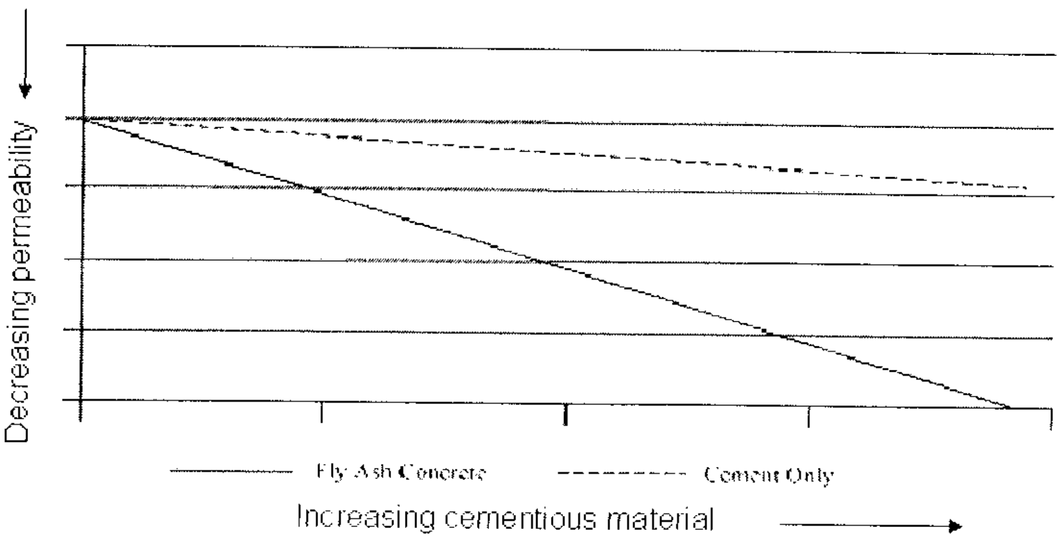


Figure 2.2: - **Permeability of fly ash concrete (23)**

c. **Improved durability.** The decrease in free lime and the resulting increase in cementitious compounds, combined with the reduction in permeability enhance concrete durability. This affords several benefits:

- Improved resistance to ASR. Fly ash reacts with available alkali in the concrete, which makes them less available to react with certain silica minerals contained in the aggregates.

- Improved resistance to sulfate attack. Fly ash induces three phenomena that improve sulfate resistance:

· Fly ash consumes the free lime making it unavailable to react with sulfate.

· The reduced permeability prevents sulfate penetration into the concrete.

· Replacement of cement reduces the amount of reactive aluminates available.

- Improved resistance to corrosion. The reduction in permeability increases the resistance to corrosion.

CHAPTER - 3

MECHANISM OF FIBER REINFORCEMENT AND ITS APPLICATION IN PAVEMENTS

3.1 GENERAL

Concrete is inherently a brittle material, with low tensile strength and limited ductility. Contribution of the conventional steel reinforcements in RCC construction in taking care of the tensile stresses is limited in its own plane. Widespread cracking due to secondary effects like temperature and shrinkage in fresh concrete is quite common, which affects its performance. Incorporation of discontinuous, discrete, uniformly dispersed fibre in the matrix of concrete or mortar improves tensile and flexural strength, ductility, toughness, and impact and fatigue resistance of the composite (FRC) manifold, compared to the plain concrete. Another weakness in common concrete construction is that water, in excess of what is needed for hydration and strength properties, is required to be added for achieving workability.

Taking advantage of these characteristics, FRC, finds application in roads and airfield pavements, runways and taxiways, overlays, industrial floors, blast-resistant structures, hydraulic structures like dams, spillways, tunnel lining, underground roof support with shotcrete, repairs and restoration and many others. Improved ductility is of advantage in earthquake resistant structures (30).

Therefore it becomes necessary to understand the mechanism of fiber reinforcement. This chapter will discuss the mechanism of fiber reinforcement.

3.2 BASIC MECHANISM OF FIBER REINFORCEMENT

Fibre influences the mechanical properties of concrete in all modes of failure, especially those that induce fatigue and tensile stresses. The strengthening mechanism of fibre involves transfer of stress from the matrix to the fiber by interfacial shear or by interlock between the fiber and the matrix. With the increase in the applied load, stress is shared by the fiber and the matrix in tension until the matrix cracks; then the total stress is progressively transferred to the fiber, till the fibre are pulled out, or yield, or break in tension (ACI 544.4, 1994). Fiber efficiency and the fiber content are the important variables controlling the performance of FRC. Fiber efficiency is controlled by the resistance to pullout, which depends on the bond at the fiber-matrix interface. Pullout resistance increases with fiber length. Since pullout resistance is proportional to interfacial area, the smaller the diameter, the larger is the interfacial area available for bond. For a given fiber length, smaller the area, more effective is the bond. The composite effect of these two variables is expressed by the 'aspect ratio' (length/diameter). Fiber efficiency increases with increase in 'aspect ratio' (30).

3.3 THE VARIABLES AFFECTING FIBRE REINFORCED CONCRETE

The contribution of fiber to the composite depends upon:

a. The fiber material and type,
b. Length (L), diameter (d) and aspect ratio (L/d), as discussed above.
c. Volume concentration of fibre in the matrix. In case of steel it varies from low volume (<0.5 %), moderate (1 – 3 %) or high volume (3% +). For low volumes indicated, normal batching, mixing and casting procedures are adopted and no additional precautions are needed. For moderate and high volumes, there may be tendency for the fibre to 'ball' and workability of concrete may be reduced. For large volumes, special techniques like slurry infiltration, shotcrete or pressures forming are adopted (30).

3.4 PROPERTIES OF CONCRETE

Concrete is an artificial conglomerate stone made essentially of Portland cement, water, and aggregates. When first mixed the water and cement constitute a paste which surrounds all the individual pieces of aggregate to make a plastic mixture. A chemical reaction called hydration takes place between the water and cement, and concrete normally changes from a plastic to a solid state in about 2 hours. Thereafter the concrete continues to gain strength as it cures.

During the first week to 10 days of curing it is important that the concrete not be permitted to freeze or dry out because either of these, occurrences would be very detrimental to the strength development of the concrete. Theoretically, if kept in a moist environment, concrete will gain strength forever, however, in practical terms; about 90% of its strength is gained in the first 28 days (14). Concrete has almost no tensile strength (usually measured to be about 10 to 15% of its compressive strength), and for this reason it is almost never used without some form of reinforcing. Its compressive strength depends upon many factors, including the quality and proportions of the ingredients and the curing environment. The single most important indicator of strength is the ratio of the water used compared to the amount of cement. Basically, the lower this ratio is, the higher the final concrete strength will be. (This concept was developed by Duff Abrams of The Portland Cement Association in the early 1920s and is in worldwide use today.) A minimum w/c ratio (water-to-cement ratio) of about 0.3 by weight is necessary to ensure that the water comes into contact with all cement particles (thus assuring complete hydration). In practical terms, typical values are in the 0.4 to 0.6 range in order to achieve a workable consistency so that fresh concrete can be placed in the forms and around closely spaced reinforcing bars (14). Typical stress-strain curves for various concrete strengths are shown in.fig

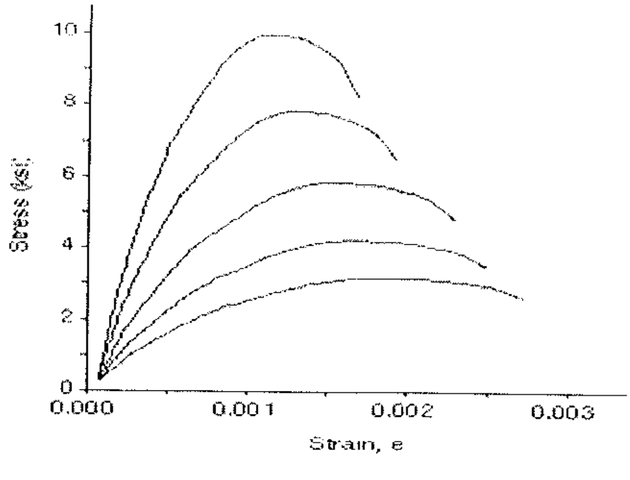


Figure 3.1: - **Stress-strain curve for different compressive strength (14)**

During hydration and hardening, concrete needs to develop certain physical and chemical properties. Among other qualities, mechanical strength, low moisture permeability, and chemical and volumetric stability are necessary.

I. Workability

Workability is the ability of a fresh (plastic) concrete mix to fill the form/mould properly with the desired work (vibration) and without reducing the concrete's quality. Workability depends on water content, aggregate (shape and size distribution), cementitious content and age (level of hydration), and can be modified by adding chemical admixtures. Raising the water content or adding chemical admixtures will increase concrete workability. Excessive water will lead to increased bleeding (surface water) and/or segregation of aggregates (when the cement and aggregates start to separate), with the resulting concrete having reduced quality. Workability can be measured by the "slump test," a simplistic measure of the plasticity of a fresh batch of concrete. Slump can be increased by adding chemical admixtures such as mid-range or high-range water reducing agents (super-plasticizers) without changing the water/cement ratio.

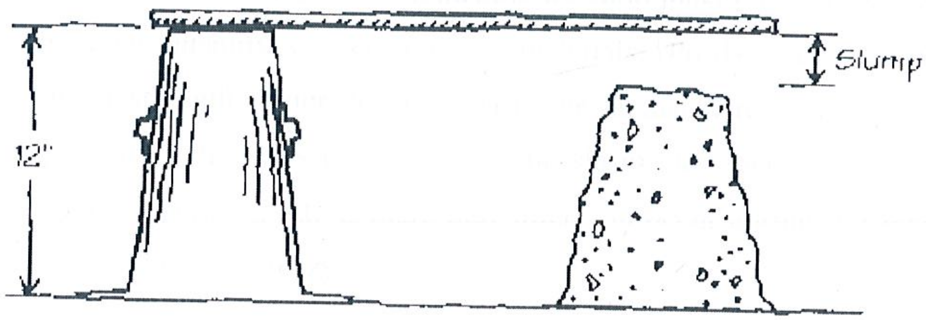


Figure 3.2: - Slump **test**

II. Curing

It is well known that the cement requires time to fully hydrate before it acquires strength and hardness, concrete must be *cured* once it has been placed and achieved initial setting. Curing is the process of keeping concrete under a specific environmental condition until hydration is relatively complete. Good curing is typically considered to provide a moist environment and control temperature. A moist environment promotes hydration, since increased hydration lowers permeability and increases strength resulting in a higher quality material. Allowing the concrete surface to dry out excessively can result in tensile stresses, which the still-hydrating interior cannot withstand, causing the concrete to crack.

III. Strength

Concrete has relatively high compressive strength, but significantly lower tensile strength (about 10% of the compressive strength). As a result, concrete always fails from tensile stresses even when loaded in compression. The practical implication of this is that concrete elements subjected to tensile stresses must be reinforced. Concrete is most often constructed with the addition of steel or fibre reinforcement. The reinforcement can be done by using bars, mesh, or fibres, producing reinforced concrete. Concrete can also be prestressed (reducing tensile stress) using internal steel cables (tendons), allowing for beams or slabs with a longer span than is practical with reinforced concrete alone.

The ultimate strength of concrete is influenced by the water-cement ratio *(w/c)*, the design constituents, and the mixing, placement and curing methods employed. All things being equal, concrete with a lower water-cement (cementitious) ratio makes a stronger concrete than a higher ratio. The total quantity of cementitious materials (Portland cement, slag cement, pozzolans) can affect strength, water demand, shrinkage, abrasion resistance and density. As concrete is a liquid which hydrates to a solid, plastic shrinkage cracks can occur soon after placement; but if the evaporation rate is high, they often can occur during finishing operations (for example in hot weather or a breezy day).

If no restraints existed the concrete would simply shrink, aggregate interlock and steel reinforcement cause tensile stresses to develop within the concrete and due to its low tensile strength, has the effect of plastic shrinkage cracking of various depths at the surface. A typical strength-gain curve is shown in Fig. 3.3. The industry has adopted the 28-day strength as a reference point, and specifications often refer to compression tests of cylinders of concrete which are crushed 28 days after they are made. The resulting strength is given in the designation Fig. 3.3.

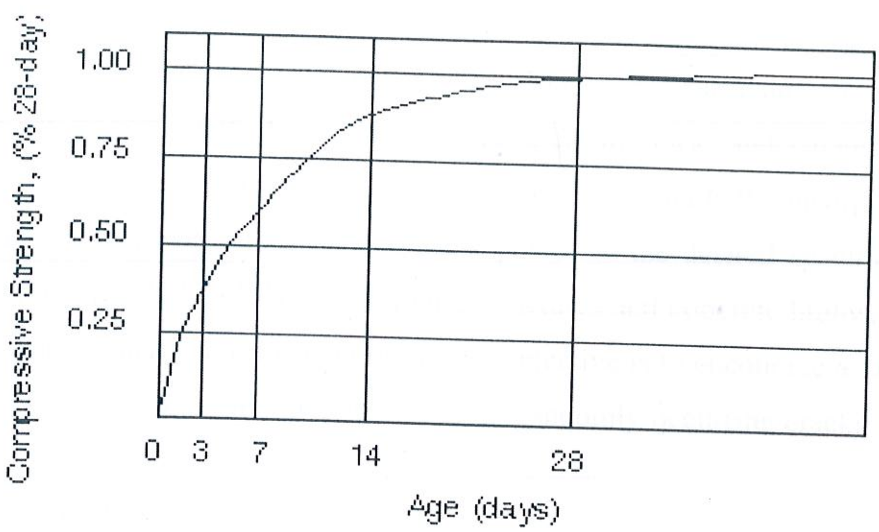


Figure 3.3: - **Typical strength-gain curve (14)**

IV. Elasticity

The modulus of elasticity of concrete is a function of the modulus of elasticity of the aggregates and the cement matrix and their relative proportions. The modulus of elasticity of concrete is relatively linear at low stress levels but becomes increasingly non-linear as matrix cracking develops.

V. Expansion and Shrinkage

Concrete has a very low coefficient of thermal expansion (0.000008 to 0.000012, per degree Celsius). However if no provision is made for expansion very large forces can be created, causing cracks in parts of the pavement not capable of withstanding the force or the repeated cycles of expansion and contraction. As concrete matures it continues to shrink, due to the ongoing reaction taking place in the material, although the rate of shrinkage falls relatively quickly and keeps reducing over time (for all practical purposes concrete is usually considered to not shrink any further after 30 years). The relative shrinkage and expansion of concrete and brickwork require careful accommodation when the two forms of construction interface. Because concrete is continuously shrinking for years after it is initially placed, it generally accepted that under thermal loading it will never expand to its originally-placed volume.

VI. Cracking

Concrete cracks due to tensile stress induced by shrinkage or by applied loading. Engineers are familiar with the tendency of concrete to crack, and where appropriate, special design precautions are taken to ensure crack control. This entails the incorporation of secondary reinforcing, for example deformed steel bars, placed at the desired spacing to limit the crack width to an acceptable level. Water retaining structures and concrete highways are examples of structures where crack control is exercised. The objective is to encourage a large number of very small cracks, rather than a small number of large, randomly-occurring cracks.

VII. Shrinkage cracking

Shrinkage cracks occur when concrete members undergo restrained volumetric changes (shrinkage) as a result of drying, autogenous shrinkage, or thermal effects. Restraint is provided either externally (i.e. supports, walls, and other boundary conditions) or internally (differential drying shrinkage, reinforcement). Once the tensile strength of the concrete is exceeded, a crack will develop. The number and width of shrinkage cracks that develop are influenced by the amount of shrinkage that occurs, the amount of restraint present, and the amount and spacing of reinforcement provided.

Concrete is placed while in a wet (or plastic) state, and therefore can be manipulated and moulded as needed. Hydration and hardening of concrete during the first three days is critical. Abnormally fast drying and shrinkage due to factors such as evaporation from wind during placement may lead to increased tensile stresses at a time when it has not yet gained significant strength, resulting in greater shrinkage cracking. The early strength of the concrete can be increased by keeping it damp for a longer period during the curing process. Minimizing stress prior to curing minimizes cracking. High early-strength concrete is designed to hydrate faster, often by increased use of cement, which increases shrinkage and cracking.

VIII. Creep

Creep is the term used to describe the permanent movement or deformation of a material in order to relieve stresses within the material. Concrete which is subjected to forces is prone to creep. Creep can sometimes reduce the amount of cracking that occurs in a concrete structure or element, but it also must be controlled. The amount of primary and secondary reinforcing in concrete structures contributes to a reduction in the amount of shrinkage, creep and cracking (10, 14).

PHYSICAL PROPERTIES

The coefficient of thermal expansion of Portland cement concrete is 0.000008 to 0.000012 (per degree Celsius). The density varies, but is around 2400 kg/m³.

3.5 PROPERTIES OF SFRC

Steel Fibre Reinforced Concrete (SFRC) is a composite material comprising of hydraulic (Portland) cement, aggregate, sand, water and a dispersion of discontinuous steel fibres. It may also contain supplementary cementious material (such as fly ash, silica fume or ground granulated blast furnace slag) and/or additives (admixtures) commonly used in conventional concrete.

The addition of steel fibres, typically from 20 to 50 kg/m^3, significantly enhances some of the engineering properties of hardened concrete including:

- **a.** Toughness,
- **b.** Impact strength,
- **c.** Flexural strength,
- **d.** Abrasion resistance.

However, the major benefit is the ability of the randomly dispersed fibres to provide a crack arrest mechanism and enhanced post-cracking performance. The steel fibres provide a ductile member in a brittle matrix which significantly improves the dynamic properties of the concrete. The fibres enable the concrete to continue to carry load after cracking has occurred. The failure mode is no longer brittle once the first crack is reached. Instead, the concrete behaves in a ductile manner with the bending moment remaining constant as deformation increases after cracking, effectively forming a plastic hinge.

The properties and performance of Steel Fibre Reinforced Concrete depend upon:

a. The concrete mix strength,

b. The fibre volume,

c. The bond between the fibre and the concrete matrix.

The uniform distribution of fibres in a well proportioned and graded concrete is essential to achieving appropriate ductility which is fundamental to the performance of Steel Fibre Reinforced Concrete (18, 19).

3.5.1 Static Mechanical Properties

I. Compressive Strength

Fibres do little to enhance the static compressive strength of concrete, with increases in strength ranging from essentially nil to perhaps 25%. Even in members which contain conventional reinforcement in addition to the steel fibres, the fibres have little effect on compressive strength. However, the fibres do substantially increase the post-cracking ductility, or energy absorption of the material as shown in Fig. 3.4.

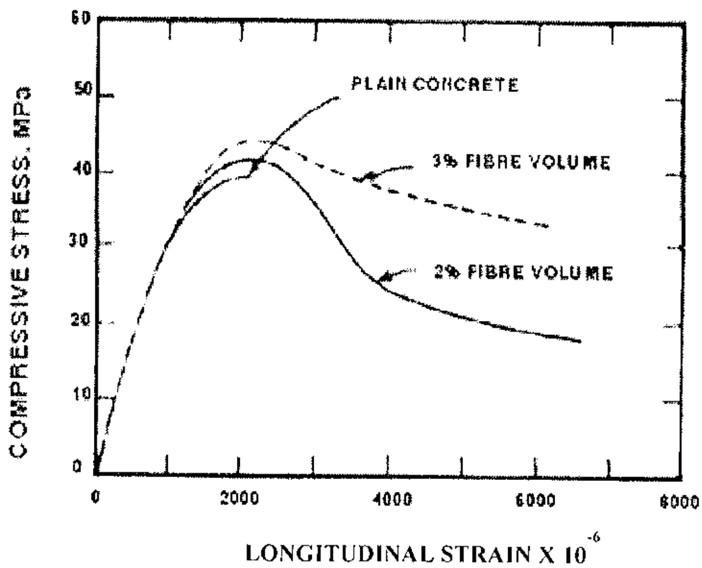


Figure 3.4: **- Stress-Strain curves in compression (4)**

II. Tensile Strength

Fibres aligned in the direction of the tensile stress may bring about very large increases in direct tensile strength, as high as 133% for 5% of smooth, straight steel fibres. However, for more or less randomly distributed fibres, the increase in strength is much smaller, ranging from as little as no increase in some instances to perhaps 60%, with many investigations indicating intermediate values, as shown in Figure 3.5. Splitting-tension test of SFRC show similar result. Thus, adding fibres merely to increase the direct tensile strength is probably not worthwhile. However, as in compression, steel fibres do lead to major increases in the post-cracking behavior or toughness of the composite.

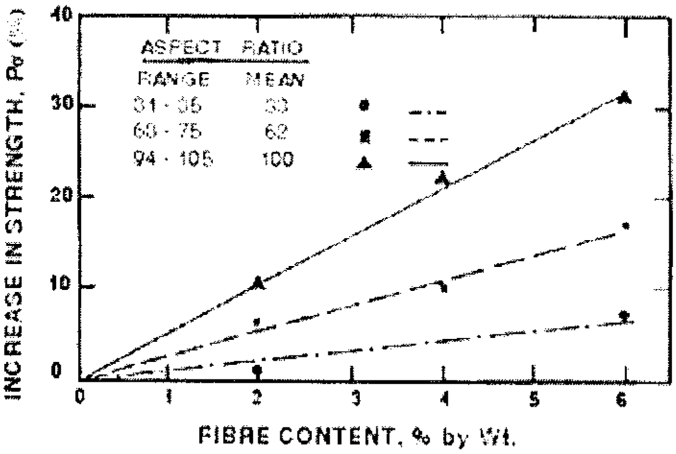


Figure 3.5: - **Influence of fibre content on tensile strength (4)**

III. Flexural Strength

Steel fibres are generally found to have aggregate much greater effect on the flexural strength of SFRC than on either the compressive or tensile strength, with increases of more than 100% having been reported. The increase in flexural strength is particularly sensitive, not only to the fibre volume, but also to the aspect ratio of the fibres, with higher aspect ratio leading to larger strength increases. Figure 3.6 describes the fibre effect in terms of the combined parameter W*l*/*d*, where *l*/*d* is the aspect ratio and W is the weight percent of fibres. It should be noted that for W*l*/*d* > 600, the mix characteristics tended to be quite unsatisfactory. Deformed fibres show the same types of increases at lower volumes, because of their improved bond characteristics.

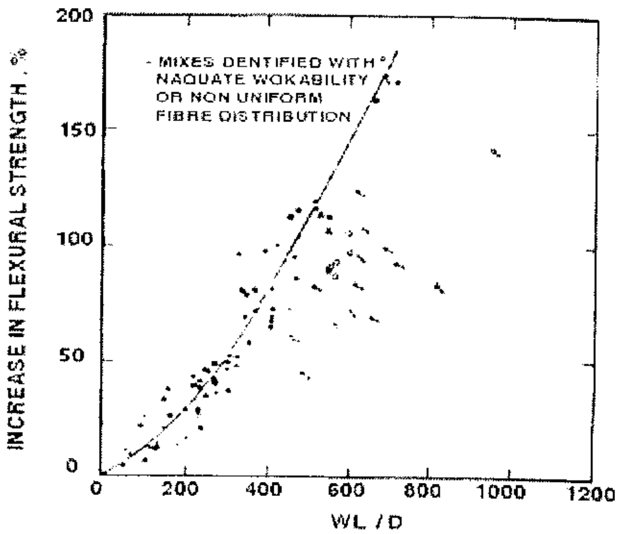


Figure 3.6: **- Effect of W./*l*/d on the flexural strength of mortar and concrete (4)**

IV. Flexural Toughness

As was indicated previously, fibres are added to concrete not to improve the strength, but primarily to improve the toughness, or energy absorption capacity. Commonly, the flexural toughness is defined as the area under the complete load-deflection curve in flexure; this is sometimes referred to as the total energy to fracture. Alternatively, the toughness may be defined as the area under the load-deflection curve out to some particular deflection, or out to the point at which the load has fallen back to some fixed percentage of the peak load. Probably the most commonly used measure of toughness is the toughness index proposed by Johnston and incorporated into ASTM C1018. As is the case with flexural strength, flexural toughness also increases at the parameter W*l*/*d* increases, as show in Figure 3.7.

The load-deflection curves for different types and volumes of steel fibres can vary enormously, as was shown previously in Figure 3.8. For all of the empirical measures of toughness, fibres with better bond characteristics (i.e. deformed fibres, or fibres with greater aspect ratio) give higher toughness values than do smooth, straight fibres at the same volume concentrations (4).

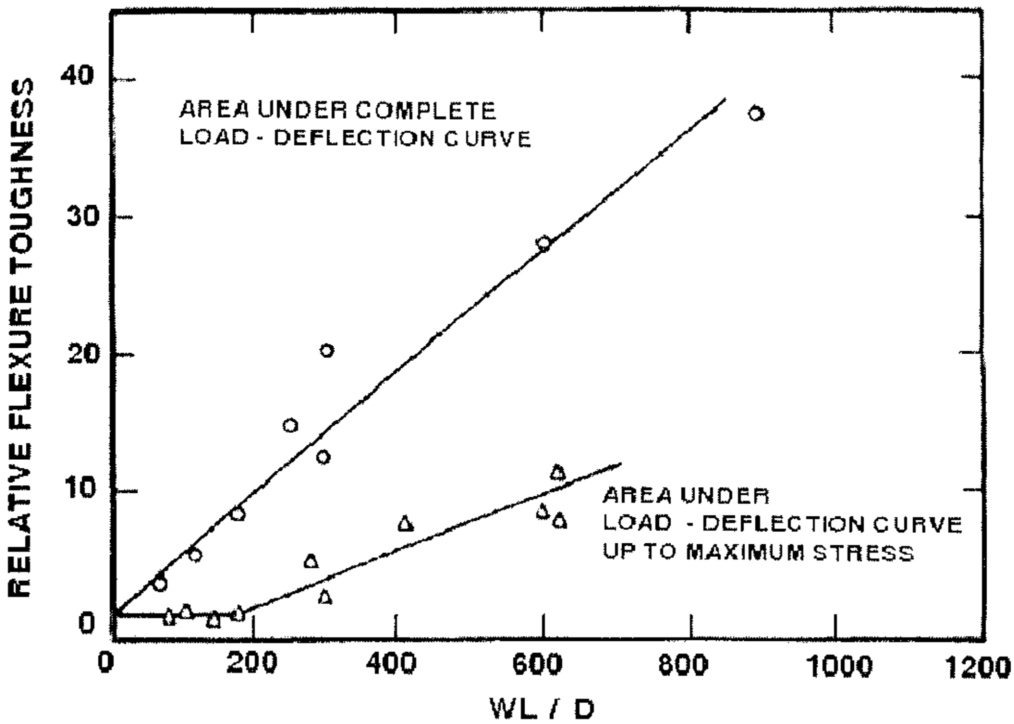


Figure 3.7: - **The effect of Wl/d on the flexural toughness of SFRC (9)**

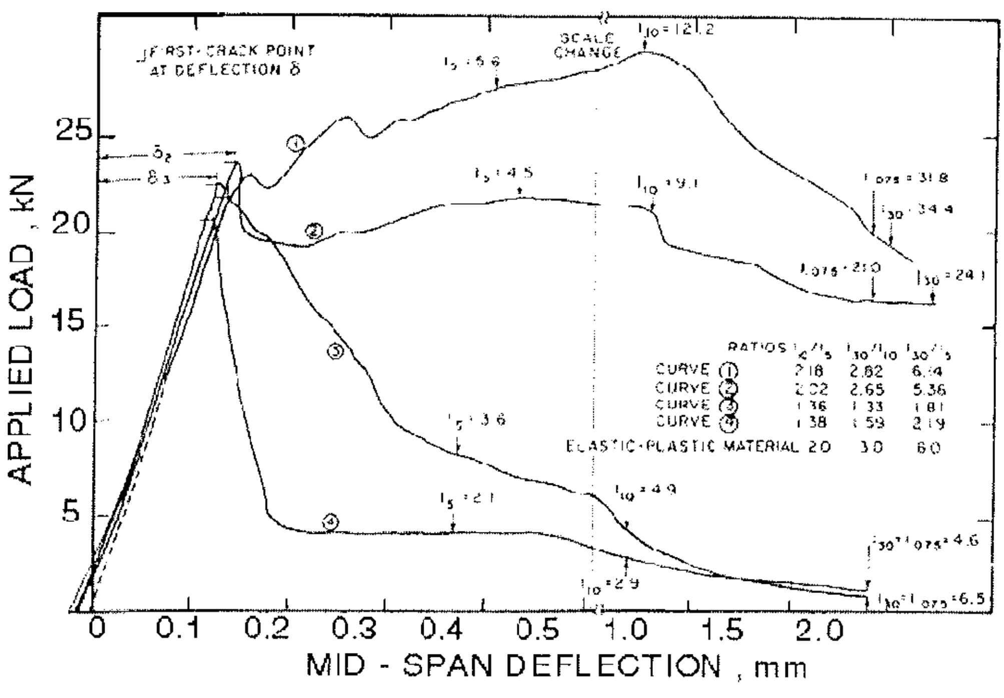


Figure 3.8: - **A range of load – deflection curves obtained in the testing of SFRC (9)**

V. Modulus of Elasticity

The modulus of elasticity of Steel Fibre Reinforced Concrete has not been determined by testing, but it has been established that the modulus of elasticity of plain concrete is not significantly affected by the incorporation of steel fibres in the concentrations normally used.

As per IS code, the following relationship may be used to determine an appropriate modulus of elasticity for design purposes, based on the characteristic compressive strength of the concrete (17).

$$E_c = 5000\sqrt{f_{ck}}$$

where, f_{ck} = Characteristics strength of concrete in N/mm^2

VI. Poisson's Ratio

For Steel Fibre Reinforced Concrete, a nominal value of *0.15* is used for Poisson's Ratio.

3.5.2 Mechanics Of Fibre Reinforcement

Fibres influence the mechanical properties of concrete in all modes of failure, especially those that induce fatigue and tensile stresses. The strengthening mechanism of fibres involves transfer of stress from the matrix to the fibre by interfacial shear or by interlock between the fibre and the matrix. With the increase in the applied load, stress is shared by the fibre and the matrix in tension until the matrix cracks; then the total stress is progressively transferred to the fibre, till the fibres are pulled out, or yield, or break in tension (5).

3.5.2.1 First Crack Loading

FRC in flexure essentially undergoes a trilinear deformation behavior as shown in figure 3.9

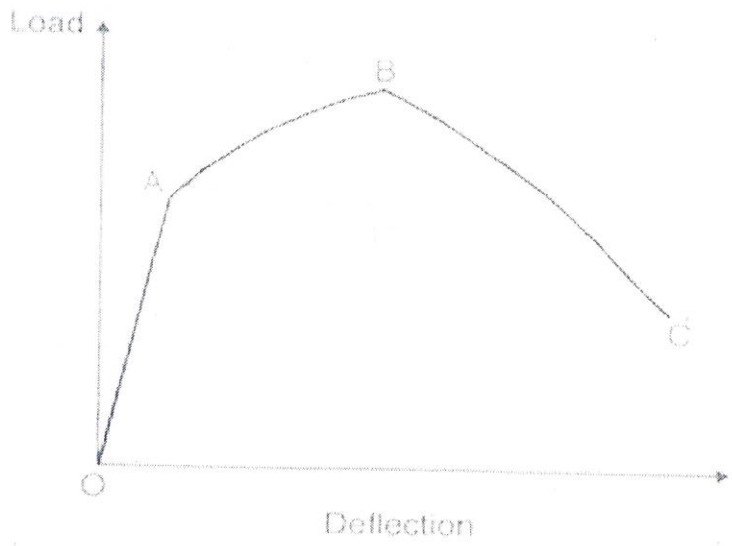


Figure 3.9: **- Schematic load-deflection relationship of FRC (12)**

Point A on the load deflection curve represent first crack load, which can be termed the first crack strength. Normally this is the same load level at which a non reinforced element cracks. Hence segment OA in the diagram would be the same and essentially have the same slope for both plain and fibre reinforced concrete.

Once the matrix is cracked, the applied load is transferred to the fibres that bridge and tie the crack to keep it from opening farther. As the fibres deforms, additional narrow cracks develop and continued cracking of the matrix takes place until the maximum load reaches point B of the load deflection curve. During this stage, debonding and pullout of some of the fibres occur. But the yield strength in most of the fibres is not reached.

In the falling branch BC of the load- deflection diagram, matrix cracking and fibre pullout continue. If the fibres are long enough to maintain their bond with the surrounding gel, they may fail by yielding or by fracture element depending on their size and spacing.

3.5.2.2 Critical Fibre Length: *Length Factor*

If l_c is the critical length of a fibre above which the fibre fractures instead of pulling out when the crack intersect the fibre at its midpoint, it can be approximated by Mindess and Young (12).

$$l_c = \frac{d_f}{v_b} \sigma_f$$

where,

d_f = fibre diameter in mm,

v_b = interfacial bond strength,

σ_f = fibre strength.

Bentur and Mindess, 1990 (12) demonstrating that the strength of a composite increases continuously with the fibre length. This is of significance as it indicates that pullout work may go through a maximum and then decrease as bond strength increase over a critical value. This loss of pull out work would be reduced to a typical range of l= 10mm in the cement based composite. If a critical v_b value of 10 MPa is chosen and a small diameter fibre d_f = 20 micro meter, namely a cementitious system with such a small diameter fibre, an increase in bond may result n reduced toughness.

3.5.2.3 Critical Fiber spacing: *Space Factor*

The spacing of the fibres considerably affects cracking development in the matrix. The closer the spacing the higher will be the first crack load of the matrix. This is owing to the fact that the fibres reduce the stress intensity factor that controls fracture. The approach taken by Romualdi and Batson (1963) to increase the tensile strength of the mortar was to increase the stress-intensity factor through decreasing the spacing of the fibres acting as crack arresters.

Figure 3.10 relates the tensile cracking stress to the spacing of the fibres for various volumetric percentages.

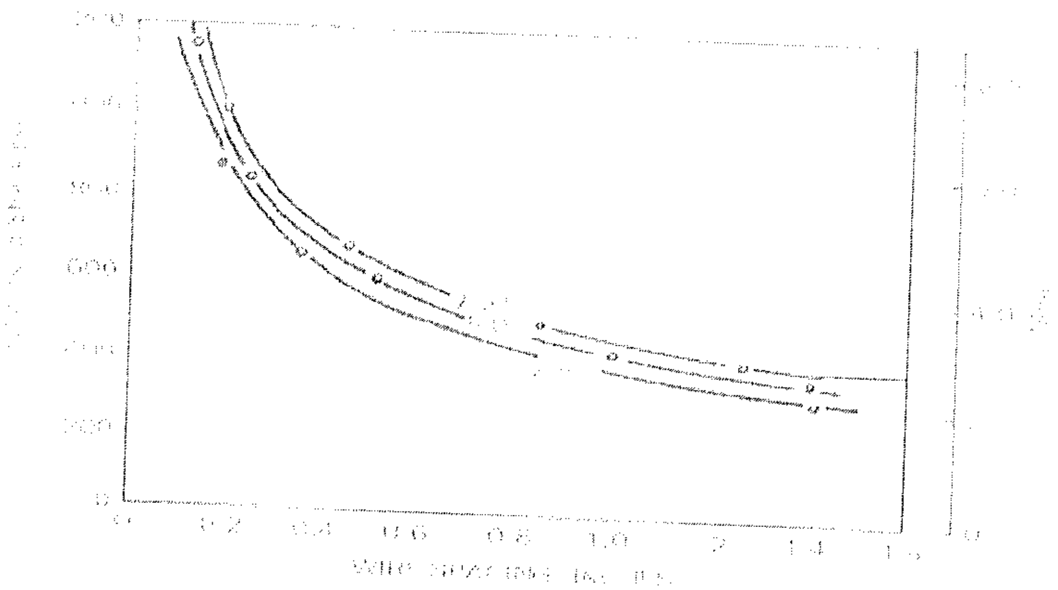


Figure 3.10: **- Effect of Steel fibre spacing on the tensile cracking stress in fibrous concrete in ρ = 2.5, 5.9 and 7.5%. (1)**

Figure 3.11 compares the theoretical and experimental values of the ratio of the first crack load to the cracking strength of PCC(strength ratio).This figure shows that the closer the spacing of the fibres the higher the strength ratio, namely, the higher the tensile strength of the concrete up to practical workability and cost effective limits. Several expressions to define the spacing of the fibres have been developed. If s is the spacing of the fibres, one expression from Romualdi and Batson,1963 gives

$$s = 13.8\, d_f \sqrt{\frac{1.0}{\rho}}$$

Where, d_f = diameter of the fibre,

ρ = fibre percentage by volume of the matrix.

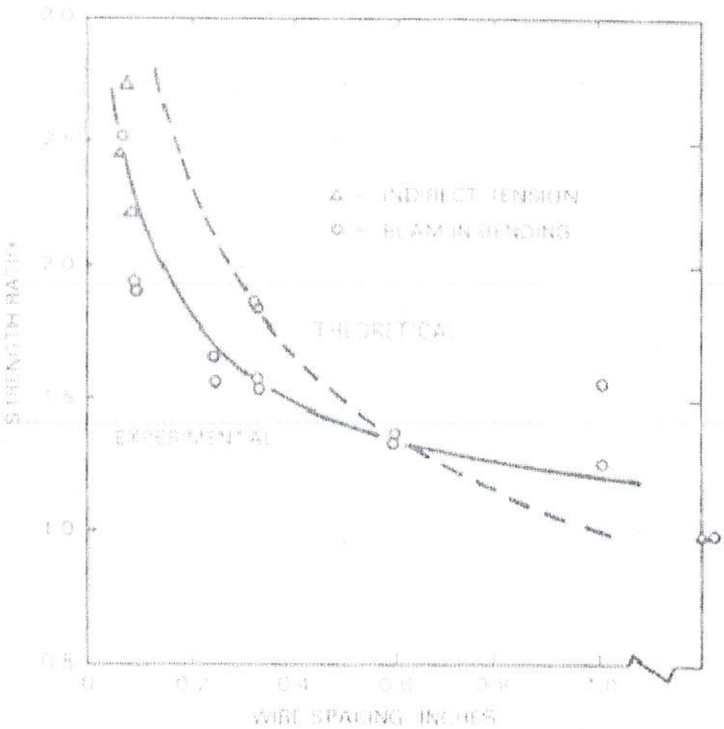


Figure 3.11: - **Effect on fibre spacing on the strength ratio (2).**

3.5.2.4 Fibre Orientation: *Fibre Efficiency Factor*

The orientation of the fibres with respect to load determines the efficiency with which tee randomly oriented fibres can resist the tensile forces in their directions. This is synonymous with the contribution of bent bars and vertical shear stirrups provide in beams to resist the inclined diagonal tension stress. If one assumes perfect randomness, the efficiency factor is 0.41, but it can vary between 0.331 and 0.651 close to the surface of the specimen as trawling or leveling can modify the orientation of fibres.

3.5.3 Steel Fibres

Steel fibres have been used in concrete since the early 1900s. The early fibres were round and smooth and the wire was cut or chopped to the required lengths. The use of straight, smooth fibres has largely disappeared and modern fibres have either rough surfaces, hooked ends or are crimped or undulated through their length. Modern commercially available steel fibres are manufactured from drawn steel wire, from slit sheet steel or by the melt extraction process which produces fibres that have a crescent-shaped cross-section. Typically steel fibres have equivalent diameters (based on cross sectional area) of from 0.15 mm to 2 mm and lengths from 30 to 50 mm. Aspect ratios generally range from 20 to 100. (*Aspect ratio* is defined as the ratio between fibre length and its equivalent diameter, which is the diameter of a circle with an area equal to the cross-sectional area of the fibre.)

Carbon steels are most commonly used to produce fibres but fibres made from corrosion-resistant alloys are available. Stainless steel fibres have been used for high-temperature applications. Some fibres are collated into bundles using water-soluble glue to facilitate handling and mixing (11).

Steel fibres have high tensile strength (0.5 -- 2 GPa) and modulus of elasticity (200 GPa), a ductile/plastic stress strain characteristic and low creep. Steel fibres have been used in conventional concrete mixes, shotcrete and slurry-infiltrated fibre concrete. Typically, the content of steel fibre ranges from 0.50 to 2.0% by volume. Fibre contents in excess of 2% by volume generally result in poor workability and fibre distribution, but can be used successfully where the paste content of the mix is increased and the size of coarse aggregate is not larger than about 10 mm.

Steel-fibre-reinforced concrete containing up to 1.5% fibre by volume has been pumped successfully using pipelines of 125 to 150 mm diameter. Steel fibre contents up to 2% by volume have been used in shotcrete applications using both the wet and dry processes. Steel fibre contents of up to 25% by volume have been obtained in slurry-infiltrated fibre concrete. Concretes containing steel fibre have been shown to have substantially improved resistance to impact and greater ductility of failure in compression, flexure and torsion (5, 11).

Table 3.1:- **Characteristics of Commonly Used Fiber (*5,9,11,13*)**								
FIBRE Type	**Diameter (μm)**	**Length (mm)**	**Specific Gravity**	**Tensile Strength Mpa**	**E, GN/m^2**	**Elongation at failure, %**	**Typical Volume in composite**	**Poisson's ratio**
Polypropylene	20 - 200	20 - 75	0.91	550 - 700	3.5-6.8	21	<2	0.29 - 0.46
Steel	500-1000	30 - 50	7.86	500 -2000	200	0.5-3.5	<2	0.5 - 2
Glass	10 - 15	10 - 50	2.7	1200 -1700	73	2-3.5	4-6	0.22
Asbestos	0.02 - 20		2.55	210 -2000	159	2-3	7-18	----
Polyester			1.4	400 - 600	8.4 - 16	11-13	~ 0.065	----
Carbon	8 - 10		1.9	2600	230	0.5-1	2-12	0.35
Sisal	10 - 50		1.5	800	----	3	----	----
Nylon	100 - 200		1.14		~ 4	13.5	0.1-6	0.40
Concrete, for comparison			2.4	2-6	20-50		0	0.15 - 0.22

3.6 APPLICATION OF SFRC IN HIGHWAY PAVEMENTS

The uses of SFRC over the past thirty years have been so varied and so widespread, that it is difficult to categorize them. Steel fiber-reinforced concrete are mostly used for on-ground floors and pavements, but can be considered for a wide range of construction parts (beams, pillars, foundations etc) either alone or with hand-tied rebar's. Concrete reinforced with fibre (which are usually steel) is less expensive than hand-tied rebar, while still increasing the tensile strength many times. The most common applications are pavements, tunnel linings, pavements and slabs, shotcrete and now shotcrete also containing silica fume, airport pavements, bridge deck slab repairs, and so on. There has also been some recent experimental work on roller-compacted concrete (RCC) reinforced with steel fibres. The list is endless, apparently limited only by the ingenuity of the engineers involved. The fibres themselves are, unfortunately, relatively expensive; a 1% steel fibre addition will approximately double the material costs of the concrete, and this has tended to limit the use of SFRC to special applications. The fibre reinforcement may be used in the form of three – dimensionally randomly distributed fibres throughout the structural member when the added advantages of the fibre to shear resistance and crack control can be further utilized. SFRC has been used in number of highway pavements and bridge deck surfaces and airport pavements with fiber content 0.75 to 1.5% by volume. SFRC has also been applied even on wooden deck and was found to be virtually crack free. Therefore, SFRC could usefully be deployed in the areas of:

a. New pavements slab for roadways, bridge decks or runways.
b. Bonded and unbonded overlays for rehabilitation purposes and for strengthening of existing pavements to increase load carrying capacity.
c. Thin repairs of damaged patches in bridge deck or pavement slabs. The minimum thickness of patch shall be 50mm.
d. Overlays for rehabilitation of existing roadways and bridge decks. The ability of SFRC to provide a thin overlay is of great value in case where overhead clearances are critical.
e. Rotaries, intersection and locations where odd shape of pavement is required to be constructed.

CHAPTER- 4

EXPERIMENTAL PROGRAMME

4.1 GENERAL

The experimental program consists of preparing the composite mixes and then finding their compressive strength. The mix that were prepared during the experimental program were plain cement concrete, plain cement concrete with fly ash and steel fiber reinforced composite. The SFRC mixes were made by varying fibre content from 0.5 to 1.5 percent by volume. It has been observed that by increasing the percentage of fibre in mix improves the compressive strength and a simultaneous increase in the ductility of the hardened concrete can be achieved and thus increases the flexural strength. Throughout the investigation, the number of samples prepared for each mix was 10 for different proportions of Cement: Fine aggregate: Coarse aggregate of concrete i.e. 1:1.5:3 and 1:1:2 at w/c ratio 0.6 and at different fiber content. The compressive strength for each mix was calculated after 7 and 28 days and the result was tabulated for each mix.

4.2 RAW MATERIALS

SFRC is usually specified by strength and fibre content. For paving works, normally flexural strength is specified. A design flexural strength of 60 to 70 kg/cm^2 is a typical range after 28 days (13). All the ingredients such as cement, aggregates, water, admixtures and steel fibres used for SFRC are conforming to the standard specifications used for conventional concrete.

4.2.1 Fibres

Steel fibres normally used are cut from mild steel wire. The wire diameter adopted is in the range of 0.456 mm and aspect ratio 80. So the length of fibre generally used is 3.6 cm. Steel fibres used are clean, free from dust, rust, oil and deleterious materials. The tensile strength and percentage elongation of mild steel wire was 82.6 kg/mm^2 and 0.95% respectively.

4.2.2 Cement

OPC-43 Grade is produced from enriched limestone most suited to make high quality clinker, which on grinding gives a cement with characteristics better than those specified in IS: 8112-1989. Table below may be referred for the same. OPC-43 Grade cement is available in 50Kg HDPE bags of a distinctive design and cover (bag shot). OPC-43 has emerged as the top choice of Construction & Civil Engineering Companies engaged in construction of mega projects such as National Highways, Bridges, Transmission lines or power plants (19).

Table 4.1:- **Properties of J.P Buland, 43 Grade Ordinary Portland Cement (19)**

S No.	Particulars	Test Results Obtained	Requirement of IS: 8112 - 1989
	PHYSICAL TESTING		
1.	Fineness (M2/Kg)	298	225 Min.
2.	Setting Time (Minutes)		
	Initial	125	30 Min.
	Final	180	600 Max.
3.	Soundness		
	Le. Chatelier, Expansion (mm)	2.00	10.00 Max.
	Auto clave, (%)	0.12	0.8 Max.
4.	Compressive Strength (MPa)		
	3 Days	34.6	23 Min.
	7 Days	44.0	33 Min.
	28 Days	Due	43 Min.

4.2.3 Aggregates

Coarse Aggregate

Aggregates used are clean and free from dust and moisture. The maximum size of aggregate used is 10 mm and are rounded or angular in shape and are confirming to IS: 383-1970. For better results, rounded aggregates should be used. The coarse aggregates were washed thoroughly and hen dries before mixing. The aggregates used were passing through 10 mm sieve and 100 percent retaining on 4.75 mm sieve.

Fine Aggregate

River sand was used as fine aggregate and free from moisture and is conforming to IS: 383-1970. The fine aggregate were sieved and any impurities in them are removed before mixing them in the mixer.

4.2.4 Water

Water used for mixing and curing should be free from injurious and deleterious materials. Throughout the investigation, portable tap water was used.

4.2.5 Fly Ash

The fly ash used in the experimental program is obtained from a thermal power plant. The fly ash is clean and free from any debris and waste materials. The main precaution while using fly ash is to protect it from moisture during the experimental program duration due to ever changing weather conditions in the region. For this purpose the bag of fly ash was kept in dry places free from moisture.

4.3 DESIGN OF SFRC/PCC MIX

It has been reported that there is a maximum increase in first crack and ultimate load for an aspect ratio of 80 and a volume percentage of 1.5 of steel fibres. Therefore, in the present investigation also, mixes having an aspect ratio 80 and fibre content varying between 0.5 to 1.5 percentage by volume indicated that the mixes could be prepared in the concrete mixer, compacted and placed conveniently. Therefore, an aspect ratio of 80 was adopted for various percentages of fibre content and the mix proportion used in the present investigation. The maximum size of aggregate was limited to 10 mm. The mix proportion for all the specimen was 1:1.5:3, w/c ratio 0.55 and cement content 355 kg/m^3 and 1:1:2, w/c ratio 0.6 and with same cement content and also with 15% by weight of fly ash for PCC and the same mix with fiber content from 0.5 to 1.5 % by volume is used for SFRC. For every mix proportions, 10 blocks of 150 x 150 x 150 mm cast iron mould cube were made.

4.3.1 Mixing Procedures

Mixing of designed SFRC mix was done in following two steps i.e. "Dry Mix" and "Wet Mix". In dry mixing procedure for mixes the aggregates were poured in the mixer. The mixer is allowed to run for 1-2 minutes and then cement is added to the mix along with fibre or fly ash if used. Excessive rotation of mixer in dry run process leads to excessive wastage which must be reduced, therefore, it has been avoided and the mixer is allowed to rotate in different intervals of time.

Then for making the mix wet and easily workable, the appropriate amount of water was added as required for the design mix. The mixer is then started for 4-5 minutes and in the due time the required mix is produced. The mix was then taken out from the mixer. Mixing and placing of SFRC mixes into the cast iron mould were made and compaction in 3 to 4 layers using both tampering rod and vibrators was achieved conveniently without any stiffening problem and segregation and bleeding of concrete.

4.3.2 Curing Procedure

The cast specimens were kept under standard moist condition for 24 hours and then demoulded and cured in water for 7 and 28 days. The blocks were taken a day before it was to be tested to ensure proper drying of the block. If the blocks were still wet then oven drying of blocks were made at a temperature of 40 degree C until it gets fully dried.

4.3.3 Mechanical Testing

The test specimens were prepared in the standard 150 x 150 x 150 mm cast iron cube moulds for compressive strength tests. The SFRC mixes containing fibre contents of 1.0% and

1.5% by weight of concrete were tested for compressive strength in the laboratory using the compressive strength test machine. Testing was performed on all specimens. Ten specimens of each type mixes were used for the test and the average compressive strength of each type of mix were calculated.

4.3.4 Mix Design

The mix design for PCC, PCC with fly ash and SRFC have been determined mathematically. The mixes were designed for 10 blocks each of 150 x 150 x 150 mm size. The wastage is kept 10% for M20 and M25. The material requirement for M25 and M20 are given in the table 4.2 and 4.3 respectively.

Table 4.2:- **Material requirement for M25 mix**

S. No.	Mix	Cement (kg)	Fine Aggregate (kg)	Coarse Aggregate (kg)	Fly ash (kg)	Fiber Content (%)			w/c ratio
						0.5	*1.0*	*1.5*	
1.	PCC	22.28	22.28	44.55		-	-	-	0.50
2.	PCC with fly ash	18.90	22.28	44.55	3.38	-	-	-	0.50
3.	SFRC	22.28	22.28	44.55		1.35	2.7	4.05	0.60

Table 4.3:- **Material requirement for M20 mix**

S. No.	Mix	Cement (kg)	Fine Aggregate (kg)	Coarse Aggregate (kg)	Fly ash (kg)	Fiber Content (%)			w/c ratio
						0.5	*1.0*	*1.5*	
1.	PCC	16.20	24.30	48.60					0.50
2.	PCC with fly ash	13.78	24.30	48.60	2.43				0.50
3.	SFRC	22.28	22.28	44.55		1.35	2.7	4.05	0.60

4.4 TEST RESULTS OF PCC AND SFRC SPECIMENS

The objective in the present study is to evaluate the SFRC mixes for their compressive strength and workability of concrete. The results of laboratory tests are discussed below.

4.4.1 Workability of PCC/SFRC Mixes

The result of the slump tests on steel fibre reinforced concrete mixes shows that all the designed SFRC mixes satisfy the IRC requirements of slump. The mixes were compacted using both, tampering rod and vibrators. The addition of fibres resulted in the decrease of slump value from 12 mm plain concrete to 3 mm for steel fiber reinforced concrete. Nearly a zero slump value for SFRC mix with 1.5% fibre by weight was obtained. No significant balling problems were encountered. Adding of fly ash increases the workability of the plain concrete mix and SFRC. No difficulty was found for mixing and placing of SFRC mix up to a fiber content of 1.5 percent and also no segregation and bleeding of concrete was seen.

4.4.2 Strength Tests

The strength characteristics of plain cement concrete and SFRC mixes containing 0.5-1.5% fibres by volume are given in table 4.2 and table 4.3. As compared from PCC the SFRC mixes showed an improvement in compressive strength for fibre contents from 0-1.5 %. Beyond 1.5% the proper compaction of the mix was not achievable hence they were avoided. A maximum increase of 11.69% and 11.14% in compressive strength was observed for a fibre volume of 1.5% over that of PCC mix for a mix 1:1:2 and 1:1.5:3 respectively after 28 days for blocks of size 150 mm. A poor workability was observed at higher fibre volume fraction beyond 1.5 percent. So to improve the workability chemical additives such as super plasticizers are required to be incorporated in the mix.

Table 4.4:- Compressive Strength Values For M25 mix					
S. No.	MIX	COMPRESSIVE STRENGTH(N/mm^2)		% INCREASE	
		7 days	*28 days*	*7 days*	*28 days*
1.	PCC	28.78	32.8	----	----
2.	PCC with fly ash	30.5	33.5	5.64	2.1
3.	SFRC with Fibre content				
a.	0.5%	32.26	35.11	10.79	6.58
b.	1.0%	33.10	36	13.05	8.89
c.	1.5%	34.12	37.14	15.65	11.69

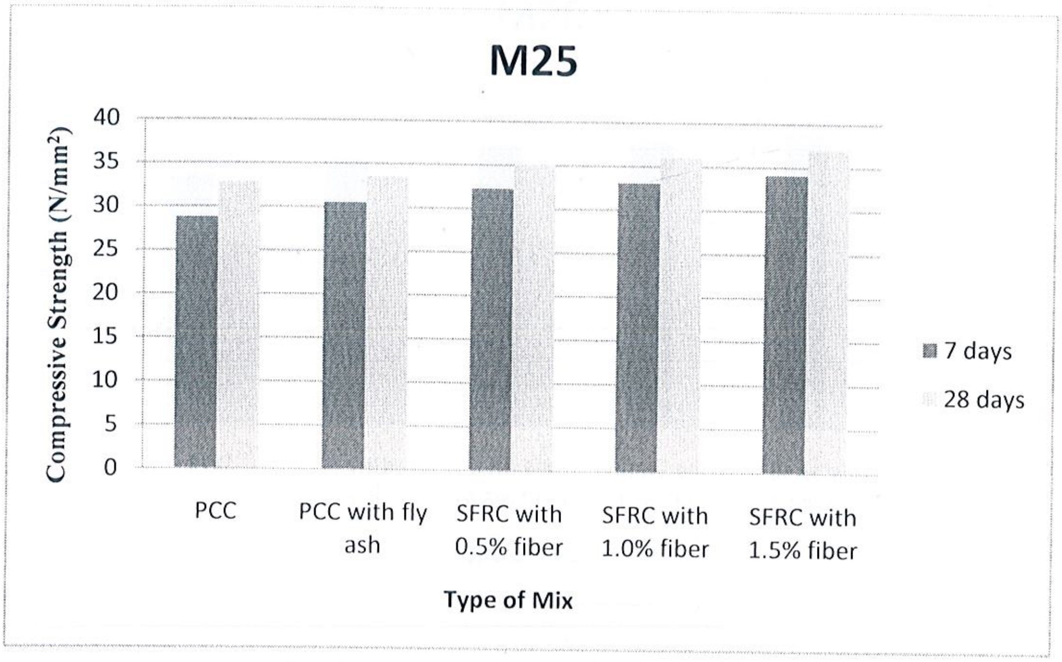


Figure 4.1:- **Compressive strength of M25 mix after 7 and 28 days**

Table 4.5:- Compressive Strength Values For M20 mix					
S. No.	**MIX**	**COMPRESSIVE STRENGTH(N/mm²)**		**% INCREASE**	
		7 days	*28 days*	*7 days*	*28 days*
1.	PCC	23.9	27.1	----	----
2.	PCC with fly ash	24.7	27.9	3.24	2.9
3.	SFRC with Fibre content				
a.	0.5%	26.57	28.4	10.04	4.58
b.	1%	27.32	29.44	12.52	7.95
c.	1.5%	28.56	30.5	16.32	11.14

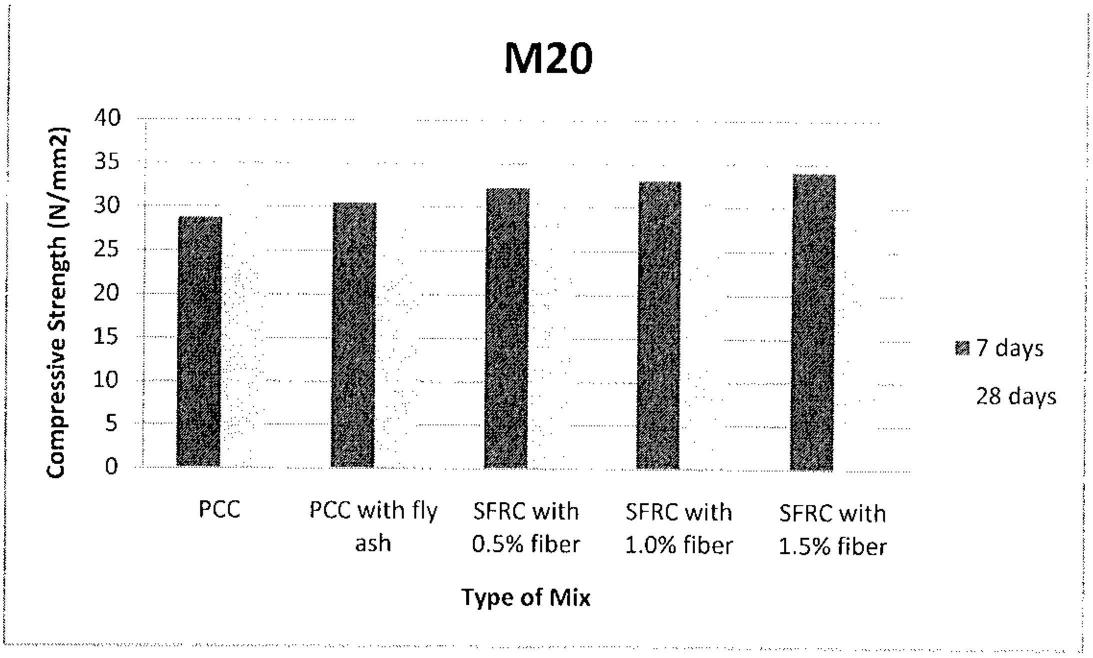


Figure 4.2:- **Compressive strength of M20 mix after 7 and 28 days**

4.5 CONCLUSION

It has been experienced throughout the experimental programme that the cement, aggregates and fibres were properly mixed without any difficulty. Further it is evident by using w/c ratio of 0.6 for SFRC that the workability was just sufficient to enable the concrete to be mixed and compacted fully without using any admixtures. Use of fly ash up to 15% by weight makes the experiment economical (as the amount of cement was decreased) without resulting in any decrease in the strength of hardened concrete and also increases the workability of the plain cement concrete mix.

It is also illustrated that the variation of cement with 15 percent fly ash does not affect the compressive strength of the concrete mix. Mild steel fibres having aspect ratio 80 were used in all SFRC mixes for better workability and strength characteristics and no clumping of fibres were found. Mixing was done by using the ball mixer throughout the work. The compressive strength of SFRC mix with ratio 1:1.5:3 and 1:1:2 at different fiber content was compared with the PCC and a maximum increase of 11.69% and 11.14% in compressive strength was observed for a fibre volume of 1.5% over that of PCC.

CHAPTER- 5

THEORETICAL ANALYSIS OF RIGID PAVEMENTS

5.1 GENERAL

The design of rigid pavements is a soil-structure interaction problem. While, formulating the mathematical model, the foundation may be idealized either by an elastic continuum or by mechanical models. Winkler in 1967 represented the subgrade by means of identical closely spaced discrete springs which later became known as Winkler's model. However, when a vertical load acts at a centre of a pavement, it produces deflections at surrounding points, and the deflected surface takes the shape of basin. For this reason, the subgrade or foundation for the pavement is treated as an elastic continuum.

The most severe stresses caused by wheel loads in a concrete pavement are flexural stresses. Changes in temperature and the moisture content in the concrete also generate stresses in the pavements. The critical locations of the wheel load are the interior, edge and corner positions. When the pavement is loaded maximum stresses are developed under the loaded area itself. As the load is increased gradually the stresses also increase till they exceed the flexural strength of concrete and a crack is developed in the slab. The load causing the first rack at the bottom is known as the yield load, and the elastic design of pavement is based on this load. As the load exceeds the yield load, the pavement passes from the elastic to the plastic state and the central deflection increases rapidly. The stresses in the pavements are compressive and tensile at the top and bottom of the pavement respectively. A small tensile stress is also developed at the top of the pavement at a certain distance from the loaded area.

In the present study, the Westergaard equations are used for stress analysis of concrete pavements considered three conditions of loading i.e. Interior, Edge and Corner. Westergaard (3) considered the rigid pavement slab as a thin elastic plate resting on soil subgrade, which is assumed as a dense liquid.

5.2 ANALYSIS FOR WHEEL LOAD

5.2.1 Westergaard Analysis

Westergaard in 1925 gave the equation for computing stresses in concrete pavements which form the basis of the elastic analysis of concrete slabs. It was Westergaard (24), who first developed the complete analysis of stresses of concrete road slabs in 1926. Since then Westergaard analysis has been regarded as a fundamental method. The following assumptions were made in analysis:

i. The concrete slab is truly elastic, uniform thick and acts as a homogenous elastic solid in equilibrium.
ii. The subgrade is elastic in one direction and behaves like a liquid or a group of vertical springs.
iii. The reaction of subgrade is solely vertical and proportional to the deflection of slab.
iv. The wheel load is uniformly distributed over a circular contact area. For the corner loading, circumference of the loaded area is tangential to the edges of the slab.
v. The slab is fully supported by the subgrade at all points.
vi. The load at the center of the slab is distributed uniformly over a semi-circular contact area, the diameter of semicircle being along the edge of the slab.

Westergaard's Modulus of Subgrade Reaction

The modulus of subgrade reaction, K is proportional to the displacement. The displacement level Δ is taken as 0.125 cm in calculating K. If p is the pressure sustained in kg/cm² by the rigid plate of diameter 75 cm deflection Δ = 0.125 cm, the modulus of subgrade reaction is given by (3):

$$K = \frac{p}{\Delta} = \frac{p}{0.125} \text{ kg/cm}^2$$

where,

p = pressure, kg/cm²

Δ = deflection, 0.125 cm

Radius of Relative Stiffness

A certain degree of resistance to slab deflection is offered by the subgrade. This is dependent upon the stiffness or pressure deformation properties of the subgrade material. The tendency of the slab to deflect is dependent upon its properties of flexural strength.

The resultant of the slab which is also the deformation of the subgrade is a direct measure of magnitude of subgrade pressure. The pressure deformation characteristic of rigid pavement is thus a function of relative stiffness of slab to that of subgrade. Radius of relative stiffness is given by (3):

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)} \right]^{\frac{1}{4}}$$

E = modulus of elasticity of cement concrete, kg/cm^2

μ = Poisson's ratio for concrete = 0.15

h = slab thickness, cm

K = subgrade modulus, kg/cm^3

Equivalent Radius of Resisting Section

Considering the case of interior loading, the maximum bending moment occurs at the loaded area and acts radially in all directions. With the load concentrated on a small area of the pavement is effective in resisting the bending moment. According to Westergaard, the equivalent radius of resisting section is approximated, in terms of radius of load distribution and slab thickness (3),

$$b = \sqrt{1.6a^2 + h^2} - 0.675h \qquad (\text{if } a > 1.724h)$$

$$b = a \qquad (\text{if } a < 1.724h)$$

where,

b = equivalent radius of resisting section, cm

a = radius of wheel load distribution, cm

h = slab thickness, cm

Goldbeck (3) in 1919 indicated that many concrete slabs failed at corners. He derived a corner load formula for stresses due to a point load at the corner of the slab and given by:

$$S_c = \frac{3P}{h^2}$$

This corner formula gives very high stresses. Westergaard in his theory of stress analysis of concrete pavements considered three conditions of loading i.e. Interior, Edge and Corner. The corners are formed by intersection of transverse joints or cracks with the slab edges or longitudinal joints. The critical stresses Si, Se and Sc at the typical locations i.e. interior, edge and corner are given by (3, 26):

Interior Loading:

$$S_i = \frac{0.316\,P}{h^2}\left[\,4\log_{10}\left(\frac{l}{b}\right) + 1.069\,\right]$$

Edge Loading:

$$S_e = \frac{0.572\,P}{h^2}\left[\,4\log_{10}\left(\frac{l}{b}\right) + 0.359\,\right]$$

Corner Loading:

$$S_c = \frac{3\,P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)\right]$$

where, S_i, S_e and S_c are the maximum stress at interior, edge and corner loading respectively, kg/cm^2

Further these equations have been modified by various investigations (25) based on their research work on cement concrete pavement slabs. The Indian Road Congress recommends the following two formulas for analysis of load stresses at the edge and corner regions and for design of rigid pavements;

i. Westergaard's edge load stress formula, modified by Teller and Sutherland for finding the load stress S_e in the critical edge region,

$$S_e = \frac{0.529\ P}{h^2}[1 + 0.54\mu]\ [\ 4\log_{10}(\frac{l}{b}) + \log_{10} b - 0.4048]$$

ii. Westergaard's corner load stress formula, modified by Kelley for finding the load stress S_c in the critical corner region,

$$S_c = \frac{3\ P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{1.2}\right]$$

where,

S_e = load stress at the edge region, kg/cm^2

S_c = load stress at the edge region, kg/cm^2

P = design wheel load, kg

h = thickness of CC pavement slab, cm

μ = Poisson's ratio of the CC slab

E = modulus of elasticity of the CC, kg/cm^2

K= reaction modulus of pavement foundation, kg/cm^2

l = radius of relative stiffness, cm

b = radius of equivalent distribution of pressure, cm

a = radius of load contact, cm (assumed circular in shape)

5.2.2 Factor of Safety

The design of Westergaard analysis takes into account the flexural fatigue endurance of the PCC and Steel Fibre Reinforced Concrete. A Load Safety Factor of slightly greater than 1 is used for rigid pavements (27) while in case of SFRC pavements; the value of load factor safety varies from 1.3 to 2. (7)

5.3 ANALYSIS USING WESTERGAARD'S METHOD

Example: - Calculate the stresses at interior, edge and corner regions of a Steel fiber reinforced concrete pavement.

Wheel load, P = 5100 kg

Modulus of Elasticity of cement concrete, E = 3.0 x 10^5 kg/cm2

Pavement thickness, h = 10.0 cm

Poisson's ratio of concrete, $\mu = 0.15$

Modulus of subgrade reaction, K = 6.0 kg/cm2

Radius of contact area, a = 10.0 cm

SFRC flexural strength, $F_s = 60$ kg/cm^2

Solution: -

Radius of Relative Stiffness

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{\frac{1}{4}}$$

$$= \left[\frac{300000*10^3}{12*6(1-.15^2)}\right]^{\frac{1}{4}} = 45.44 \text{ cm}$$

Equivalent Radius of Resisting Section

a/h = 10/10= 1.0<1.74

So, b $= \sqrt{1.6a^2 + h^2} - 0.675h$

$= \sqrt{1.6 \times (10)^2 + 10^2} - 0.675 \times 10$

= 9.37 cm

Stress at the Interior, (S_i)

$$S_i = \frac{0.316\,P}{h^2}\left[\,4\log_{10}\left(\frac{l}{b}\right) + 1.069\,\right]$$

$$= \frac{0.316*5100}{10^2}\left[\,4\log_{10}\left(\frac{45.44}{9.37}\right) + 1.069\,\right] = 61.43 \text{ kg/cm}^2$$

Stress at the Edg,e (S_e)

$$S_e = \frac{0.572\,P}{h^2}\left[\,4\log_{10}\left(\frac{l}{b}\right) + 0.359\,\right]$$

$$= \frac{0.572*\,5100}{10^2}\left[\,4\log_{10}\left(\frac{45.44}{9.37}\right) + 0.359\,\right] = 90.485 \text{ kg/cm}^2$$

Stress at the Corner, (S_c)

$$S_c = \frac{3\,P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

$$= \frac{3*5100}{10^2}\left[1 - \left(\frac{10\sqrt{2}}{45.44}\right)^{0.6}\right] = 77.048 \text{ kg/cm}^2$$

Residual Concrete strength for supporting load is given by (27):

$F_L = F_s$ – temperature stresses.

Since no temperature stresses are taken in the problem, therefore, residual strength of concrete,

$F_L = F_s = 60$ kg/cm^2

Factor of Safety:

Since the maximum stress is at edge, therefore,

F.O.S = F_L / S_e = 60 / 90.485

= 0.663 < 1.3

Since for the given example the value of F.O.S is less than 1.3, therefore the pavement fails and need to be redesigned by taking higher pavement thickness.

Now taking h = 16 cm and considering the other values same:

Radius of Relative Stiffness

$$l = \left[\frac{Eh^3}{12K(1-\mu^2)}\right]^{\frac{1}{4}}$$

$$= \left[\frac{300000*16^3}{12*6(1-.15^2)}\right]^{\frac{1}{4}} = 64.64 \text{ cm}$$

Equivalent Radius of Resisting Section

a/h = 10/16= 0.625<1.74

So, b $= \sqrt{1.6a^2 + h^2} - 0.675h$

$= \sqrt{1.6 \times (10)^2 + 16^2} - 0.675 \times 16$

= 9.60 cm

Stress at the Interior, (S_i)

$$S_i = \frac{0.316\,P}{h^2}\left[\,4\,\log_{10}\left(\frac{l}{b}\right) + 1.069\,\right]$$

$$= \frac{0.316*5100}{16^2}\left[\,4\,\log_{10}\left(\frac{64.64}{9.6}\right) + 1.069\,\right] \;=\; 27.586 \text{ kg/cm}^2$$

Stress at the Edge, (S_e)

$$S_e = \frac{0.572\,P}{h^2}\left[\,4\,\log_{10}\left(\frac{l}{b}\right) + 0.359\,\right]$$

$$= \frac{0.572*\,5100}{16^2}\left[\,4\,\log_{10}\left(\frac{64.64}{9.6}\right) + 0.359\,\right] = 41.843 \text{ kg/cm}^2$$

Stress at the Corner, (S_c)

$$S_c = \frac{3\,P}{h^2}\left[1 - \left(\frac{a\sqrt{2}}{l}\right)^{0.6}\right]$$

$$= \frac{3*5100}{16^2}\left[1 - \left(\frac{10\sqrt{2}}{64.64}\right)^{0.6}\right] \qquad = 35.752 \text{ kg/cm}^2$$

Since no temperature stresses are taken in the problem, therefore, residual strength of concrete,

$F_L = F_s = 60$ kg/cm^2

Factor of Safety:

Since the maximum stress is at edge, therefore,

F.O.S = F_L / S_e = 60 / 41.843

= 1.43 > 1.3

Since F.O.S is greater than 1.3, therefore the designed pavement using Westergaard analysis is safe.

5.4 CONCLUSION

In conclusion it is summarized that before designing any pavement, all the stresses due to wheel loads should be calculated using the Westergaard theory. The stresses can also be calculated using the IRC charts for different load stresses. In the present study, the stress analysis for interior, edge and corner loading using Westergaard method, was determined for pavement thickness 10 cm and 16 cm for comparing it with the MATLAB analysis results which are discussed in the chapter 6.

CHAPTER-6

FINITE ELEMENT ANALYSIS

6.1 GENERAL

The finite element analysis is a numerical technique. In this method all the complexities of the problems, like varying shape, boundary conditions and load are maintained as they are but the solution obtained are approximate. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. The fast improvement in computer hardware technology and slashing of cost of computers has boosted this method, since the computer is the basic need for the application of this method. Some of the popular packages are STAAD-PRO, GT-STRUDEL, NASTRAN, NISA and ANSYS. Using these packages one can analyze complex structures. In the present study MATLAB has been used to analysis a highway pavement.

The basic unknowns which are encountered in the engineering problems are displacement in solid mechanics. In a continuum these unknown are infinite. The finite element procedure reduces such unknowns to a finite number by dividing the solution regions into small parts called elements and by expressing these unknowns in terms of approximating functions (shape functions) within the element. The approximate functions are defined in terms of field variables of specific points called nodes or nodal points. Thus in the finite element analysis unknowns are the field variables of the nodal points.

After all this the next step in a finite element program is to assemble element properties of each element. These element properties are used to assemble global stiffness properties to get system equations. Then the boundary conditions are imposed. The solution of these simultaneous equations gives the nodal unknowns. Using the nodal values additional calculations are made to get the required values e.g. stresses, strains, moments etc. in solid mechanic problems. A number of FEM models for the analysis of concrete pavements have been developed like Winkler's

Model, Elastic Continuum Model, Axi- Symmetric Model, etc. In this study an elastic continuum model is used (32).

6.2 FOUNDATION MODELS

A number of FEM models for the analysis of concrete pavements have been developed and are summarized below:

6.2.1 Winkler's Model

Rigid plate slab is generally designed as an elastic plate lying on an elastic subgrade whose reaction at any point are assumed to be purely vertical and proportional to the magnitude of deflection at that point. This assumption was introduced by Winkler in 1867. The simplest representation of the behavior of subgrade material is done by closely spaced linear springs. The physical representation of Winkler model is shown in fig. 6.1. The pressure deflection relationship at any point is given by

$$P = k.\ \Delta$$

where,

P = pressure

k = spring constant, also called as the modulus of subgrade reaction

Δ = deflection.

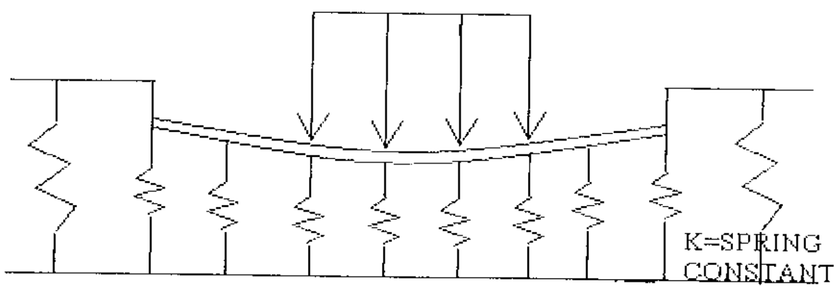


Figure 6.1:- **Winkler's Foundation (7)**

Westergaard, in 1926, analyzed the structural action of slab on ground by considering the subgrade to be acting like a series of parallel, vertical independent springs supporting the concrete slab. The elastic properties of the subgrade are expressed in terms of spring constant obtained from the plate bearing test. The solution is given in the form of equations which enables the tensile stresses at three locations in the upper plate to be calculated. The equations were further modified to bring the calculated stress into better agreement with measured stresses.

For the case of edge and corner loads, the extent of significant deflections produced at the surrounding points by vertical load acting at or near the edge or corner is rather limited in which case a Winkler's type model is assumed for the foundation.

6.2.2 Elastic Continuum Model

Vesic and Saxena (30) on the basis of investigation carried out in 1961 indicated that the contact pressure between the beams or slabs and subgrade may be quite different from that obtained by conventional analysis based on Winkler's model. It is an elastic solid which is mathematically described by Boussinesq equations. Sargious and Wang (31) have indicated that when a vertical load acts at a centre of a pavement, it produces deflections at the surrounding points, and the deflected surface takes the shape of basin. For this reason, pavement or foundation is considered as elastic continuum for the case of central load. The stiffness of each subgrade element is represented by a stiffness matrix pavement model incorporating thickness variation and other boundary condition.

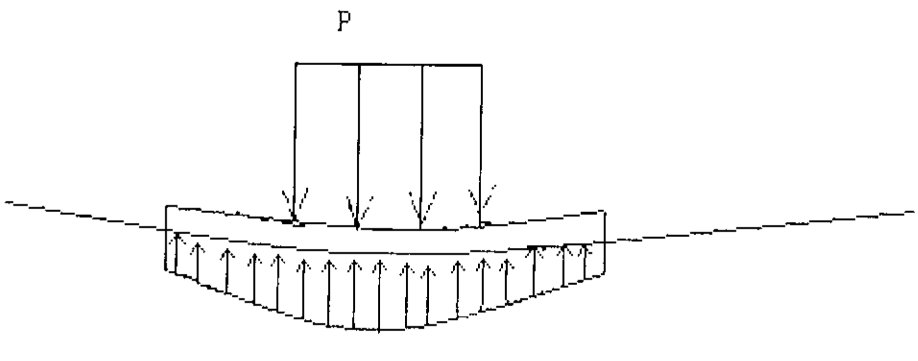


Figure 6.2:- **Elastic Continuum (32)**

6.2.3 Axi-symmetric Model

In this model, a pavement system is idealized as a multiple layered cylindrical system loaded symmetrically at the centre line. The axi-symmetric problem (32) of a semi-infinite half space loaded by circular area i.e. a circular footing of soil mass is shown in fig. 6.3. If cylindrical structures are subjected to axisymmetric loadings like uniform internal or external pressures, uniform self weight or live load uniform over the surface, there exists symmetry about any axis and this may made the analysis simple. It is convenient to express problem in terms of cylindrical coordinates. Because of symmetry the stress components are independent of the angular (θ) coordinate; hence all the derivatives w.r.t θ vanishes.

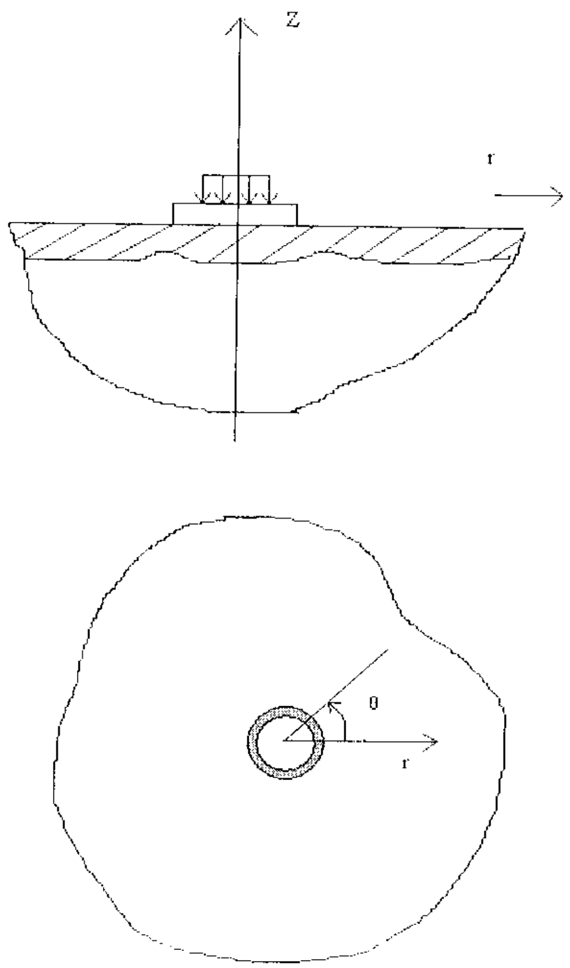


Figure 6.3: - **Axisymmetric Problem of a Semi-Infinite half space loaded by a circular area (7)**

6.3 STEPS IN FINITE ELEMENT ANALYSIS

6.3.1 Discretization of the continuum

In any continuum, the actual number of degrees of freedom is infinite and, unless a closed form solution is available, an exact analysis is impossible. An approximate solution is therefore, attempted by assuming that the behavior of the continuum can be represented by a finite number of unknowns. The continuum is therefore divided into a series of element which are connected at a finite number of points known as nodal points. This process is known as discretization (32). The shape of the element is decided depending upon the geometry of the body or the structure or the number of independent space coordinates (i.e. x, y or z) necessary to describe the problem. In present analysis, a two dimensional 8 noded isoparametric element has been used.

In idealization using finite element method, the mesh size is successively reduced so that the compatibility requirements are completely satisfied and the finite element solution converges to an exact solution.

The foundation is considered as consisting of series of rectangular pressure areas whose centers coincide with and remain in contact with the nodal points of the slab. The pressure is assumed to be constant within each rectangle. All the external forces applied to the system are replaced by equivalent nodal forces which may be done either by allocating a specific area to each node or on the basis of energy considerations.

6.3.2 Isoparametric Element

Isoparametric elements are elements for which the geometry and displacement formulations are of the same order. Stated more precisely, the interpolation of the element coordinates and element displacements use the same interpolation functions, which are defined in a natural coordinate system (Bathe 1982). A natural coordinate system is a local coordinate system which specifies the location of any point within the element by a set of dimensionless numbers whose magnitudes never exceed unity (Desai and Able 1972). An interpolation function

N_i must be formulated such that its value in the natural coordinate system is unity at node I and zero at all other nodes.

Isoparametric elements satisfy the following necessary and sufficient conditions for completeness and compatibility (33):

a. The displacement models must be continuous within the elements, and the displacements must be compatible between adjacent nodes.
b. The displacement models must include the rigid body displacements of the elements.
c. The displacement models must include the constant strain states of the element.

The isoparametric concept is a powerful generalized technique for constructing complete and conforming elements of any order (33). In the first-order or linear element, the interpolation functions of the elements are linear with respect to the natural coordinates. Similarly, for a quadratic or second-order element, the interpolation functions are quadratic with respect to the natural coordinates.

Simple Isoparametric linear and quadratic element in natural coordinates is shown in figure below.

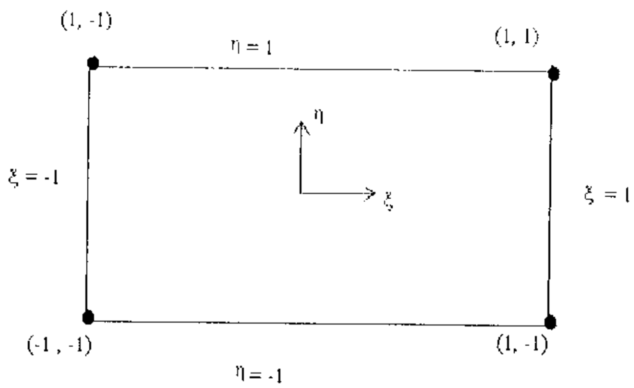


Figure 6.4: **- Linear Isoparametric Element (32)**

The shape functions for linear element can be written as:

For Corner nodes:

$$N_i = \frac{1}{4}(1 + \xi\,\xi_i)(1 + \eta\eta_i)$$

For example, for top right corner node, the shape function can be written in the form:

$$N_3 = \frac{1}{4}(1 + \xi)(1 + \eta)$$

As $\xi_i = \eta_i = 1$

Similarly for quadratic element, the shape functions at various nodes can be written as:

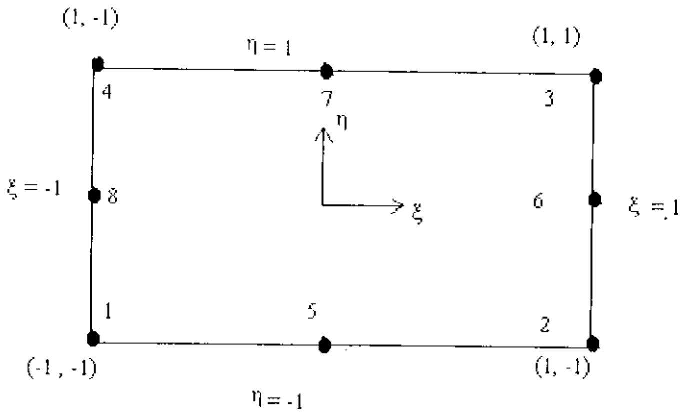


Figure 6.5: **- Quadratic Isoparametric Element (32)**

For Corner nodes: (i = 1,2,3,4)

$$N_i = \frac{1}{4}(1 + \xi\,\xi_i)(1 + \eta\eta_i)(\xi\,\xi_i + \eta\eta_i - 1)$$

For mid side nodes:

If $\xi_i = 0$, then $N_i = \frac{1}{4}(1 - \xi^2)(1 + \eta\eta_i)$ (i.e. for 5, 7)

If $\eta_i = 0$, then $N_i = \frac{1}{4}(1 - \eta^2)(1 + \xi\,\xi_i)$ (i.e. for 6, 8)

6.3.3 Displacement Function

In the finite element displacement method, displacement is assumed to have unknown value only at the nodal points. The displacement function is such that the displacement variation within any element is described in terms of nodal values by means of interpolation function. The displacement { f_i } at any point I within the element boundaries is given by

$$\{ f_i \} = \begin{Bmatrix} u_i \\ v_i \end{Bmatrix}$$

where u_i and v_i are the nodal displacements at node i

and the element displacement $\{f^e\}$ is given as:

$$\{f^e{}_i\} = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \\ . \\ . \\ f_n \end{Bmatrix}$$

where, N denotes the number of nodes per element. The displacement within the element is given by:

$$u = \sum_{i=1}^{n} N_1 u_1 + N_2 u_2 + \cdots + N_n u_n = \sum_{i=1}^{n} N_i u_i$$

$$v = \sum_{i=1}^{n} N_1 v_1 + N_2 v_2 + \cdots + N_n v_n = \sum_{i=1}^{n} N_i v_i$$

where, the functions N_1, N_2, ... N_n are the shape functions to define the nodal variables. The shape functions for finite elements used in present study are discussed in clause 5.3.2

Displacement function can be therefore expressed as:

$$\{F\} = \begin{bmatrix} u \\ v \end{bmatrix} = [N] \{ f \}^e$$

where,

{F} = Displacement of any point within the element boundaries

{f^e} = Vector of nodal displacement

[N] = Displacement Function

Indeed, it is quite easy to generate serendipity shape functions for elements with different number of nodes along each side by a symmetric algorithm. With this mode of generating shape functions, a fewer degrees of freedom are necessary for a given polynomial expansion as compared to Lagrangian family.

6.3.4 Strain Displacement Relationship

The stress components in the vertical, radial, circumferential and component of shear stresses in case of axisymmetric problem under consideration are given by σ_z, σ_r, σ_θ and τ_{rz} respectively. The corresponding strain components are given by ε_z, ε_r, ε_θ and γ_{rz} respectively. If {ε} is the component of strain at a point which contributes to internal work, then

$$\varepsilon = \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial r} \\ \frac{\partial v}{\partial z} \\ \frac{u}{r} \\ \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r} \end{Bmatrix}$$

$$= \begin{bmatrix} \frac{\partial}{\partial r} & 0 \\ 0 & \frac{\partial}{\partial z} \\ \frac{1}{r} & 0 \\ \frac{\partial}{\partial z} & \frac{\partial}{\partial r} \end{bmatrix} \begin{Bmatrix} u \\ v \end{Bmatrix}$$

$$= [L]\{F\}$$

Therefore,

$$\{\varepsilon\} = [L][N]\{f^e\} \qquad (6.1)$$

$$= [B]\{f^e\} \qquad (6.2)$$

The matrix [B] of equation 6.2 will be of form

$$[B]_i = \begin{bmatrix} \frac{\partial N_i}{\partial r} & 0 \\ 0 & \frac{\partial N_i}{\partial z} \\ \frac{N_i}{r} & 0 \\ \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial r} \end{bmatrix}$$

where, *i* denote the quantities related to node *i* of the element.

6.3.5 Transformation between Global and Local Coordinate System

In case of Isoparametric elements shape functions defining geometry will be similar to the shape functions defining displacement functions.

Therefore for geometry,

$$r = \sum_{i=1}^{n} N_i(\xi, \eta) r_i \qquad (6.3)$$

$$z = \sum_{i=1}^{n} N_i(\xi, \eta) z_i$$

Now,

$$dr = \frac{\partial r}{\partial \xi} d\xi + \frac{\partial r}{\partial \eta} d\eta$$

$$dz = \frac{\partial z}{\partial \xi} d\xi + \frac{\partial z}{\partial \eta} d\eta$$

or $$\begin{Bmatrix} dr \\ dz \end{Bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial r}{\partial \eta} \\ \frac{\partial z}{\partial \xi} & \frac{\partial z}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \partial \xi \\ \partial \eta \end{Bmatrix} \tag{6.4}$$

$$= [J]^T \begin{Bmatrix} \partial \xi \\ \partial \eta \end{Bmatrix} \tag{6.5}$$

where [J] is known as Jacobian matrix from equation (6.3) we get,

$$\frac{\partial r}{\partial \xi} = \frac{\partial N_i}{\partial \xi} r_i$$

$$\frac{\partial z}{\partial \xi} = \frac{\partial N_i}{\partial \xi} z_i$$

Substituting these values in equation 6.4, we get,

$$\begin{Bmatrix} dr \\ dz \end{Bmatrix} = \begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} r_i & \sum \frac{\partial N_i}{\partial \eta} r_i \\ \sum \frac{\partial N_i}{\partial \xi} z_i & \sum \frac{\partial N_i}{\partial \eta} z_i \end{bmatrix} \begin{Bmatrix} \partial \xi \\ \partial \eta \end{Bmatrix} \tag{6.6}$$

From equation 6.5 and 6.6, we get,

$$[J]^T = \begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} r_i & \sum \frac{\partial N_i}{\partial \eta} r_i \\ \sum \frac{\partial N_i}{\partial \xi} z_i & \sum \frac{\partial N_i}{\partial \eta} z_i \end{bmatrix} \tag{6.7}$$

6.3.6 Transformation for gradient of function

From the principle of partial differentiation,

$$\frac{\partial N_i}{\partial \xi} = \frac{\partial N_i}{\partial r} \cdot \frac{\partial r}{\partial \xi} + \frac{\partial N_i}{\partial z} \cdot \frac{\partial z}{\partial \xi}$$

and

$$\frac{\partial N_i}{\partial \eta} = \frac{\partial N_i}{\partial r} \cdot \frac{\partial r}{\partial \eta} + \frac{\partial N_i}{\partial z} \cdot \frac{\partial z}{\partial \eta}$$

Writing in the matrix form,

$$\begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial r}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial r}{\partial \eta} & \frac{\partial z}{\partial \eta} \end{bmatrix} \begin{Bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix}$$

$$= [J] \begin{Bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} \qquad (6.8)$$

To find out, the global derivative equation (6.8) can be inverted as:

$$\begin{Bmatrix} \frac{\partial N_i}{\partial r} \\ \frac{\partial N_i}{\partial z} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \frac{\partial N_i}{\partial \xi} \\ \frac{\partial N_i}{\partial \eta} \end{Bmatrix} \qquad (6.9)$$

From the equation 6.6, [J] in terms of shape function can be written as

$$[J] = \begin{bmatrix} \sum \frac{\partial N_i}{\partial \xi} r_i & \sum \frac{\partial N_i}{\partial \eta} r_i \\ \sum \frac{\partial N_i}{\partial \xi} z_i & \sum \frac{\partial N_i}{\partial \eta} z_i \end{bmatrix}$$

6.3.7 Stress-Strain Matrix [D]

Figure 6.3 shows a semi-infinite half space loaded by a circular area. It is convenient to express such problem of axially symmetric loading in terms of the cylindrical coordinates. The stress components are independent of the angular (θ) coordinate, hence all derivatives w.r.t θ vanish. The nonzero stress components are σ_z, σ_r, σ_θ and τ_{rz}. The strain displacement relation for the non-zero strain becomes

$$\varepsilon_r = \frac{\partial u}{\partial r}$$

$$\varepsilon_\theta = \frac{u}{r}$$

$$\varepsilon_z = \frac{\partial v}{\partial z}$$

$$\gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial v}{\partial r}$$

So the constitutive stress-strain relationship is

$$\begin{Bmatrix} \sigma_r \\ \sigma_z \\ \sigma_\theta \\ \tau_{rz} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1-\nu & \nu & \nu & 0 \\ 0 & 1-\nu & \nu & 0 \\ 0 & 0 & 1-\nu & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_r \\ \varepsilon_z \\ \varepsilon_\theta \\ \gamma_{rz} \end{Bmatrix}$$

The stress-strain relationship can be therefore be written as:

$$\{\sigma\} = [D][\varepsilon] \qquad (6.10)$$

So from equation 6.2 and 6.10

$$\{\sigma\} = [D][B]\{f^e\} \qquad (6.11)$$

where, [D] is the elasticity matrix containing the appropriate material properties i.e. Modulus of Elasticity (E) of the material and Poisson's ratio ν . The matrix resulting from the product of

[D][B] is known as stress matrix. Thus the stresses are completely defined in terms of strains through the [D] matrix. Once the nodal displacements are known, the displacements, strains and stresses at any point can be determined.

6.3.8 Stiffness Matrix

Considering a single element which is acted upon by nodal loads and body forces resulting in an equilibrium stress field σ . The element, when subjected to an arbitrary virtual displacement pattern $\{f^e\}$ results in compatible internal displacements and strain distributions. The stiffness matrix [K]e of the element w.r.t nodal displacement {f} is evaluated from principle of virtual work by equalizing the internal virtual work done to the external work associated with the virtual displacements. The sum of the product of the displacement and corresponding load component represents truly the external work done, while that of the strain and corresponding stress components results in the total internal work (7, 32, 33).

Now internal work done,

$$dW_i = \{\bar{\varepsilon}\}^T \{\sigma\}\, dv \qquad (6.12)$$

In which $\{\bar{\varepsilon}\}^T$ is the transpose of the column vector of virtual strains and thus represents the row vector of the virtual strains $\{\bar{\varepsilon}\}$ and $\{\sigma\}$ represents the column vector of the actual $\{\sigma\}$.

from equation 6.4, $\{\varepsilon\} = [B]\,\{f^e\}$

$$\{\bar{\varepsilon}\} = [B]\,\{\bar{f}^e\}$$

Where $\{\bar{f}\}^e$ is the virtual nodal displacement of the element. Therefore,

$$\{\bar{\varepsilon}\}^T = \{\bar{f}^e\}^T [B]^T$$

Equation 6.12 can be therefore written as

$$dW_i = \{\bar{f}^e\}^T [B]^T \{\sigma\}\, dv$$

Substituting the value of $\{\sigma\}$ from equation 6.11,

$$dW_i = \{\bar{f}^e\}^T [B]^T [D]\,[B]\,\{f^e\}dv \qquad (5.13)$$

Integrating the equation 6.13 over the total volume of the element, the total internal virtual work done W_i is given as

$$W_i = \{\bar{f}^e\}^T [\int_{vol}[B]^T[D][B]dv] \{f^e\}$$

where $\{\bar{f}\}^e$ and $\{f^e\}$, being an independent of the coordinates of nodal points of elements, have been kept out of the integral. The external work W_e associated with the virtual displacements $\{f^e\}$ is obtained by the product of virtual forces {S} associated with the generalized displacements {f} and is given by:

$$W_e = \{\bar{f}^e\}^T \{S\}$$

Therefore, from the principle of virtual work,

$$\{\bar{f}^e\}^T [\int_{vol}[B]^T[D][B]dv] \{f^e\} = \{\bar{f}^e\}^T \{S\}$$

Therefore,

$$\{S\} = [\int_{vol}[B]^T[D][B]dv] \{f^e\} \qquad (6.14)$$

which can be further written as

$$\{S\} = [K]^e \{f^e\}$$

So the stiffness matrix can be represented as $[K]^e$ and equation 6.14 represents the force displacement equation. Therefore,

$$[K]^e = \int_{vol}[B]^T[D][B]dv \qquad (6.15)$$

For an axisymmetric case,

$dv = 2\pi r\, dr\, dz$

By substituting the value of dv in equation 6.15, the general formula of the stiffness matrix $[K]^e$ becomes

$$[K]^e = \int_{vol}[B]^T[D][B]\, 2\pi r\, dr\, dz$$

So the stiffness matrix $[K]^e$ is evaluated by numerical integration with the help of Gaussian Quadrature formula.

6.3.9 Numerical Integration

It may be seen that exact integration of complex expression for element stiffness matrix $[K]^e$ as given in equation 6.15 could be troublesome and numerical integration is essential for such functions. In case of Gauss-Legendre quadrature, the positions of sampling points are allowed to be located at points to be determined. So as to achieve best accuracy consider the given integral.

$$I = \int_{-1}^{1} \int_{-1}^{1} f(\xi,\eta)\, d\xi\, d\eta$$

where f is given function of ξ and η .

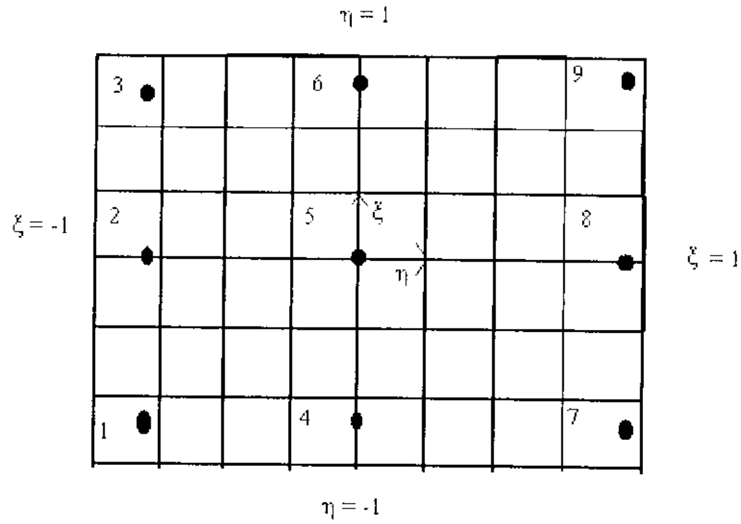
For obtaining the integral, first inner integral is keeping η as constant i.e.

$$\int_{-1}^{1} f(\xi,\eta) d\xi = \sum_{j=1}^{n} W_j\, f(\xi_j,\eta)$$

The outer integral is then, evaluated as

$$\begin{aligned} I &= \int_{-1}^{1} \sum_{j=1}^{n} W_j\, f(\xi_j,\eta)\, d\eta \\ &= \sum_{i=1}^{m} W_i \sum_{j=1}^{n} W_j\, f(\xi_j,\eta_i) \\ &= \sum_{i=1}^{m} \sum_{j=1}^{n} W_i\, W_j\, f(\xi_j,\eta_i) \end{aligned}$$

Where W_i and W_j are the weight coefficients corresponding to ξ_j and η_i respectively, m and n are the number of sampling points in ξ and η directions respectively. In the above integration, the number of integration points in each direction have been assumed to be same (m = n). It may be noted that double summation can be readily interpreted as a single one over n x n points for rectangle. In the present study 3 x 3 Gaussian integrals have been applied over the finite elements. Sometime it may be advantageous to use different number of points in each direction of integration (32).



PARENT ELEMENT (Integrating Points for n = 3 in a square region) (7)

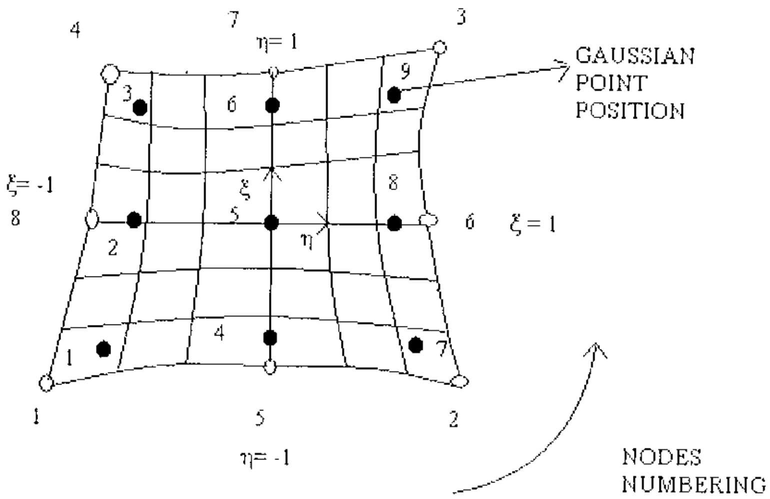


Figure 6.6:- **2-D Parabolic Isoparametric Quadratic Element (Showing orientation of local axes ξ, η and order of Gauss Point numbering.) (7, 32)**

6.3.10 Stiffness Matrix of Total Assemblage

The stiffness matrix [K] of the whole assembly is obtained by superimposing the individual element stiffness contributing to each nodal point of the assemblage. This is done by simple computer process by calling the element stiffness one after another and adding to previously called stiffness values of the corresponding nodal points.

6.3.11 Solution of Equilibrium Equation

From the stiffness matrix of the whole structure assembly, the equations giving relationship between nodal points force and the corresponding nodal displacement is expressed as:

$$\{S\} = [K]\ \{f\}$$

6.3.12 Determination of Stresses

From the nodal displacements so obtained by solving simultaneous equilibrium equations, the stresses in the elements are evaluated from the matrix resulting from the product of [D][B] which relates element stresses to the nodal displacements. The stresses and the deflection can be calculated using the computer program also.

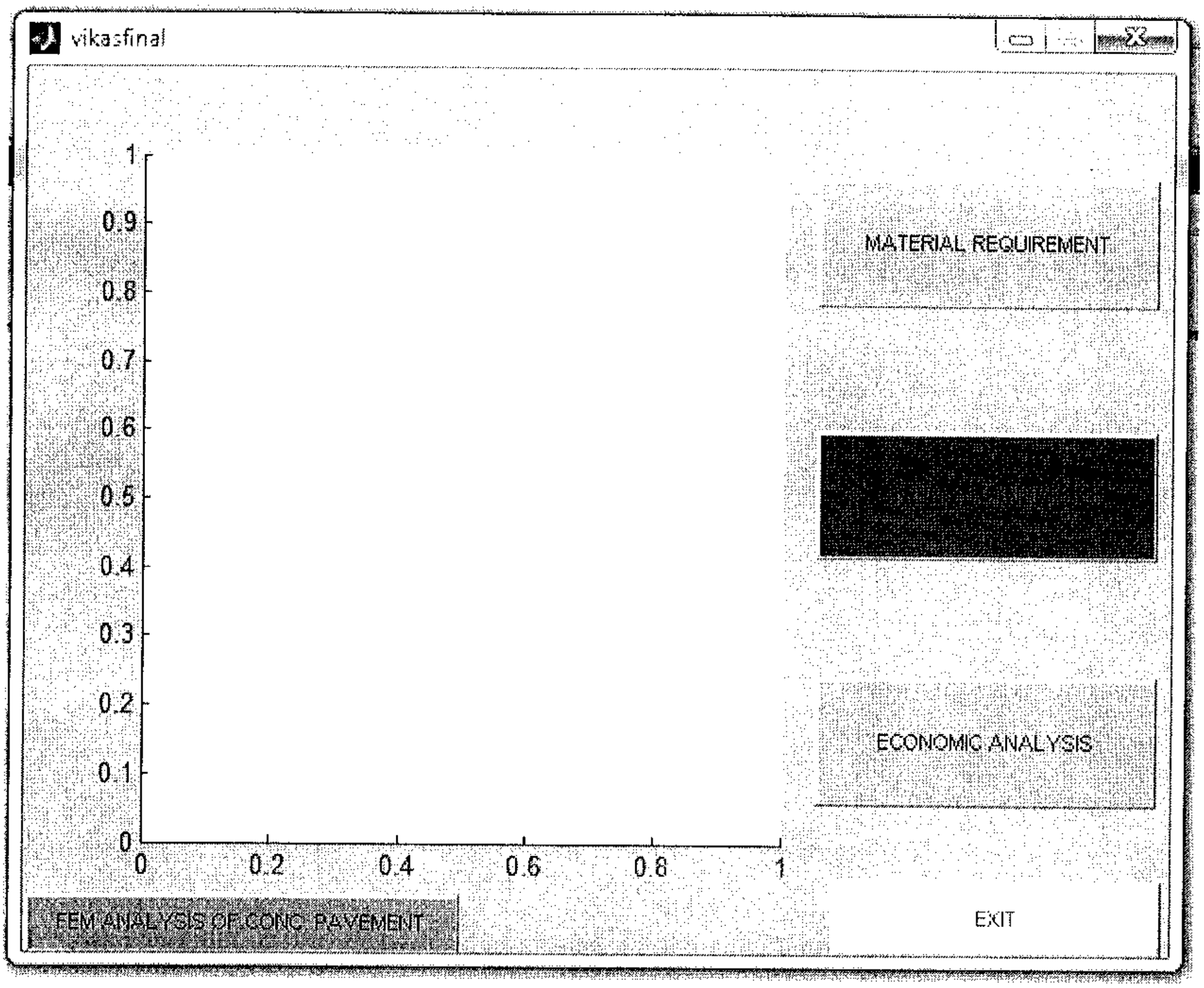
6.4 MATLAB PROGRAM

The program used for finite element analysis of highway pavement is a line coding which also include material requirement calculation, strength analysis and economic analysis of PCC, PCC with fly ash and SFRC. This section illustrates the executon of this program.

STEP 1:-

The MATLAB program developed in the present study executes with the display of a GUI window.

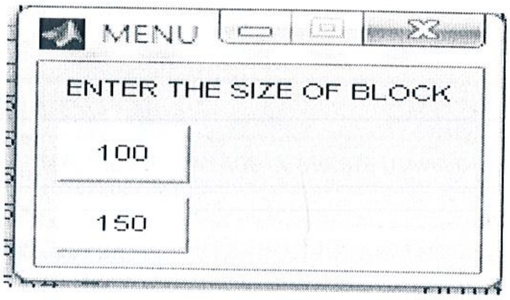
GUI stands gor graphical user display and is used just to enhance the asthetics of the program execution. The GUI window has several push buttons like Material Requirement, Strength Analysis, Economic Analysis, Finite Element Analysis and an Exit button. It also contains a graph sheet to display the charts and figures.



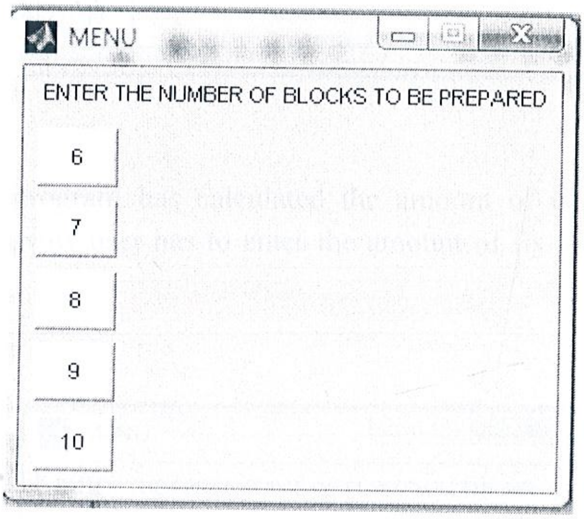
STEP 2: -

This step shows the execution of the program on pressing the material requirement button. On pushing Material requirement button the program executes in the following way:

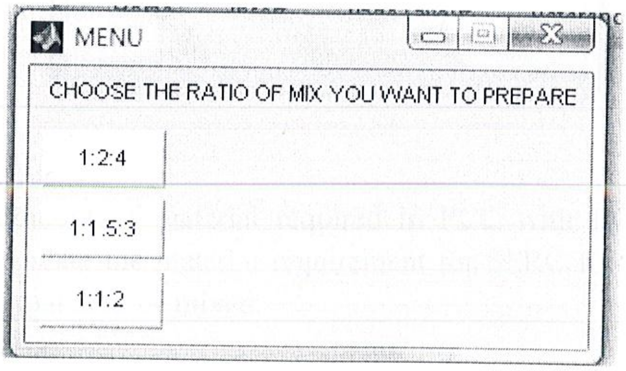
(I) Enter the size of the block that is to be prepared in the laboratory.



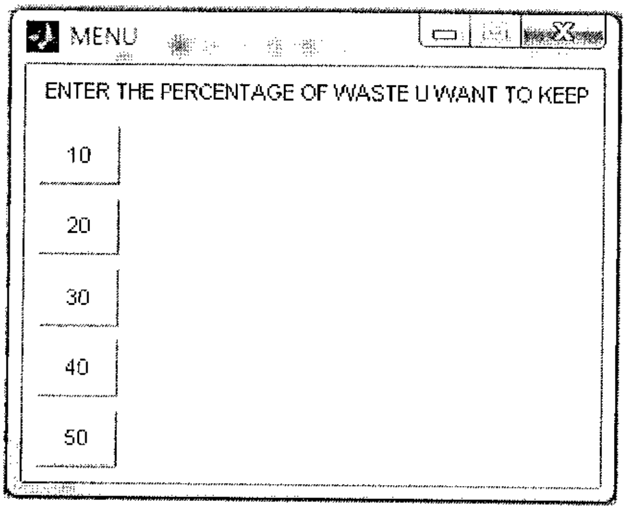
(II) Once you click on the size of the block the program ask you about the number of blocks to be prepared.



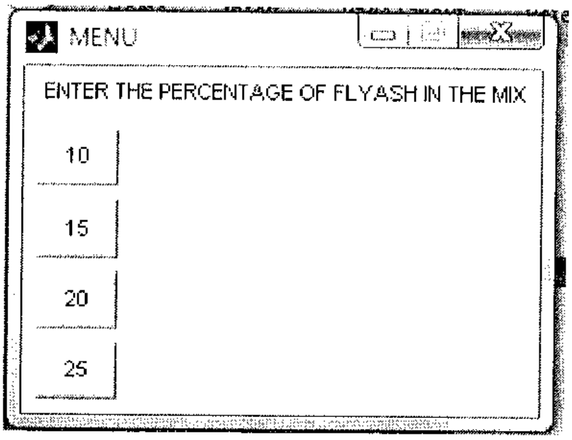
(III) After you decide the number of blocks user ask you to enter the ratio of mix that is to be prepared and the following menu is displayed.



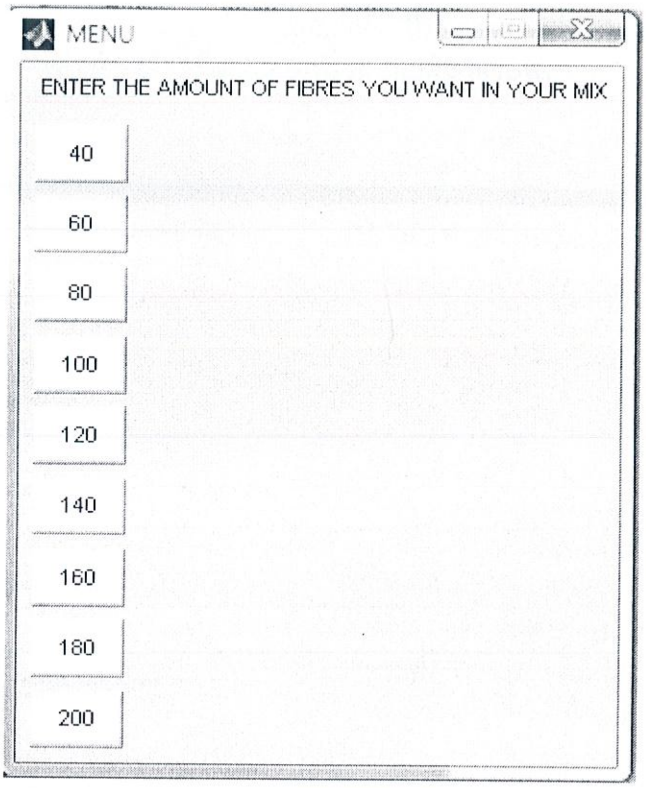
(IV) Once the ratio is entered the user is required to tell the amount of wastage to be considered while preparing the mix.



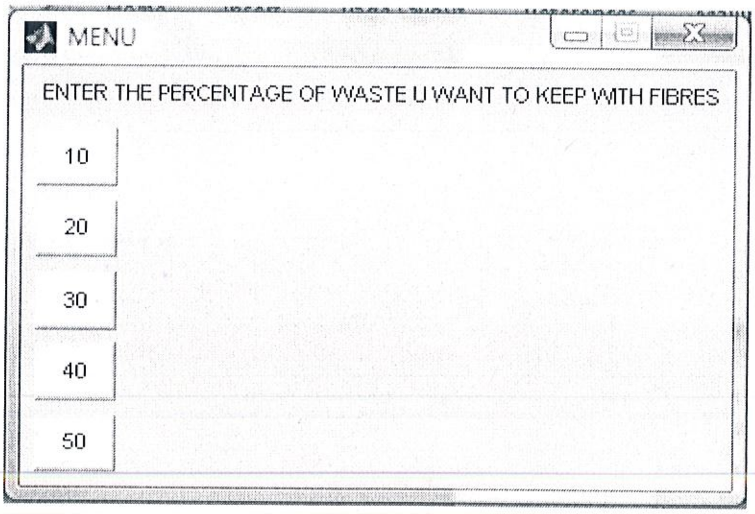
(V) By this time the program has calculated the amount of cement, aggregate and sand required for PCC. Now user has to enter the amount of fly ash that is to be varied with cement.



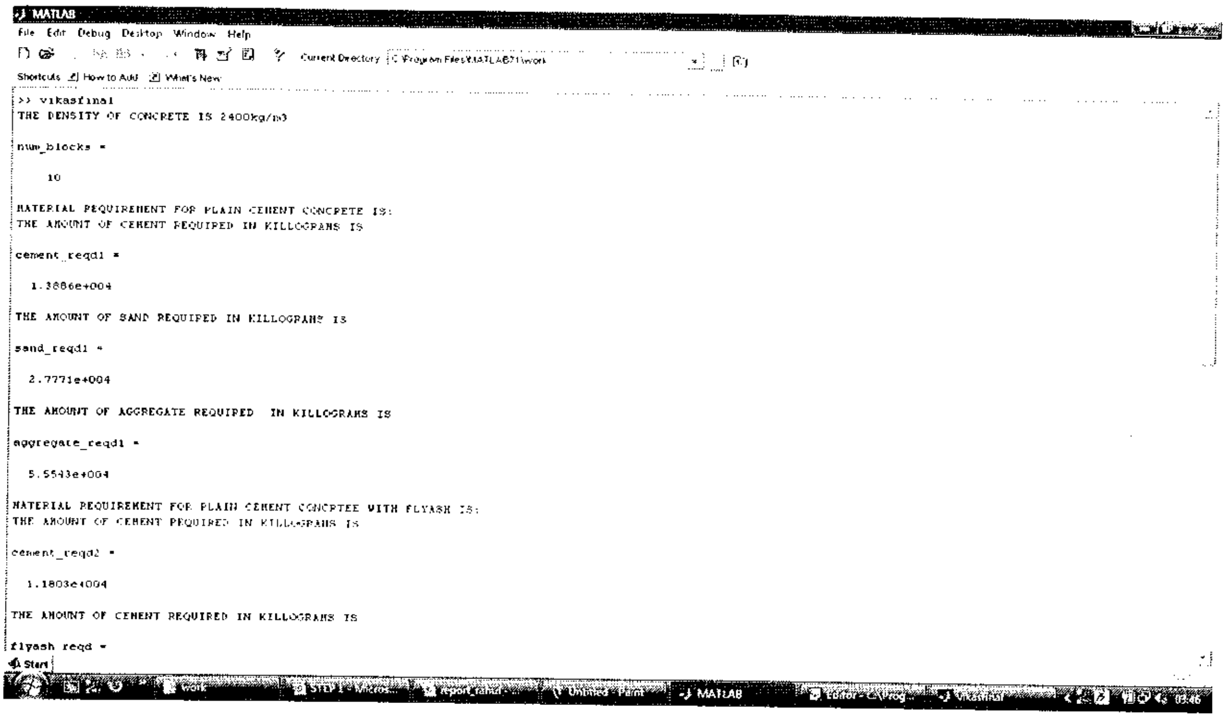
(VI) After this the amount of material required in PCC with fly ash is calculated. Now program will calculate the material requirement for SFRC for this user has to enter the amount of fibres that is to be mixed.

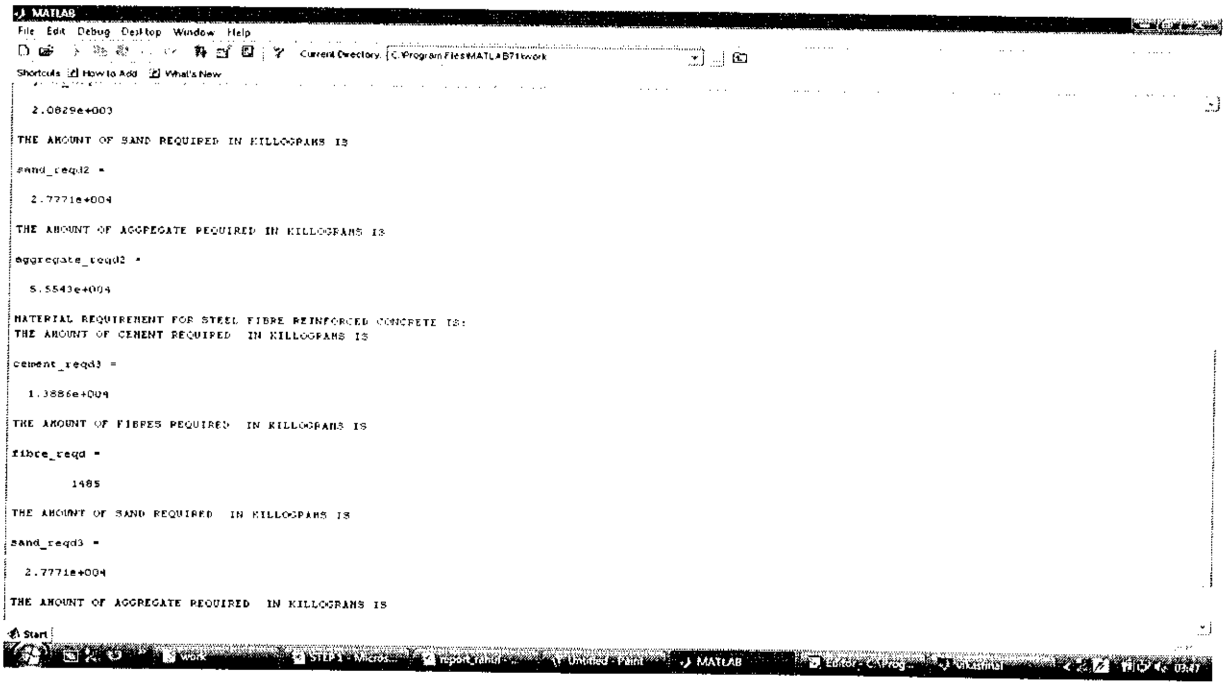


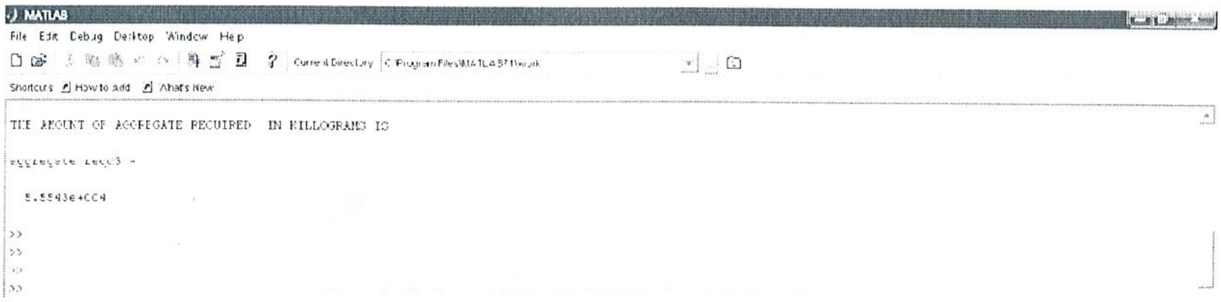
(VII) As fibres are very small in size and light weight, therefore a certain percentage of waste is added to calculate the total amount of fibres required.



(VIII) The amount of cement, sand, aggregate, fly ash and fibres are displayed in the command window of MATLAB as shown in figure below.

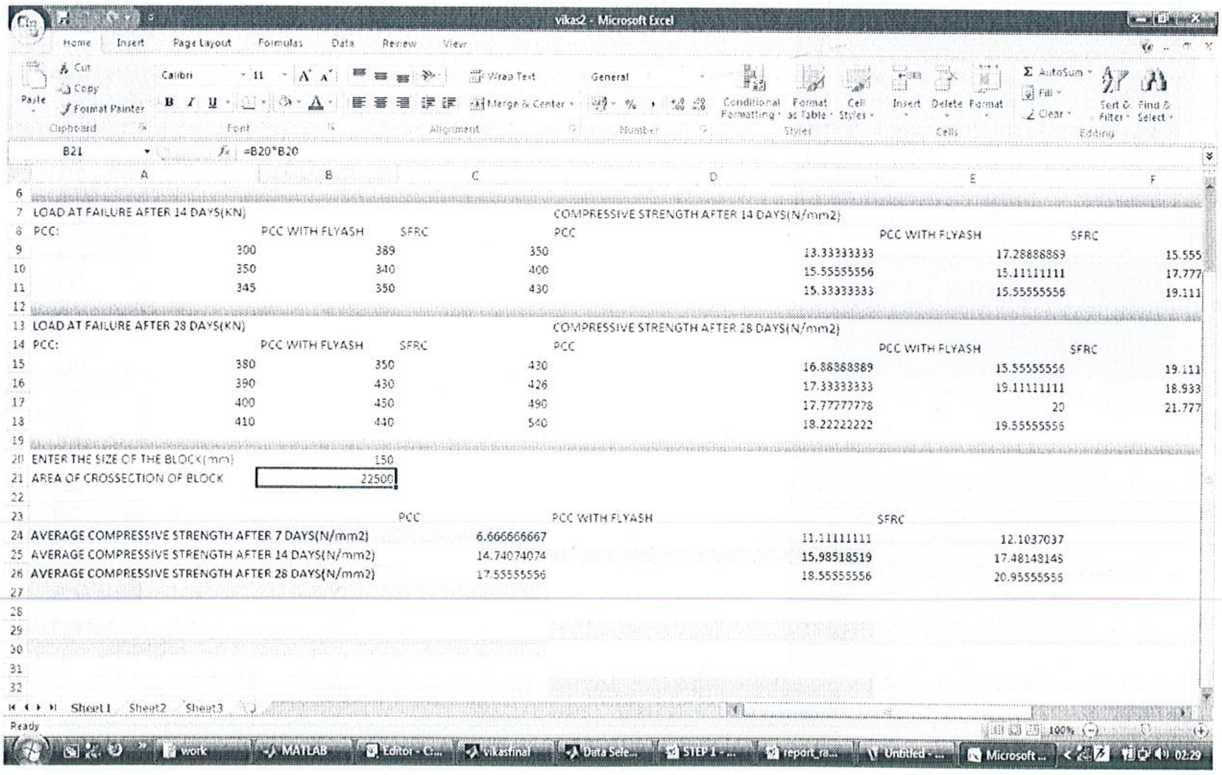




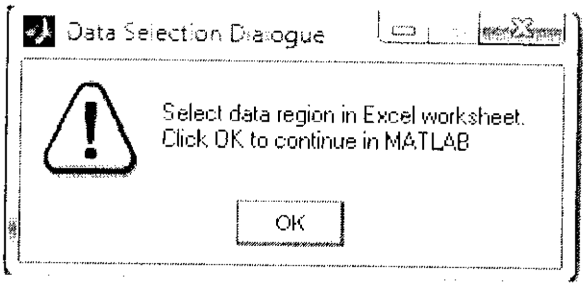


STEP 3: -

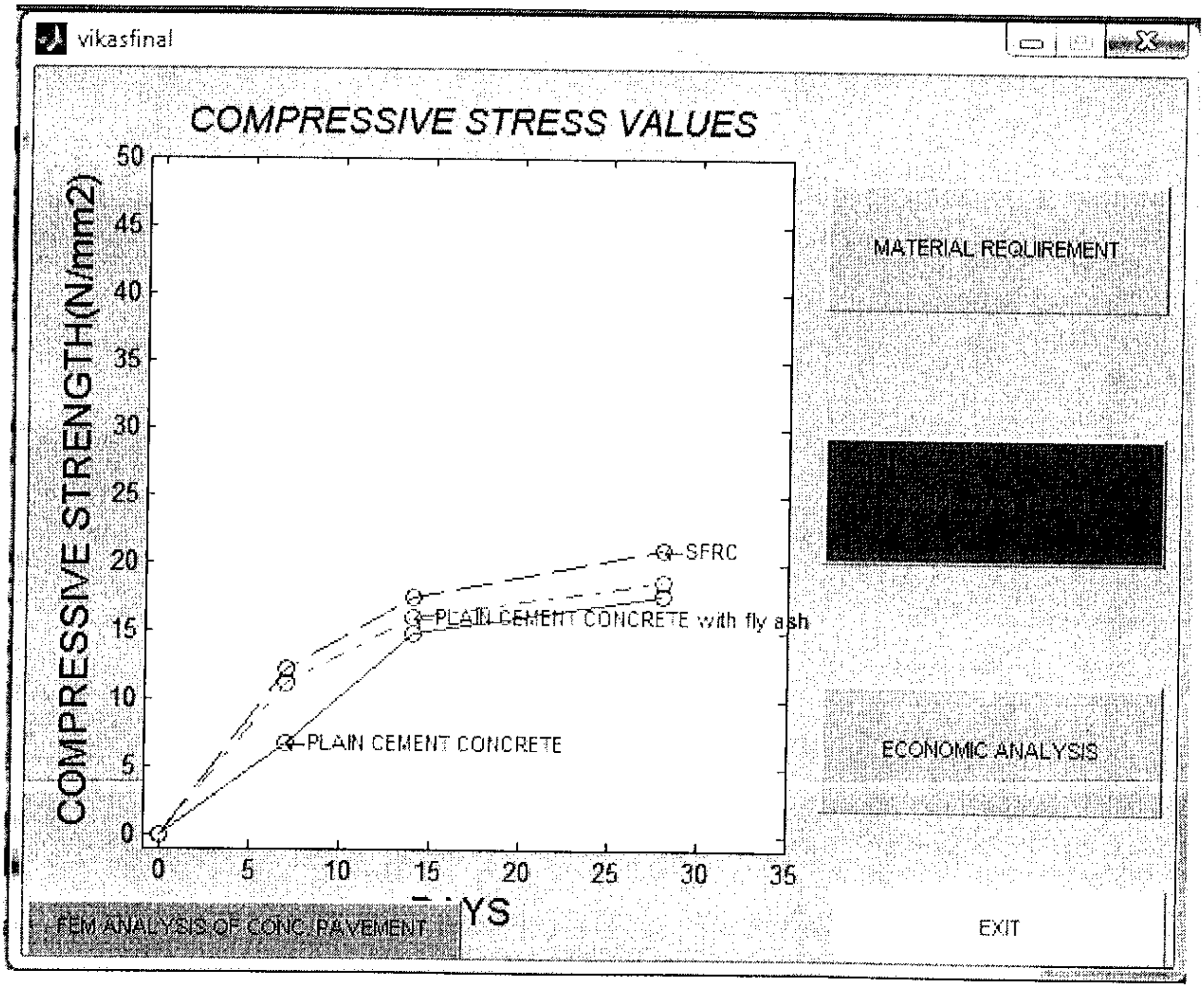
When user clicks on the strength analysis push button than the program opens an excel sheet, in which the values of lab test are to be entered. The sheet comprises of a table in which the load carrying capacity data as obtained from the compressive strength machine of PCC, PCC with fly ash and SFRC are to be entered for 7 days, 14 days and 28 days. Also the size of block has to be entered with its unit in mm. Once the user has entered all the values the compressive strength for all corresponding load values are calculated. The average compressive strength values are calculated in the lower part of the sheet. The inputs for the program in this case are the experimental breaking load values obtained from the compressive strength test. The output generated in this step is the average compressive strength values of the three mixes after 7, 14 and 28 days.



The user is required to select all these average compressive strength values and then press ok in the given menu.

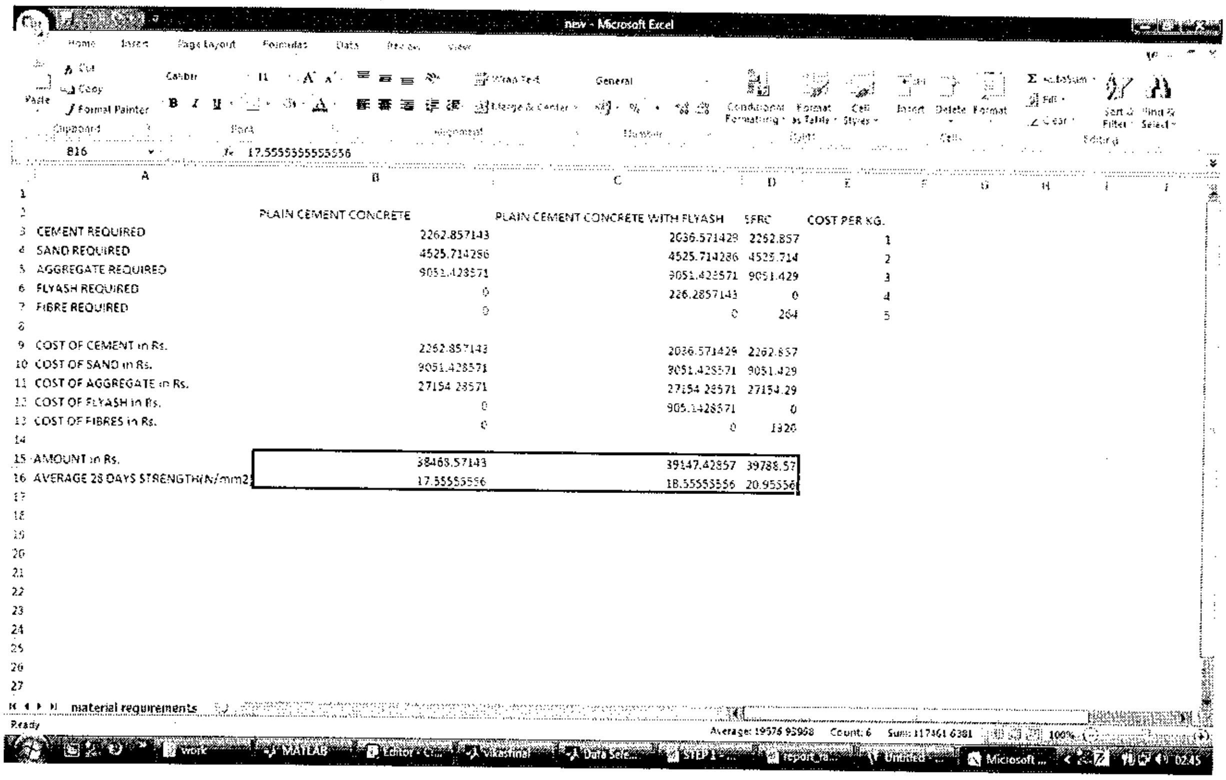


Once the user has pressed ok a graph showing relative compressive strength of PCC, PCC with fly ash and SFRC is displayed on the GUI chart menu.

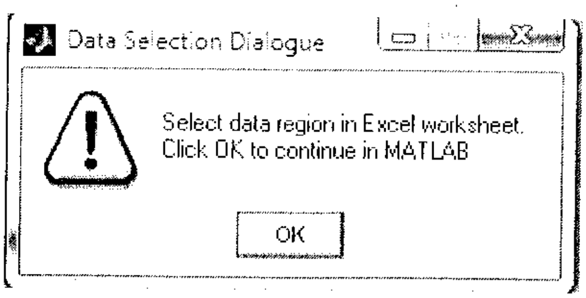


STEP 4: -

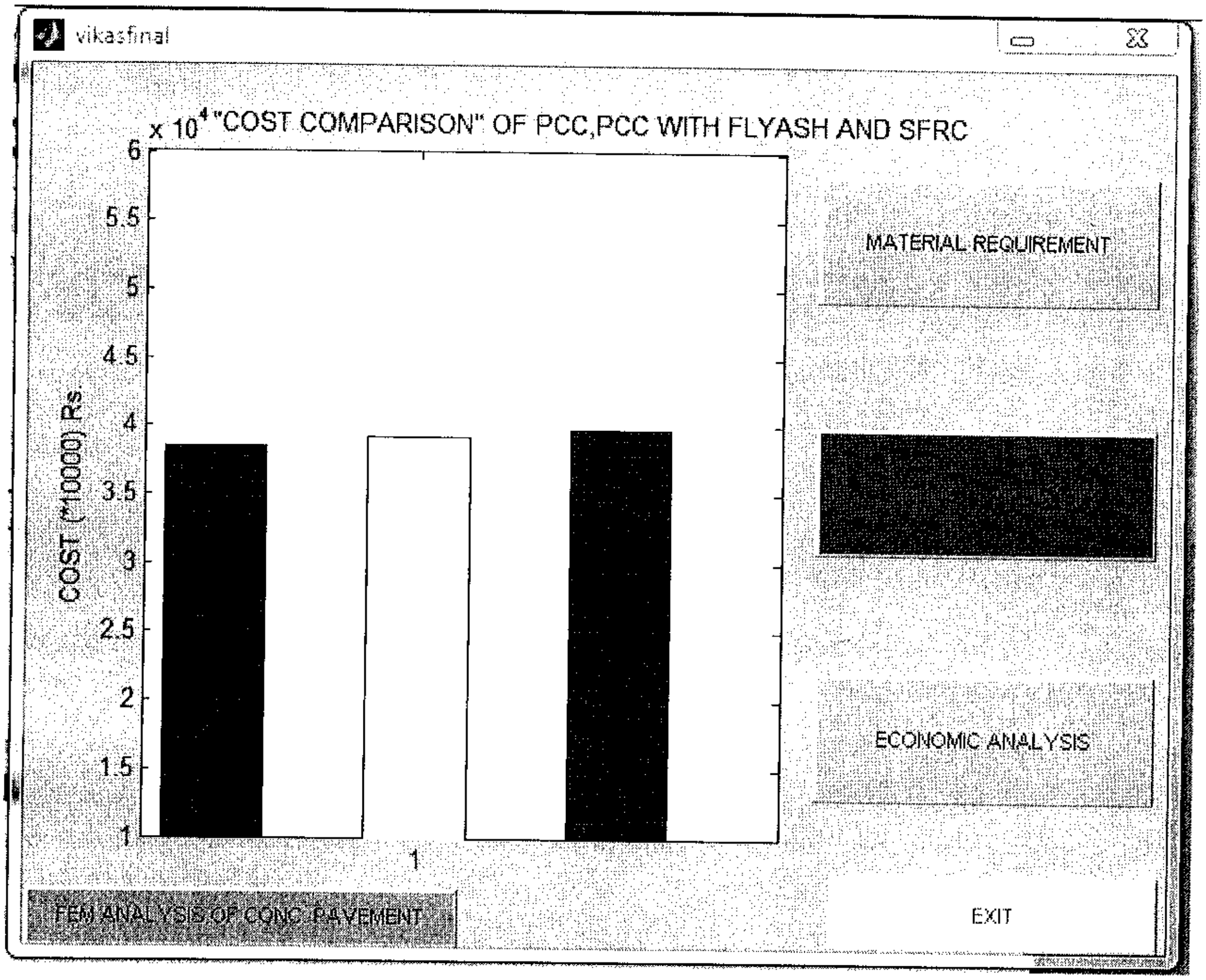
This part of the program takes in the values displayed in the material requirement as input in the economic analysis of the mixes. On clicking this push button all the material requirements for PCC, PCC with fly ash and SFRC are written on a new excel sheet. This sheet has a column which requires the user to input the current market value of cement, sand, coarse aggregate, fly ash and fibres in the unit specified. Once these values are inputted the excel program calculates the cost of preparing the three mixes separately. The user is required to select these values from the sheet. The inputs in this step are the compressive strength values of the mixes after 7, 14 and 28 days. The material requirement for designing the mix are also incorporated in this excel sheet. The user is required to input the market rates of all the materials used in the mix. The output of this step is the comparative cost and strength analysis of the three mixes.



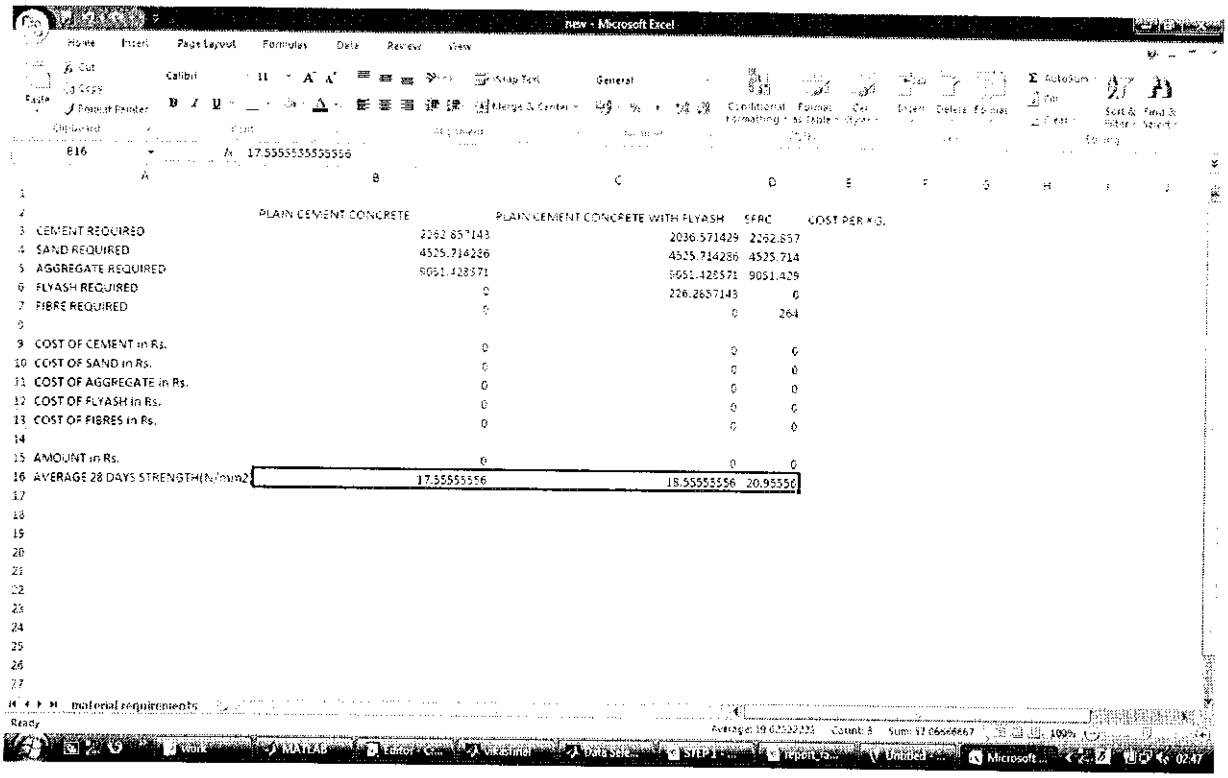
After selecting the entire values user is required to click ok on the following dialogue box.



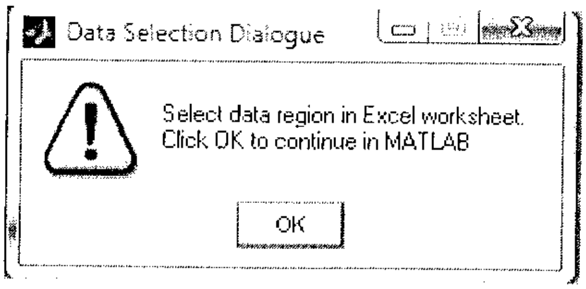
Once user has selected these values the GUI menu prepares a bar chart showing the relative cost of the three mixes.



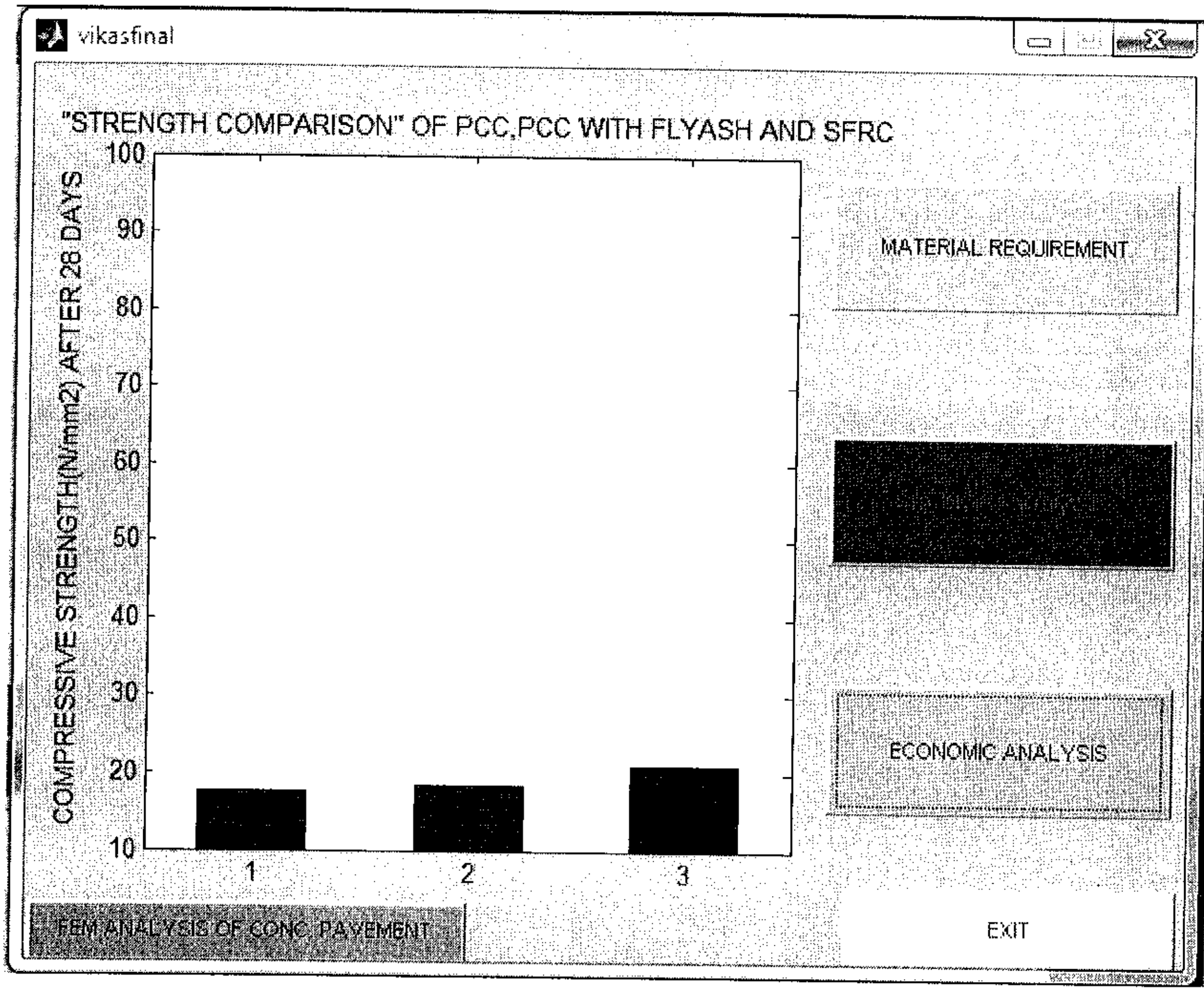
As cost alone is of no use in determining the advantages of using a mix, hence to show the relative strength that can be generated by this mix the program opens an another excel sheet and this sheet takes in as input the average compressive strength of three mixes after 28 days.



After selecting all the values user is required to click ok on the given dialogue box that also opens with the excel sheet.

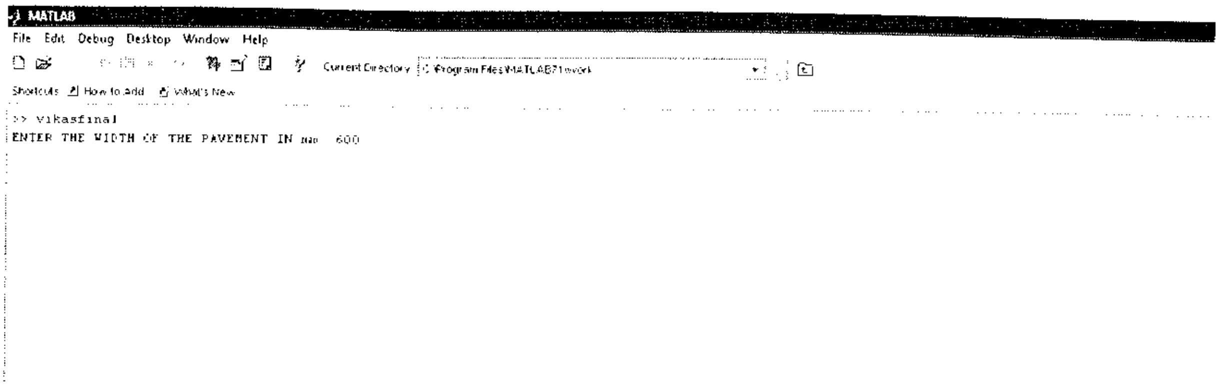


The user has to select these values and the graphical user interface draws an another bar chart showing the relative strength of these mixes.

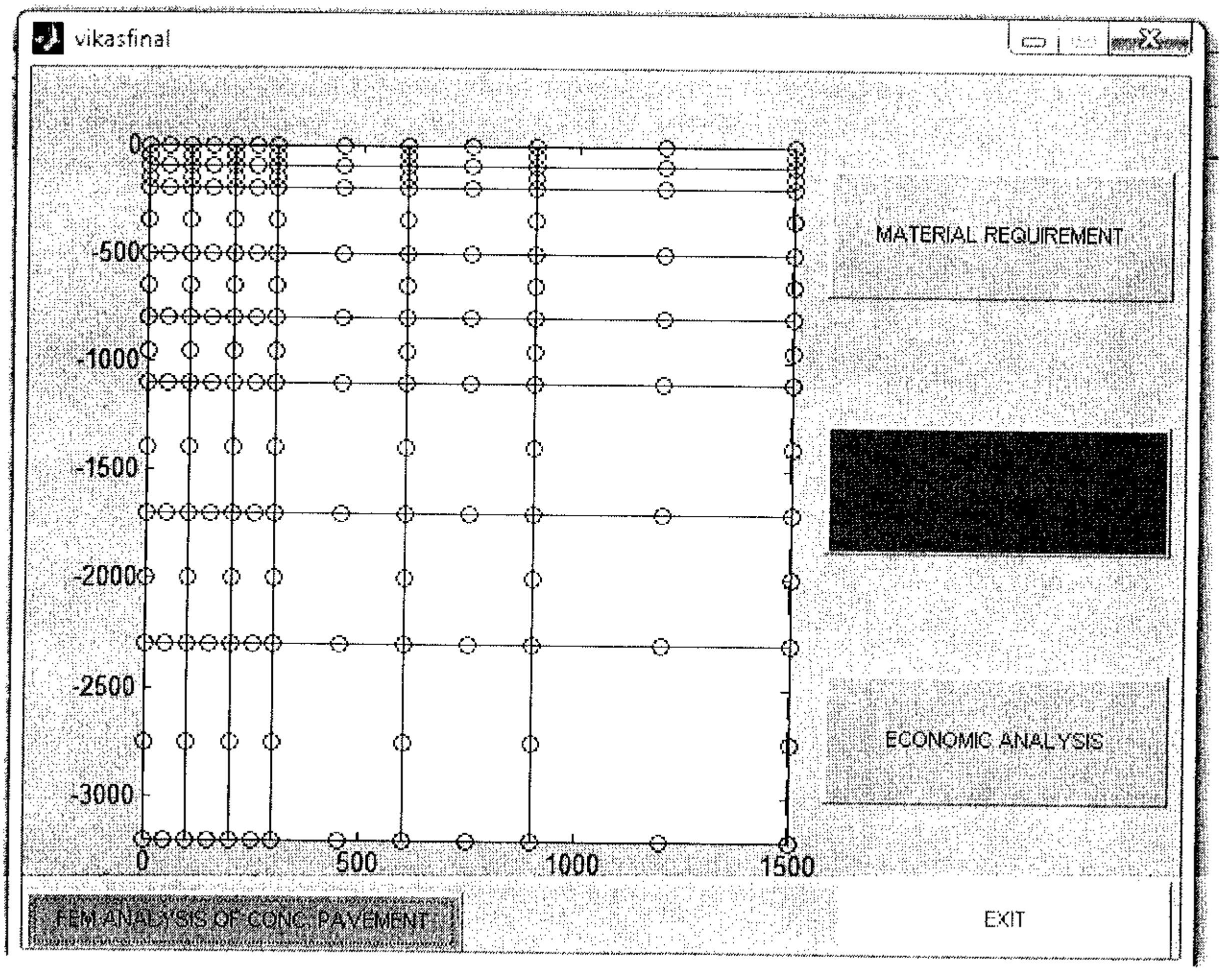


STEP 5: -

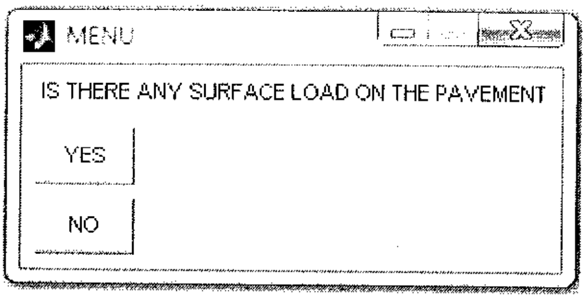
In this step the main part of the program is executed by pressing of the FEM Analysis push buttons. On clicking this button user is asked to input the thickness of the pavement in mm.



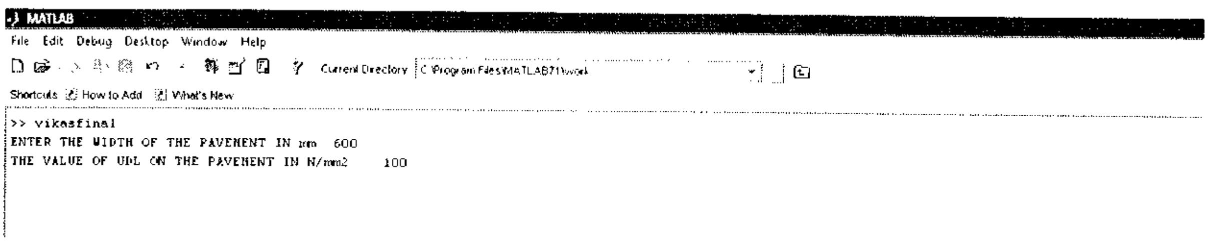
Once all the loads are entered the graphical user interface chart displays a mesh. This mesh can be used to see the discretization of the subgrade. There are all together 48 elements and each containing 8 nodes making a total of 173 nodes.The total length of mesh in horizontal direction is 5 times of the half of the pavement thickness and in vertical direction is 11 times the half of the pavement thickness (32).



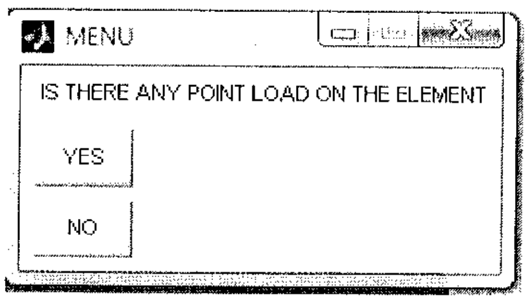
As soon as the value for pavement thickness is entered a dialogue box appears on the screen. Its ask the user to verify whether there is any surface load on the pavement or not.



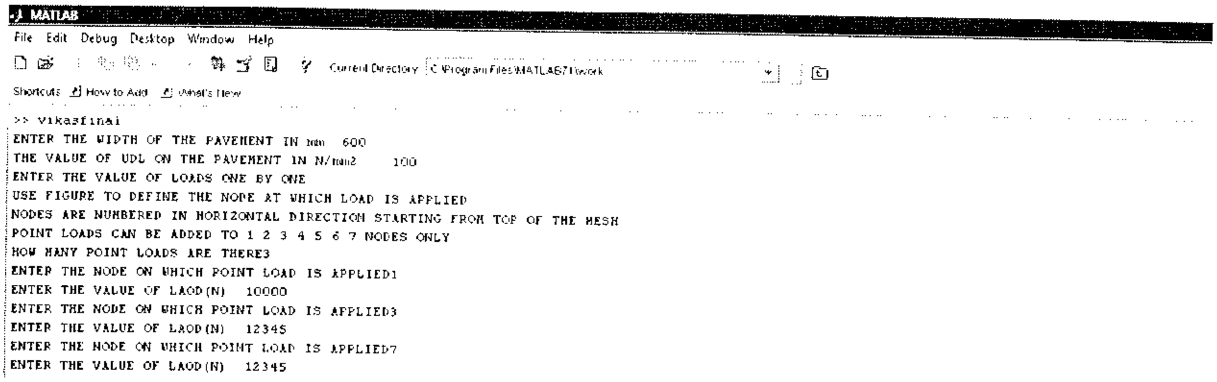
If the user press yes than he is asked for value of this load in N/mm^2.



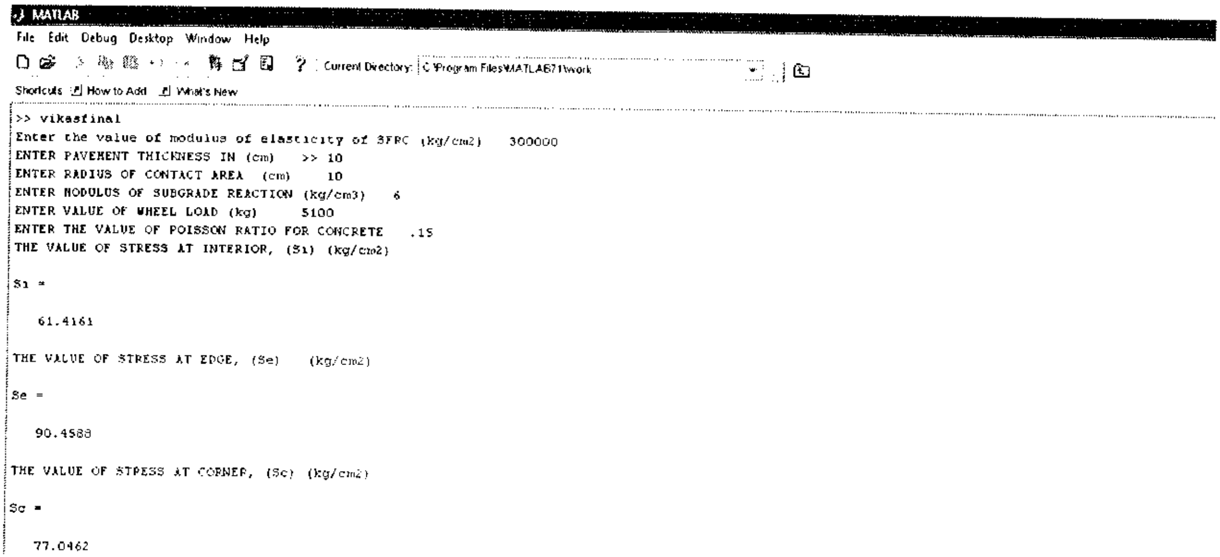
If the user enter no than a new dialogue box appears asking the user if there is any point load on the pavement or not.



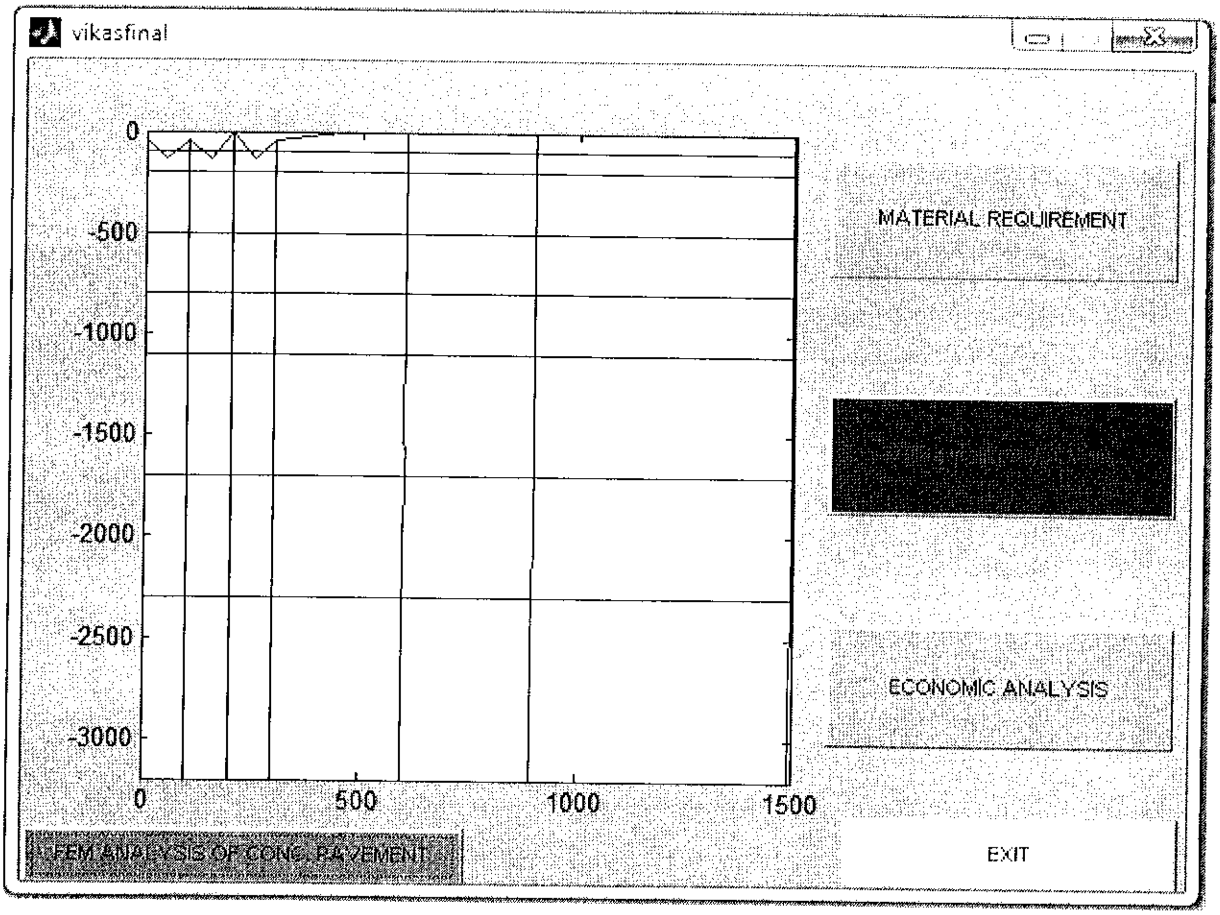
If there is any point load then user has to press yes. On pressing yes the user is asked how many point loads are there and the node number and value of each load in kN has to be entered by the user.



The output of the Stresss due to corner, edge and interior laoding as obtained from the program for thickness of pavement, h = 10cm is shown below.



The displacement in the compacted subgrade is very small and so to get the deflected shape all the values of displacement were multiplied with 50 to get the deflected shape. The deflected shape of the finite element mesh is as shown here.



The flow chart for the above MATLAB Program is given below:

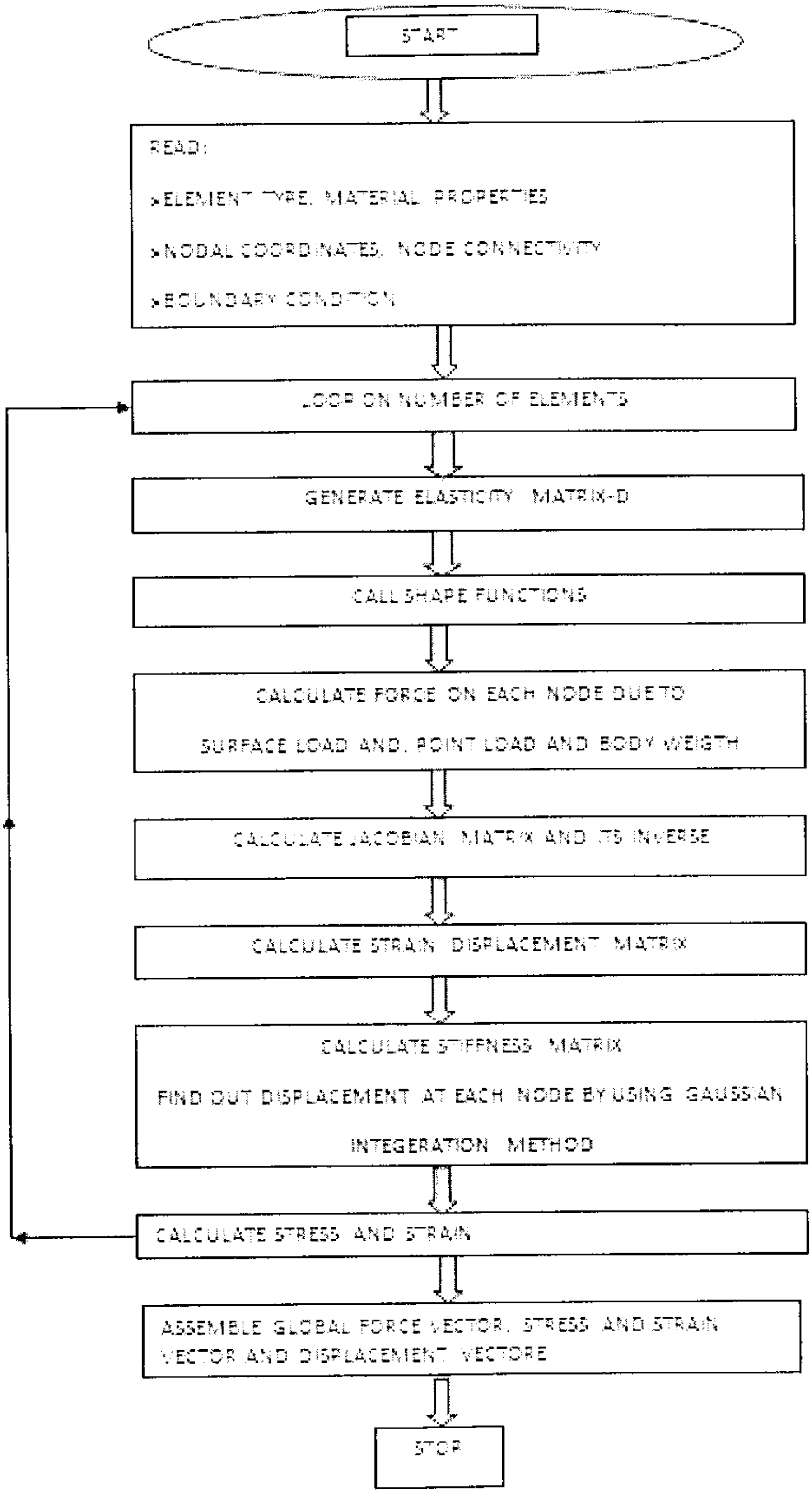


Figure 6.7: - Flow Chart for MATLAB Program

6.5 ANALYSIS OF RESULTS USING MATLAB

The results of MATLAB program using Westergaard theory of pavement analysis are verified using the results of theoretical analysis as per clause 5.3. The data for the program and the Westergaard theory was taken as follows and the difference between the two results are shown in the table below.

Given:-

Modulus of Elasticity of concrete, E (kg/cm^2) = 300000

Enter pavement thickness, h (cm.) = 10

Enter Radius of Contact Area, a (cm.) = 10

Enter Value of Subgrade Reaction, K (kg/cm^3) = 6

Enter Value of Wheel Load, P (kg) = 5100

Enter Value of Poisson's Ratio for Concrete, μ = 0.15

Table 6.1:- **Comparison of theoretical value and value obtained from the program; h = 10cm**

S. No.	Stress	Westergaard Theory (A)	MATLAB Analysis (B)	Difference in the result (A-B)
1.	Interior, S_i (kg/cm^2)	61.43	61.416	0.014
2.	Edge, S_e (kg/cm^2)	90.485	90.458	0.027
3.	Corner, S_c (kg/cm^2)	77.048	77.046	0.002

So from the above table it is observed that the difference in results is very minute and approximately equals to zero.

The factor of safety for the above pavement thickness is 0.633which is less than 1.3 as determined in the chapter 5 (clause 5.3). Therefore the above pavement has to be redesigned for a higher pavement thickness.

Taking h = 16 cm,

The output of the MATLAB Program is shown in the figure below and compared with the theoretical value in table 6.2

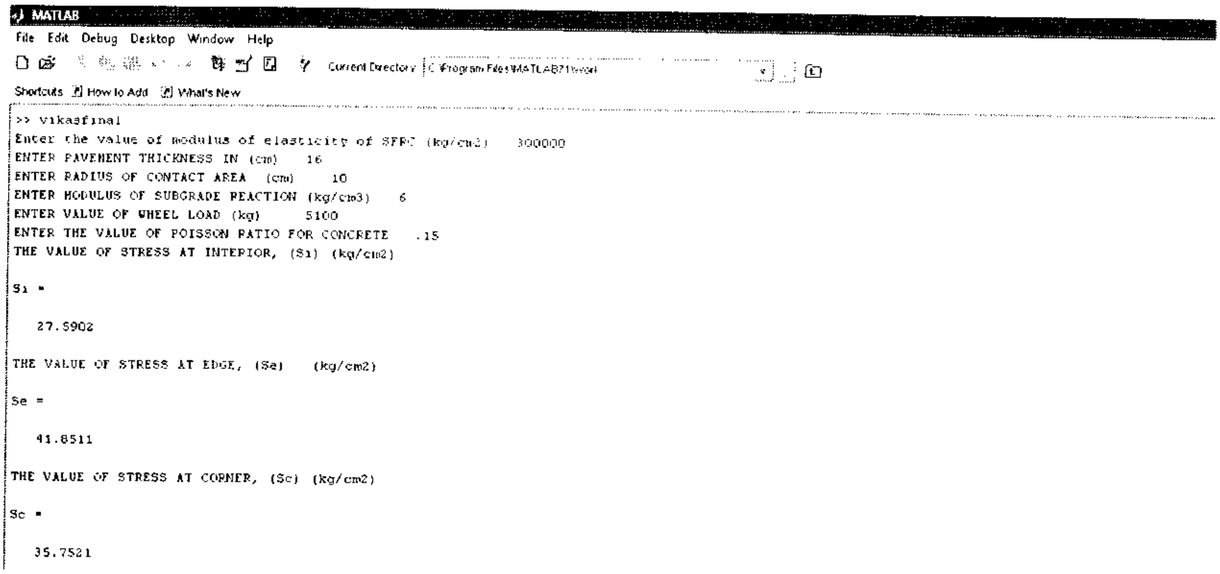


Table 6.2:- **Comparison of theoretical value and value obtained from the program; h = 16cm**

S. No.	Stress	Westergaard Theory (A)	MATLAB Analysis (B)	Difference in the result (A-B)
1.	Interior, S_i (kg/cm^2)	27.586	27.590	-0.004
2.	Edge, S_e (kg/cm^2)	41.843	41.851	-0.008
3.	Corner, S_c (kg/cm^2)	37.752	35.752	0

The factor of safety for the pavement thickness, h = 16 cm 1.43 which is greater than 1.3. So the pavement is safe for maximum wheel load of 5100 kg. These results are expected to be design the pavement for flexural strength in the range of 60 kg/cm^2 to 70 kg/cm^2.

6.6 CONCLUSION

In the present study, the finite element method has been used to evaluate the stresses and deflections in PCC and SFRC pavements resting over subgrade under central loading conditions using MATLAB Program with the use of an isoparametric quadratic serendipity element. The results have been compared with Westergaard theory for a pavement of thickness 100 mm and 160 mm.

The values for central loading condition were used as input in the MATLAB program and the values of stress, strain and deflection along a finite depth of subgrade have been calculated. The theoretical values of stresses using Westergaard method are compared with the values given by program and the difference between the values obtained from both is found to be zero. This concludes that the program made for the analysis of pavements design is validated. The pavement was found to be safe for a pavement thickness of 160 mm and results are expected to design the pavement for flexural strength in the range of 60 to 70 kg/cm^2.

CHAPTER - 7

CONCLUSIONS

7.1 GENERAL

Steel fibres are the most commonly used out of a wide range of fibre in use. As a result of numerous experimental investigations, SFRC has been recognized as a promising composite material possessing greatly improved strength characteristics and its potential use for highway and airfield pavements. When the fibre reinforcement is in the form of short discrete fibres, they act effectively as rigid inclusions in the concrete matrix. Physically, they have thus the same order of magnitude as aggregate inclusions; steel fibre reinforcement cannot therefore be regarded as a direct replacement of longitudinal reinforcement in reinforced and prestressed structural members. However, because of the inherent material properties of fibre concrete, the presence of fibres in the body of the concrete or the provision of a tensile skin of fibre concrete can be expected to improve the resistance of conventionally reinforced structural members to cracking, deflection and other serviceability conditions. The results obtained from the present study are discussed earlier and the precaution taken during the experimental program life time and the coding is discussed in the present chapter.

7.2 PCC AND SFRC MIXES AND THEIR PERFORMANCE

The steel fiber reinforced concrete mixes with fiber volume percentage of 0.5%, 1.0% and 1.5% showed higher compressive strength after 28 days as compared to plain cement concrete mixes. Beyond 1.5% the proper compaction of mix was not achievable, hence they were avoided. A maximum increase of 11.69% and 11.14% in compressive strength was observed for a fiber volume of 1.5% over that of PCC mix for a mix 1:1:2 and 1:1.5:3 respectively after 28 days for a block of size 150 x 150 x 150 mm. The w/c ratio was taken as 0.6 for SFRC mixes and it was found that the workability was just sufficient to enable the concrete to be mixed and compacted fully without using any admixtures. The comparative compressive strength values for PCC and SFRC mixed after 7 days and 28 days are given in table 4.4 and 4.5.

7.3 MIXES USING FLY ASH

The cement content was varied with 15% of fly ash for the mixes. This was basically done to prove that addition of fly ash not only reduces the cost but also the strength of the mix is unaffected by this variation in cement content. The fly ash mixes shows a very slight increase in compressive strength of 2.1% over that of plain cement concrete mixes after 28 days. I t was found that the use of fly ash in the mix was found to increase the workability of the mix and it also reduces the w/c ratio. No problem was faced in mixing the concrete mixes using fly ash.

7.4 ANALYSIS USING WESTERGAARD

The Westergaard method was used to determine the stresses on a pavement due to maximum wheel load application. The stresses were calculated at interior, corner and edge. The pavement thickness of 10 cm. was used for the Westergaard analysis under maximum wheel load of 5100 kg and for flexural strength of 60 kg/cm^2. It was inferred from the analysis that the pavement thickness was not appropriate for carrying residual concrete strength and hence was unsafe for designing the pavement. The analysis was again done for higher pavement thickness of 16 cm. and it was found that it was safe for carrying maximum wheel load.

7.5 ANALYSIS USING MATLAB

A MATLAB source code has been prepared for finite element analysis of rigid pavements and subgrade beneath. The program calculates the stresses for edge, corner and interior loading of a pavement and also distributes the stresses to the subgrade beneath. The deflection, stress and strain caused due to these stresses on the pavement on each node of the isoparametric quadratic serendipity element is calculated in the pavement. The result for deflection, stress and strain on the subgrade has been shown in Appendix II.

The MATLAB code is also verified with the stresses obtained from the Westergaard analysis for pavement thickness of 10 and 16 cm. The results of verification have been tabulated in table 6.1

and 6.2. It was found that the differences in results were very less and approximately equals to zero.

7.6 PRECAUTIONS

The present study deals with analysis of steel fibre reinforced concrete and its economic analysis. The pavement was analyzed theoretically, with experiments carried out on blocks of size 150mm and not on pavement slabs. The major precautions to be kept during the experimental program were regarding the weather conditions. The ever changing climatic condition makes it difficult to prevent moisture ingress in the materials and hence a lot of care was required to handle the moisture from entering the materials components. The other necessity was that the components of the mix were free from dust and other impurities. To make sure that the mix components cement, sand and fly ash were cleaned and sieved properly, whereas the coarse aggregate were washed thoroughly and dried before mixing. The amount of waste to be counted with each mix preparation was also difficult to calculate. The waste percentage should be appropriate to prevent the wastage of material. To obtain the compressive strength the mix should be properly compacted. To obtain proper compaction the mix was rammed with a steel rod and then kept on the motor of the mixer for 10-20 seconds. This allows the proper compaction of the mix.

The second most important feature of this experiment study was the MATLAB program. The program was efficient for material requirement, economic and strength analysis but the main problem is to analyze the pavement using MATLAB. This is due to the fact that probably MATLAB is not the proper software for this analysis and has many limitations. Other software's that can be used for this analysis are ANSYS 5.0, ANSYS 10.0, etc. The deflection of the subgrade could not be shown in the form of a graph and this was basically due to the fact that the element taken is a 2-d element and there were no provisions (in the best of my knowledge) of plotting a 2-D element displacement in form of a surface plot. The results were verified for central loading condition of Westergaard's theory.

REFERENCES

1. Mechanics of crack arrest in concrete, Romualdi, J.P. and Batson, G.B. 1963.
2. Romualdi, J.P. and Batson, G.B. 1964, ACI Journal, Vol. 61, No. 6
3. Highway Engineering, By S.K. Khanna and C.E.G Justo (1971).
4. C.D. Johnston, "Steel fiber reinforced mortar and concrete", a review of mechanical properties. In fiber reinforced concrete ACI – SP 44 – Detroit 1974.
5. American Concrete Institute (ACI), 1982, State of the Art Report on Fiber Reinforced Concrete, ACI 544.1R – 82, 22p.
6. ASTM C1018 – 89, Standard Test Method for Flexural Toughness and First Crack Strength of Fibre Reinforced Concrete,1991 Book of ASTM Standards, Part 04.02, American Society for Testing and Materials, Philadelphia, pp.507 – 513
7. Vasan, R.M., Steel Fiber Reinforced Concrete, PhD Thesis (1989).
8. Fibre Reinforced Cement Composites , Perumalsamy N. Balaguru & Surendra P. Shah(1992), McGraw-Hill, Inc.
9. Carbon Fiber Reinforced Concrete, Dr. Deborah D.L Chung, SHRP-ID/UFR-92-605, 1992.
10. Olley, P. 1992. Cork. In: Doran, D.K. (Ed.). Construction Materials Reference Book, Chapter 26. Butterworth, Oxford. pp. 26/1-5.
11. Maidl, B. R. Steel fibre reinforced concrete, Berlin: Ernst & Sohn (1995).
12. Concrete Construction Engineering Handbook, Edward G. Nawy (1997), the State University of New Jersey.
13. IRC:SP:46-1997 Steel Fibre Reinforced Concrete for Pavements
14. Gibson, L.J. & Ashby, M.F. (1999). Cellular Solids: Structure and Properties; 2nd Edition (Paperback), Cambridge University.
15. Colin D. Johnston, "Fiber reinforced cements and concretes" Advances in concrete technology volume 3 – Gordon and Breach Science publishes – 2001
16. D.J. Fiber cements and Fiber concrete, Hannant.
17. Reinforced Concrete Design, S N SINHA
18. Steel Fibre Reinforced Concrete, NGUYEN VAN CHANH
19. www.encylopedia.com: The World Book Encyclopedia

20. Desai and Abel, Finite Element Analysis, 1972.
21. Flexible Pavement Manual, American Coal Ash Association, Washington, DC.
22. Use of Coal Ash in Embankments and Bases, U.S. Department of Transportation, Federal Highway Administration, Washington, DC
23. www.trb.org: Research papers
24. Westergaard, H.M., Theory of Concrete Pavement Design, HRB proc. publication.
25. Teller, L.W. and Sutherland, E.C., The Structural Design of Concrete Pavements, Public roads, Washington.
26. Westergaard, H.M., Stresses in Concrete Pavements Computed by Theoretical Analysis, Public roads, vol. 7, 1927.
27. IRC:58-1988, Guidelines for Design of Rigid Pavements.
28. IRC:15-2002, Standard Specification and Code of Practice for Construction of Concrete roads.
29. Seehra, S S, Investigation on Rollcrete Pavement Layers.
30. A K Mullick, Pramod Walia and S N Sharma applications of polypropylene fiber reinforced concrete (pfrc) with vacuum processing.
31. Sargious Michael and Wang, S.K., Rigid Pavement Design Charts based on Finite Element Analysis, ACI Publication, SP-51, Detroit Michigan, USA, 1974.
32. Bhavikati, S.S., Finite Element Analysis, New Age International Publishers.

APPENDIX-I

The MATLAB source code for FEM Analysis of Rigid pavements and subgrade beneath:

```
%PROGRAM STARTS

function varargout = analysis(varargin)

gui_Singleton = 1;
gui_State = struct('gui_Name',       mfilename, ...
                   'gui_Singleton',  gui_Singleton, ...
                   'gui_OpeningFcn', @analysis_OpeningFcn, ...
                   'gui_OutputFcn',  @analysis_OutputFcn, ...
                   'gui_LayoutFcn',  [], ...
                   'gui_Callback',   []);
if nargin && ischar(varargin{1})
   gui_State.gui_Callback = str2func(varargin{1});
end

if nargout
    [varargout{1:nargout}] = gui_mainfcn(gui_State, varargin{:});
else
    gui_mainfcn(gui_State, varargin{:});
end
function analysis_OpeningFcn(hObject, eventdata, handles, varargin)
handles.output = hObject;

% Update handles structure
guidata(hObject, handles);

% --- Outputs from this function are returned to the command line.
function varargout = analysis_OutputFcn(hObject, eventdata, handles)
varargout{1} = handles.output;

% --- Executes on button press in pushbutton6 (strength analysis)
function pushbutton6_Callback(hObject, eventdata, handles)
global output;
global num;
num = xlsread('EXCEL2.xlsx', -1)
s=[0 7 14 28 ]

for i=1:3
   for j=1:3
      str(1,j)=0;c=i+1;
      str(c,j)=num(i,j);
   end
end

r=str(:,1);plot (s,r,'-ob')

c=str(2,1);d=str(3,2);e=str(4,3);
text(7,c,...
   '\leftarrowPLAIN CEMENT CONCRETE',...
   'FontSize',8)
text(14,d,...
   '\leftarrowPLAIN CEMENT CONCRETE with fly ash',...
   'FontSize',8)
text(28,e,...
   '\leftarrowSFRC',...
```

```
'FontSize',8)

xlim([-1 35])
ylim([-1 50])
xlabel('DAYS','FontSize',14)
ylabel('COMPRESSIVE STRENGTH(N/mm2)','FontSize',14)
title('\it{COMPRESSIVE STRESS VALUES}','Fontsize',14)

hold on
r1=str(:,2);r2=str(:,3);
plot(s,r1,'-or')

plot(s,r2','--ok')

% --- Executes on button press in pushbutton9 (finte element analysis)
function pushbutton9_Callback(hObject, eventdata, handles)
global output;
clear
num_elements=48;num_nodes_elements=8;num_dof_node=2;total_nodes=173;
total_syst_dof=num_dof_node*total_nodes;
single_elem_dof=num_nodes_elements*num_dof_node;

% Westergaard Analysis

E=input('Enter the value of modulus of elasticity of SFRC (kg/cm2)   ');
h=input('ENTER PAVEMENT THICKNESS IN (cm)   ');
a=input('ENTER RADIUS OF CONTACT AREA  (cm)    ');
K=input('ENTER MODULUS OF SUBGRADE REACTION (kg/cm3)   ');
P=input('ENTER VALUE OF WHEEL LOAD (kg)     ');
U=input('ENTER THE VALUE OF POISSON RATIO FOR CONCRETE   ');

l23=E*h*h*h;u1=U*U;u2=1-u1;den1=12*K*u2;rat1=l23/den1;l=(rat1)^(1/4);

va1=h*h;va2=1.6*a*a;va3=va2+va1;va4=sqrt(va3);va5=.675*h;c32=a/h;
if c32<1.724
   b=va4-va5;
end
if c32>=1.724
   b=a;
end

cal1=.316*P;cal2=l/b;cal3=log10(cal2);cal4=4*cal3;
cal5=cal4+1.069;cal6=cal1/va1;Si=cal6*cal5;
disp('THE VALUE OF STRESS AT INTERIOR, (Si) (kg/cm2) ');
Si
cal7=cal4+.359;cal8=.572*P;cal9=cal8/va1;Se=cal9*cal7;
disp('THE VALUE OF STRESS AT EDGE, (Se)   (kg/cm2)   ');
Se

cal10=3*P;cal11=cal10/va1;cal12=a*sqrt(2);cal13=(cal12/l);cal14=cal13^.6;cal15=1-cal14;
 Sc=cal11*cal15;
disp('THE VALUE OF STRESS AT CORNER, (Sc) (kg/cm2)   ');
Sc

width=input('ENTER THE WIDTH OF THE PAVEMENT IN mm  ');
a=width/2;

%COLUMNS
c1=0;c2=a/6;c3=2*c2;c4=3*c2;c5=4*c2;c6=5*c2;c7=a;c8=3*a/2;c9=2*a;c10=5*a/2;
c11=3*a;c12=4*a;c13=5*a;

%ROWS
```

```
r1=0;r2=-50;r3=-100;r4=-150;r5=-200;r6=-((a/2)+200);r7=-(a+200);
r8=-((3*a/2)+200);r9=-(2*a+200);r10=-((5*a/2)+200);r11=-(3*a+200);
r12=-(4*a+200);r13=-(5*a+200);r14=-(6*a+200);r15=-(7*a+200);
r16=-(8.5*a+200);r17=-(10*a+200);

%INPUTTING DATA FOR NODAL COORDINATE VALUES
%GLOBAL CORDINATE(i,j)where i ------> the node no. and j ----------->x or y

global_coordinate=[c1 r1;c2 r1;c3 r1;c4 r1;c5 r1;c6 r1;c7 r1;c8 r1;c9 r1;c10 r1;c11 r1;c12 r1;c13 r1;
            c1 r2;c3 r2;c5 r2;c7 r2;c9 r2;c11 r2;c13 r2;
            c1 r3;c2 r3;c3 r3; c4 r3;c5 r3;c6 r3;c7 r3;c8 r3;c9 r3;c10 r3;c11 r3;c12 r3;c13 r3;
            c1 r4;c3 r4;c5 r4;c7 r4;c9 r4;c11 r4;c13 r4;
            c1 r5;c2 r5;c3 r5;c4 r5;c5 r5;c6 r5;c7 r5;c8 r5;c9 r5;c10 r5;c11 r5;c12 r5;c13 r5;
            c1 r6;c3 r6;c5 r6;c7 r6;c9 r6;c11 r6;c13 r6;
            c1 r7;c2 r7;c3 r7;c4 r7;c5 r7;c6 r7;c7 r7;c8 r7;c9 r7;c10 r7;c11 r7;c12 r7;c13 r7;
            c1 r8;c3 r8;c5 r8;c7 r8;c9 r8;c11 r8;c13 r8;
            c1 r9;c2 r9;c3 r9;c4 r9;c5 r9;c6 r9;c7 r9;c8 r9;c9 r9;c10 r9;c11 r9;c12 r9;c13 r9;
            c1 r10;c3 r10;c5 r10;c7 r10;c9 r10;c11 r10;c13 r10;
            c1 r11;c2 r11;c3 r11;c4 r11;c5 r11;c6 r11;c7 r11;c8 r11;c9 r11;c10 r11;c11 r11;c12 r11;c13 r11;
            c1 r12;c3 r12;c5 r12;c7 r12;c9 r12;c11 r12;c13 r12;
            c1 r13;c2 r13;c3 r13;c4 r13;c5 r13;c6 r13;c7 r13;c8 r13;c9 r13;c10 r13;c11 r13;c12 r13;c13 r13;
            c1 r14;c3 r14;c5 r14;c7 r14;c9 r14;c11 r14;c13 r14;
            c1 r15;c2 r15;c3 r15;c4 r15;c5 r15;c6 r15;c7 r15;c8 r15;c9 r15;c10 r15;c11 r15;c12 r15;c13 r15;
            c1 r16;c3 r16;c5 r16;c7 r16;c9 r16;c11 r16;c13 r16;
            c1 r17;c2 r17;c3 r17;c4 r17;c5 r17;c6 r17;c7 r17;c8 r17;c9 r17;c10 r17;c11 r17;c12 r17;c13 r17];
%INPUT DATA FOR NODAL CONNECTIVITY OF EACH ELEMENT
%NODE(i,j) WHERE i------>THE ELEMENT NUMBER AND j----->CONNECTED NODE
node_connectivity=[21 23 3 1 22 15 2 14;23 25 5 3 24 16 4 15;25 27 7 5 26 17 6 16;27 29 9 7 28 18 8 17;29 31 11 9 30 19 10
18;31 33 13 11 32 20 12 19;
            41 43 23 21 42 35 22 34;43 45 25 23 44 36 24 35;45 47 27 25 46 37 26 36;47 49 29 27 48 38 28 37;49 51 31 29 50
39 30 38;51 53 33 31 52 40 32 39;
            61 63 43 41 62 55 42 54;63 65 45 43 64 56 44 55;65 67 47 45 66 57 46 56;67 69 49 47 68 58 48 57;69 71 51 49 70
59 50 58;71 73 53 51 72 60 52 59;
            81 83 63 61 82 75 62 74;83 85 65 63 84 76 64 75;85 87 67 65 86 77 66 76;87 89 69 67 88 78 68 77;89 91 71 69 90
79 70 78;91 93 73 71 92 80 72 79;
            101 103 83 81 102 95 82 94;103 105 85 83 104 96 84 95;105 107 87 85 106 97 86 96;107 109 89 87 108 98 88
97;109 111 91 89 110 99 90 98;111 113 93 91 112 100 92 99;
            121 123 103 101 122 115 102 114;123 125 105 103 124 116 104 115;125 127 107 105 126 117 106 116;127 129 109
107 128 118 108 117;129 131 111 109 130 119 110 118;131 133 113 111 132 120 112 119;
            141 143 123 121 142 135 122 134;143 145 125 123 144 136 124 135;145 147 127 125 146 137 126 136;147 149 129
127 148 138 128 137;149 151 131 129 150 139 130 138;151 153 133 131 152 140 132 139;
            161 163 143 141 162 155 142 154;163 165 145 143 164 156 144 155;165 167 147 145 166 157 146 156;167 169 149
147 168 158 148 157;169 171 151 149 170 159 150 158;171 173 153 151 172 160 152 159];

%PLOTING THE FINITE ELEMENT MESH
node_connectivity_plooting=[21 22 23 15 3 2 1 14;23 24 25 16 5 4 3 15;25 26 27 17 7 6 5 16;27 28 29 18 9 8 7 17;29 30 31 19
11 10 9 18;31 32 33 20 13 12 11 19;
            41 42 43 35 23 22 21 34;43 44 45 36 25 24 23 35;45 46 47 37 27 26 25 36;47 48 49 38 29 28 27 37;49 50 51 39 31
30 29 38;51 52 53 40 33 32 31 39;
            61 62 63 55 43 42 41 54;63 64 65 56 45 44 43 55;65 66 67 57 47 46 45 56;67 68 69 58 49 48 47 57;69 70 71 59 51
50 49 58;71 72 73 60 53 52 51 59;
            81 82 83 75 63 62 61 74;83 84 85 76 65 64 63 75;85 86 87 77 67 66 65 76;87 88 89 78 69 68 67 77;89 90 91 79 71
70 69 78;91 92 93 80 73 72 71 79;
            101 102 103 95 83 82 81 94;103 104 105 96 85 84 83 95;105 106 107 97 87 86 85 96;107 108 109 98 89 88 87
97;109 110 111 99 91 90 89 98;111 112 113 100 93 92 91 99;
            121 122 123 115 103 102 101 114;123 124 125 116 105 104 103 115;125 126 127 117 107 106 105 116;127 128 129
118 109 108 107 117;129 130 131 119 111 110 109 118;131 132 133 120 113 112 111 119;
            141 142 143 135 123 122 121 134;143 144 145 136 125 124 123 135;145 146 147 137 127 126 125 136;147 148 149
138 129 128 127 137;149 150 151 139 131 130 129 138;151 152 153 140 133 132 131 139;
            161 162 163 155 143 142 141 154;163 164 165 156 145 144 143 155;165 166 167 157 147 146 145 156;167 168 169
158 149 148 147 157;169 170 171 159 151 150 149 158;171 172 173 160 153 152 151 159];
```

```
for i=1:48
  row=node_connectivity_plooting(i,:);first_element=row(1);
  x(9)=global_coordinate(first_element,1);y(9)=global_coordinate(first_element,2);
  for j=1:8
     a=row(j);x(j)=global_coordinate(a,1);y(j)=global_coordinate(a,2);
  end
            plot(x,y,'-ob')xlim([-1 1500])
     ylim([-3200 0])
     hold on
     x=zeros();y=zeros();
 end
hold off

%INPUTTING THE PAVEMENT THICKNESS

self_weight_concrete=2.4*10^-6;self_weight_soil=1.5*10^-6; point_load=zeros(7,1);

%ASKING USER TO ENTER THE SURFACE LOAD VALUES

z3=menu('IS THERE ANY SURFACE LOAD ON THE PAVEMENT','YES','NO');
if z3==1
   udl=input('THE VALUE OF UDL ON THE PAVEMENT IN N/mm2    ');
end

if z3==2
   disp('---------------THERE ARE NO SURFACE LOADS ON THIS ELEMENT--------------      ');
end

%ASKING USER TO ENTER THE POINT LOAD VALUES
z4=menu('IS THERE ANY POINT LOAD ON THE ELEMENT','YES','NO');
  if z4==2
      disp(' ------------THERE ARE NO POINT LOADS----------------  ');

  end
  if z4==1
    disp('ENTER THE VALUE OF LOADS ONE BY ONE   ');
      disp('USE FIGURE TO DEFINE THE NODE AT WHICH LOAD IS APPLIED   ');
      disp('NODES ARE NUMBERED IN HORIZONTAL DIRECTION STARTING FROM TOP OF THE MESH    ');
      disp('POINT LOADS CAN BE ADDED TO 1 2 3 4 5 6 7 NODES ONLY   ');
      num=input('HOW MANY POINT LOADS ARE THERE    ');
      for j=1:num
        number=input('ENTER THE NODE ON WHICH POINT LOAD IS APPLIED    ');
         L1=input('ENTER THE VALUE OF LAOD(N)    ');
         point_load(number,1)=L1;
      end
  end

%INITIALIZING MATRICES AND VECTORS
element_force_vector=zeros(single_elem_dof,1);
system_force_vector=zeros(total_syst_dof,1);
global_stiffness=zeros(total_syst_dof,total_syst_dof);
displacement=zeros(total_syst_dof,1);index=zeros(single_elem_dof,1);
transf_mat=zeros(single_elem_dof,single_elem_dof);

force_vector1=zeros(single_elem_dof,1);force_vector2=zeros(single_elem_dof,1);
force_vector3=zeros(single_elem_dof,1);force_vector4=zeros(single_elem_dof,1);
force_vector5=zeros(single_elem_dof,1);

%SHAPE FUNCTION FOR EACH NODE OF EACH ELEMENT

for i=1:48
   h=0;
```

```
h1=0;
%INITIALIZING MATRIXES
force_vector1=zeros(single_elem_dof,1);force_vector2=zeros(single_elem_dof,1);
force_vector3=zeros(single_elem_dof,1);force_vector4=zeros(single_elem_dof,1);
force_vector5=zeros(single_elem_dof,1);
element_force_vector=zeros(single_elem_dof,1);
element_displacemet=zeros(single_elem_dof,1);
constitutive_mat=zeros(4,4); dof=zeros(total_nodes,2);
syms g n

%SHAPE FUNCTION of SINGLE ELEMENT
element=node_connectivity(i,:);
shape_function(i,1)=(1/4)*(1-g)*(1-n)*(-g-n-1);
shape_function(i,2)=(1/4)*(1+g)*(1-n)*(g-n-1);
shape_function(i,3)=(1/4)*(1+g)*(1+n)*(g+n-1);
shape_function(i,4)=(1/4)*(1-g)*(1+n)*(-g+n-1);
shape_function(i,5)=(1/2)*(1-g)*(1+g)*(1-n);
shape_function(i,7)=(1/2)*(1-g)*(1+g)*(1+n);
shape_function(i,6)=(1/2)*(1-n)*(1+n)*(1+g);
shape_function(i,8)=(1/2)*(1-n)*(1+n)*(1-g);

nodes3=node_connectivity(i,:);fourth=nodes3(4);
third=nodes3(3);point1=global_coordinate(fourth,:);
point2=global_coordinate(third,:);diff1=point2(1)-point1(1);
diff2=point2(2)-point1(2);sum1=((diff1*diff1)+(diff2*diff2));
length1=sqrt(sum1); area1=length1*1;
first=nodes3(1);point3=global_coordinate(first,:);
sum2=-(point3(2)-point1(2));length_element=sqrt(sum2);

%1 FOR DOF IN HORIZONTAL DIRECTION AND %2 FOR DOF IN VERTICAL DIRECTION
v1=0;

%DOF OF THE ELEMENT
for j=1:8
    xe=nodes3(j);v1=v1+1;dof(xe,1)=v1;v1=v1+1;dof(xe,2)=v1;
end

%FOR CALCULATING LENGTH AND AREA OF EACH ELEMENT
% %FORCE VECTOR
% %FORCES DUE TO SURFACE LOAD APPLICATION

if z3==1

    if i<=3
        node_num=node_connectivity(i,:);
        for j=1:8
            if j==3||j==4||j==7
                syms g n
                fourth=nodes3(j);point4=global_coordinate(fourth,:);
                diff3=point4(1)-point1(1); diff4=point4(2)-point1(2);
                sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
                nodes23=shape_function(i,j); x1=node_num(j);
                %for edge 3-4 for each element n=1
                n=1; h=subs(nodes23); integration_value=int(h,-1,1);
                v=area1*length2* integration_value*udl/2;
                VAL1=dof(x1,2); force_vector1(VAL1,1)=v;
                end
        end
    end

end
```

```
        if z4==1
          for j=1:8
            nodal=nodes3(j);
            if
      nodal==1||nodal==2||nodal==3||nodal==4||nodal==5||nodal==6||nodal==7
                VAL2=dof(nodal,2);
                force_vector2(VAL2,1)=point_load(nodal,1);
            end
          end
      end
      if i<=30
          syms g n

                    n=-1; z8=shape_function(1); h=subs(z8);
          nodes3=node_connectivity(i,:);fourth=nodes3(4);
       third=nodes3(3); point1=global_coordinate(fourth,:);
       point2=global_coordinate(third,:); diff1=point2(1)-point1(1);
       diff2=point2(2)-point1(2); sum1=((diff1*diff1)+(diff2*diff2));
       length1=sqrt(sum1); area1=length1*1;
       first=nodes3(1);
       first
       point3=global_coordinate(first,:);
       point3
       sum2=-(point3(2)-point1(2));
       sum2
       length_element7=sum2;
       length_element7
          integration_value=int(h,-1,1);
          length_element1=length_element7;
          force_add1=area1*length_element1*integration_value*self_weight_concrete;
          force_add2=area1*length_element1*integration_value*self_weight_soil;
      end
      if 30<i<=36

          syms g n;
          n=-1;
    z8=shape_function(1); nodes3=node_connectivity(i,:);
    fourth=nodes3(4); third=nodes3(3);
    point1=global_coordinate(fourth,:); point2=global_coordinate(third,:);
    diff1=point2(1)-point1(1); diff2=point2(2)-point1(2);
    sum1=((diff1*diff1)+(diff2*diff2)); length1=sqrt(sum1);
    area1=length1*1; first=nodes3(1); point3=global_coordinate(first,:);
    sum2=-(point3(2)-point1(2)); length_element8=sum2; h=subs(z8);
    length_element3=length_element8; integration_value=int(h,-1,1);
    length_element2=length_element1;
force_add1=area1*length_element1*integration_value*self_weight_concrete;
force_add2=area1*length_element1*integration_value*self_weight_soil;
      end
      if 36<i<=42
        syms g n;
        n=-1; z8=shape_function(1); h=subs(z8); nodes3=node_connectivity(i,:);
        fourth=nodes3(4); third=nodes3(3);point1=global_coordinate(fourth,:);
        point2=global_coordinate(third,:);
        diff1=point2(1)-point1(1); diff2=point2(2)-point1(2);
        sum1=((diff1*diff1)+(diff2*diff2)); length1=sqrt(sum1);
        area1=length1*1; first=nodes3(1);
        point3=global_coordinate(first,:); sum2=-(point3(2)-point1(2));
        length_element9=sum2; length_element4=length_element9;
        length_element1=length_element3; integration_value=int(h,-1,1);
        length_element2=length_element1;
```

```
 force_add1=area1*length_element1*integration_value*self_weight_concrete;
force_add2=area1*length_element1*integration_value*self_weight_soil;
    end
    if 42<i<=48
        syms g n;
        n=-1; z8=shape_function(1);h=subs(z8); length_element1=length_element4;
        integration_value=int(h,-1,1); length_element2=length_element1;
        force_add1=area1*length_element1*integration_value*self_weight_concrete;
force_add2=area1*length_element1*integration_value*self_weight_soil;
    end

   %FORCES DUE TO BODY WEIGHT
   %DUE TO TWO CONCRETE LAYERS
   if i<=6
    for j=1:8
       syms g n
       z5=nodes3(j); z8=shape_function(i,j);
       if j==1||j==8
          third=nodes3(j); point3=global_coordinate(third,:);
          diff3=point3(1)-point1(1); diff4=point3(2)-point1(2);
          sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
       end
       if j==5
          third=nodes3(j); second=nodes3(7);point4=global_coordinate(second,:);
          point3=global_coordinate(third,:);
          diff3=point3(1)-point4(1); diff4=point3(2)-point4(2);
          sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
       end
       if j==2||j==6
          third=nodes3(j); point3=global_coordinate(third,:);
          diff3=point3(1)-point2(1); diff4=point3(2)-point2(2);
          sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
       end
       if j==3||j==4||j==7
          length2=0;
       end
          if j==1||j==2||j==5
          n=-1;
       end
       if j==3||j==4||j==7
          n=1;
       end
       if j==6||j==8
          n=0;
       end

       h=subs(z8); integration_value=int(h,-1,1);
       force=area1*length2*integration_value*self_weight_concrete;
       VAL3=dof(z5,2);force_vector3(VAL3,1)=force;
    end
    h=0;
    z8=0;
 end
 if 6<i<=12
   for j=1:8
     syms g n
      z5=nodes3(j);
      z8=shape_function(i,j);
      if j==1||j==8
         third=nodes3(j);point3=global_coordinate(third,:);
         diff3=point3(1)-point1(1); diff4=point3(2)-point1(2);
         sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
```

```
        end
        if j==5
           third=nodes3(j);second=nodes3(7);point4=global_coordinate(second,:);
           point3=global_coordinate(third,:);
           diff3=point3(1)-point4(1); diff4=point3(2)-point4(2);
           sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
        end
        if j==2||j==6
           third=nodes3(j);point3=global_coordinate(third,:);
           diff3=point3(1)-point2(1); diff4=point3(2)-point2(2);
           sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
        end
        if j==3||j==4||j==7
           length2=0;
        end
        if j==1||j==2||j==5
           n=-1;
        end
        if j==3||j==4||j==7
           n=1;
        end
        if j==6||j==8
           n=0;
        end

        h=subs(z8);

        integration_value=int(h,-1,1);
        force1=area1*length2*integration_value*self_weight_concrete;
        force=force1+force_add1; VAL3=dof(z5,2);force_vector3(VAL3,1)=force;
    end
    h=0;
    z8=0;
  end

% DUE TO SOIL LAYERS
  if 12<i<=18

        for k=1:8
          if k==1||k==8
             third=nodes3(k); point3=global_coordinate(third,:);
             diff3=point3(1)-point1(1); diff4=point3(2)-point1(2);
             sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
          end
          if k==5
             third=nodes3(k);second=nodes3(7);
             point4=global_coordinate(second,:);
             point3=global_coordinate(third,:);
             diff3=point3(1)-point4(1); diff4=point3(2)-point4(2);
             sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
          end
          if k==2||k==6
             third=nodes3(k); point3=global_coordinate(third,:);
             diff3=point3(1)-point2(1); diff4=point3(2)-point2(2);
             sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
        end

          if k==3||k==4||k==7
             length2=0;
          end
          z9=shape_function(i,k);
          if k==1||k==2||k==5
```

```
            n=-1;
         end
         if k==3||k==4||k==7
              n=1;
         end
         if k==6||k==8
            n=0;
         end

         h1=subs(z9);

         integration_value=int(h1,-1,1);
         force1=area1*length2*integration_value*self_weight_soil;
         force=force1+(2*force_add1); z5=nodes3(k);
         VAL4=dof(z5,1); force_vector4(VAL4,1)=force;
      end
      h1=0;
      z9=0;
   end
   if 18<i<=24

      for k=1:8
         if k==1||k==8
            third=nodes3(k); point3=global_coordinate(third,:);
            diff3=point3(1)-point1(1); diff4=point3(2)-point1(2);
            sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
         end
         if k==5
            third=nodes3(k); second=nodes3(7);
            point4=global_coordinate(second,:);
            point3=global_coordinate(third,:);
            diff3=point3(1)-point4(1); diff4=point3(2)-point4(2);
            sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
         end
         if k==2||k==6
            third=nodes3(k); point3=global_coordinate(third,:);
            diff3=point3(1)-point2(1); diff4=point3(2)-point2(2);
            sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
         end
         if k==3||k==4||k==7
            length2=0;
         end

         z9=shape_function(i,k);
         if k==1||k==2||k==5
            n=-1;
         end
         if k==3||k==4||k==7
              n=1;
         end
         if k==6||k==8
            n=0;
         end

         h1=subs(z9);

         integration_value=int(h1,-1,1);
         force1=area1*length2*integration_value*self_weight_soil;
         force=force1+(2*force_add1)+force_add2;
         z5=nodes3(k); VAL4=dof(z5,1); force_vector4(VAL4,1)=force;
      end
      h1=0; z9=0;
```

```
    end
  if 24<i<=30
    for k=1:8
      if k==1||k==8
        third=nodes3(k); point3=global_coordinate(third,:);
        diff3=point3(1)-point1(1); diff4=point3(2)-point1(2);
        sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
      end

      if k==5
        third=nodes3(k); second=nodes3(7);
        point4=global_coordinate(second,:);
        point3=global_coordinate(third,:);
        diff3=point3(1)-point4(1); diff4=point3(2)-point4(2);
        sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
      end

      if k==2||k==6
        third=nodes3(k); point3=global_coordinate(third,:);
        diff3=point3(1)-point2(1); diff4=point3(2)-point2(2);
                        sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
      end
      if k==3||k==4||k==7
        length2=0;
      end
      z9=shape_function(i,k);

      if k==1||k==2||k==5
        n=-1;
      end
      if k==3||k==4||k==7
         n=1;
      end
      if k==6||k==8
        n=0;
      end

      h1=subs(z9);integration_value=int(h1,-1,1);
      force1=area1*length2*integration_value*self_weight_soil;
      force=force1+(2*force_add1)+(2*force_add2);
      z5=nodes3(k); VAL4=dof(z5,1); force_vector4(VAL4,1)=force;
    end
    h1=0;
    z9=0;
end
  if 30<i<=36
    for k=1:8
      if k==1||k==8
        third=nodes3(k); point3=global_coordinate(third,:);
        diff3=point3(1)-point1(1); diff4=point3(2)-point1(2);
        sum2=((diff3*diff3)+(diff4*diff4)); length2=sqrt(sum2);
      end
      if k==5
        third=nodes3(k);second=nodes3(7);
        point4=global_coordinate(second,:);
        point3=global_coordinate(third,:);
        diff3=point3(1)-point4(1);diff4=point3(2)-point4(2);
        sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
      end
      if k==2||k==6
        third=nodes3(k);point3=global_coordinate(third,:);
        diff3=point3(1)-point2(1);diff4=point3(2)-point2(2);
```

```
            sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
          end
          if k==3||k==4||k==7
             length2=0;
          end
          z9=shape_function(i,k);
          if k==1||k==2||k==5
             n=-1;
          end
          if k==3||k==4||k==7
              n=1;
          end
          if k==6||k==8
             n=0;
          end

          h1=subs(z9);integration_value=int(h1,-1,1);
          force1=area1*length2*integration_value*self_weight_soil;
          force=force1+(2*force_add1)+(3*force_add2);
          z5=nodes3(k);VAL4=dof(z5,1);force_vector4(VAL4,1)=force;
       end
       h1=0;
       z9=0;
end
  if 36<i<=42

       for k=1:8
          if k==1||k==8
             third=nodes3(k);point3=global_coordinate(third,:);
             diff3=point3(1)-point1(1);diff4=point3(2)-point1(2);
             sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
          end
          if k==5
             third=nodes3(k);second=nodes3(7);
             point4=global_coordinate(second,:);
             point3=global_coordinate(third,:);
             diff3=point3(1)-point4(1);diff4=point3(2)-point4(2);
             sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
          end
          if k==2||k==6
             third=nodes3(k);point3=global_coordinate(third,:);
             diff3=point3(1)-point2(1);diff4=point3(2)-point2(2);
             sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
          end
          if k==3||k==4||k==7
             length2=0;
          end

          z9=shape_function(i,k);
          if k==1||k==2||k==5
             n=-1;
          end
          if k==3||k==4||k==7
              n=1;
          end
          if k==6||k==8
             n=0;
          end
          h1=subs(z9);integration_value=int(h1,-1,1);
          force1=area1*length2*integration_value*self_weight_soil;
          force=force1+(2*force_add1)+(4*force_add2);
          z5=nodes3(k);VAL4=dof(z5,1);force_vector4(VAL4,1)=force;
```

```
            end
            h1=0;
            z9=0;
    end
      if 42<i<=48
            for k=1:8
              if k==1||k==8
                 third=nodes3(k);point3=global_coordinate(third,:);
                 diff3=point3(1)-point1(1);diff4=point3(2)-point1(2);
                 sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
              end
              if k==5
                 third=nodes3(k);second=nodes3(7);
                 point4=global_coordinate(second,:);
                 point3=global_coordinate(third,:);
                 diff3=point3(1)-point4(1);diff4=point3(2)-point4(2);
                 sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
              end
                 if k==2||k==6
                 third=nodes3(k);point3=global_coordinate(third,:);
                 diff3=point3(1)-point2(1);diff4=point3(2)-point2(2);
                 sum2=((diff3*diff3)+(diff4*diff4));length2=sqrt(sum2);
              end
              if k==3||k==4||k==7
                 length2=0;
              end
              z9=shape_function(i,k);

              if k==1||k==2||k==5
                 n=-1;
              end
              if k==3||k==4||k==7
                   n=1;
              end
              if k==6||k==8
                 n=0;
              end

              h1=subs(z9);integration_value=int(h1,-1,1);
              force1=area1*length2*integration_value*self_weight_soil;
              force=force1+(2*force_add1)+(5*force_add2);
              z5=nodes3(k);VAL4=dof(z5,1);force_vector4(VAL4,1)=force;
           end
           h1=0;
           z9=0;
     end
% ADDING ALL THE FORCES IN A SINGLE MATRIX
   for j=1:16

element_force_vector(j,1)=force_vector1(j,1)+force_vector2(j,1)+force_vector3(j,1)+force_vector4(j,1)+element_force_vector(j,
1);

   end

% JACOBIAN MATRIX OF THE ELEMENT
      sum1=0;sum3=0;sum2=0;sum4=0;nodes3=node_connectivity(i,:);
      first=nodes3(4);third=nodes3(1);point1=global_coordinate(first,:);
      point3=global_coordinate(third,:);diff1=point3(1)-point1(1);
      diff2=point3(2)-point1(2);sum1=((diff1*diff1)+(diff2*diff2));
      length=sqrt(sum1);syms g n ;
      for j=1:8
         N1=shape_function(i,j);d7=diff(N1,g);first=nodes3(j);
```

```
    point1=global_coordinate(first,:);x_coordinate=point1(1);
    y_coordinate=point1(2);sum1=sum1+d7*x_coordinate;
    sum2=sum2+d7*y_coordinate;
  end
       jacobian(1,1)=sum1;jacobian(1,2)=sum2;
  for j=1:8
    N1=shape_function(i,j);d7=diff(N1,n);
                 nodes3=node_connectivity(i,:);first=nodes3(j);
    point1=global_coordinate(first,:);x_coordinate=point1(1);
    y_coordinate=point1(2);
    sum3=sum3+d7*x_coordinate;sum4=sum4+d7*y_coordinate;
  end
  syms g n ;
  jacobian(2,1)=sum3;jacobian(2,2)=sum4;
  a=sum1;b=sum2;c=sum3;d=sum4;
  jac_inv= [ -d/(-a*d+b*c)  b/(-a*d+b*c);
          c/(-a*d+b*c)  -a/(-a*d+b*c)];

  transf=[jac_inv(1,1) jac_inv(1,2)   0          0    ;
       0         0          jac_inv(2,1) jac_inv(2,2);
       0         0          0          0   ;
       jac_inv(2,1) jac_inv(2,2) jac_inv(1,1)   jac_inv(1,2)];

%  strain displacement matrix
  for j=1:8
    N1=shape_function(i,j);d1=diff(N1,g);
                 strain_displacement(1,j)=d1;strain_displacement(1,8+j)=0;
  end
  for j=1:8
    N1=shape_function(i,j);d2=diff(N1,n);
    strain_displacement(2,j)=0;strain_displacement(2,j+8)=d2;
  end

  for j=1:8
    N1=shape_function(i,j);first=nodes3(j);
    point1=global_coordinate(first,:);x_coordinate=point1(1);
    if x_coordinate==0
      d3=0;
    end
    if x_coordinate~=0
      d3=1/x_coordinate;
    end
                 strain_displacement(3,j)=d3;strain_displacement(3,8+j)=0;
  end

  for j=1:8
    N1=shape_function(i,j);d4=diff(N1,n);d1=diff(N1,g);
                 strain_displacement(4,j)=d4;strain_displacement(4,j+8)=d1;
  end

%strain_displacement=roundn(strain_displacement,-2);
% CONSTITUTIVE MATRIX
 if i<=6
       poissons_ratio=.15; modulus_elasticity=30210;
 end
 if 6<i&&i<=12
      poissons_ratio=.20;modulus_elasticity=27310;
 end
 if i>12
                 poissons_ratio=.305;modulus_elasticity=106.816;
 end
```

```
val1=(1+poissons_ratio)*(1-2*poissons_ratio);
val2=modulus_elasticity/val1;
D=[1-poissons_ratio   poissons_ratio    poissons_ratio        0;
   poissons_ratio   1-poissons_ratio   poissons_ratio        0;
   poissons_ratio   poissons_ratio    1-poissons_ratio       0;
   0                0                 0                (1-(2*poissons_ratio))/2];
constitutive_mat=(val2*D);

%STIFFNESS MATRIX
MAT1=strain_displacement'*constitutive_mat*strain_displacement;
syms g n
for j=1:1
   for k=1:1
      c=MAT1(j,k);
      points=[-.8; -.3 ;.3;.8];weights=[.35;.6;.6;.35];
       summation=0;val4=0;get1=0;get2=0;
       for l=1:1
          g=points(l);re=subs(c);val4=re*weights(l);
          summation=val4+summation;get1=summation;g=0;
       end
       for l=1:1
          n=points(l);h=subs(get1);val4=h*weights(l);get2=val4+get2;
                 end
               stiffness_matrix(j,k)=get2;
   end
 end
element_displacement=inv(stiffness_matrix)*element_force_vector;

%  strain=strain_displacement*displacement
%FINDING ELEMENT STRAIN AND ELEMENT STESSES
 bounty=1;
 for j=1:8
    x3=nodes3(j);
    for k=1:4
       for l=1:16
            s2=strain_displacement(k,l);
            g=global_coordinate(x3,1);q1=subs(s2);
            n=global_coordinate(x3,1);q2=subs(q1);
            strain11(k,l)=q2;
       end
    end

         element_strain1=strain11*element_displacement;
    element_stress1=constitutive_mat*element_strain1;
    for k=1:4
       element_stress(bounty,1)=element_stress1(k,1);
       element_strain(bounty,1)=element_strain1(k,1);
       bounty=bounty+1;
    end
         element_stress1=zeros(4,1);
 end

% GLOBAL FORCE VECTOR
 for j=2:2:16
    x9=element_force_vector(j);l9=j/2;node_num=nodes3(l9);x1=2*node_num;
    system_force_vector(x1)=system_force_vector(x1)+x9;
 end


%  GLOBAL DISPLACMENT VECTOR
 cc=1;nodes4=node_connectivity(i,:);
```

```
  for j=1:16
    if j==2||j==4||j==6||j==8||j==10||j==12||j==14||j==16
      x5=element_displacement(j);l10=j/2;
      node_num1=nodes4(l10);a2=2*node_num1;displacement(a2)=x5;
    end
    if j==1||j==3||j==5||j==7||j==9||j==11||j==13||j==15
      node_num2=nodes4(ce);ce=ce+1;a1=(2*node_num2)-1;
      displacement(a1)=element_displacement(j);
    end
end

%GLOBAL STRESS AND STRAIN MATRIX
  strain=zeros(4*total_nodes,1);stress=zeros(4*total_nodes,1);
  for j=1:32
    x12=element_stress(j,1);x13=element_strain(j,1);
    if j==1||j==2||j==3||j==4
      nod_num2=nodes3(1);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
    end
    if j==5||j==6||j==7||j==8
      nod_num2=nodes3(2);global_node_num1=4*nod_num2;
      strain(global_node_num1)=strain(global_node_num1)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num1=global_node_num1+1;
    end
    if j==9||j==10||j==11||j==12
      nod_num2=nodes3(3);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
    end
    if j==13||j==14||j==15||j==16
      nod_num2=nodes3(4);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
    end
    if j==20||j==19||j==18||j==17
      nod_num2=nodes3(5);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
    end
    if j==21||j==22||j==23||j==24
      nod_num2=nodes3(6);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
    end
    if j==25||j==26||j==27||j==28
      nod_num2=nodes3(7);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
    end
    if j==29||j==30||j==31||j==32
      nod_num2=nodes3(8);global_node_num=4*nod_num2;
      strain(global_node_num)=strain(global_node_num)+x13;
      stress(global_node_num)=stress(global_node_num)+x12;
      global_node_num=global_node_num+1;
```

```
      end
    end
  end

  %PLOTTING OF THE DEFLECTED SHAPE OF THE MESH
  for i=1:48
    row=node_connectivity_plooting(i,:);firstel=row(1);
    x2=global_coordinate(firstel,1);y2=global_coordinate(firstel,2);
    set1=2*firstel;y1=displacement(set1,1);set2=set1-1;
    x1=displacement(set2,1);first_element=row(1);x(9)=x2-x1;y(9)=y2-y1;
  for j=1:8
      a=row(j);x2=global_coordinate(a,1);y2=global_coordinate(a,2);
      b1=2*a;y1=displacement(b1,1);b2=b1-1;x1=displacement(b2,1);
      x(j)=x2-x1;y(j)=y2-y1;
    end
          plot(x,y,'-K')xlim([-1 1500])
      ylim([-3200 0])
      hold on
      x=zeros();y=zeros();
  end
  hold on


  %
  % displacement
  % stress
  % strain
  % system_force_vector



  % --- Executes on button press in pushbutton13 (exit).
  function pushbutton13_Callback(hObject, eventdata, handles)
  if isfield(handles,'figure1') & ishandle(handles.figure1),
    close(handles.figure1);
  end


  % --- Executes on button press in pushbutton8 (material requirement)
  function varargout=pushbutton8_Callback(hObject, eventdata, handles)
  global output;
  disp('THE DENSITY OF CONCRETE IS 2400kg/m3');
  si=menu('ENTER THE SIZE OF BLOCK ','100','150');
  if si==1
    size=100;
  end
  if si==2
    size=150;
  end
  num=menu('ENTER THE NUMBER OF BLOCKS TO BE PREPARED','6','7','8','9','10');
  if num==1
    num_blocks=6;
  end
  if num==2
    num_blocks=7;
  end
  if num==3
    num_blocks=8;
  end
  if num==4
    num_blocks=9;
  end
```

```
if num==5
   num_blocks=10;
end
amt_conc=( 24*size*size*size/10000);
num_blocks
ch=menu('CHOOSE THE RATIO OF MIX YOU WANT TO PREPARE','1:2:4','1:1.5:3','1:1:2');
if ch==1
   cement=1; sand=2;aggregate=4;
end
if ch==2
   cement=1; sand=1.5;aggregate=3;
end
if ch==3
   cement=1; sand=1;aggregate=2;
end

choice=menu('ENTER THE PERCENTAGE OF WASTE U WANT TO KEEP','10','20','30','40','50');
if choice==1
   waste=10;
end
if choice==2
   waste=20;
end
if choice==3
   waste=30;
end
if choice==4
   waste=40;
end
if choice==5
   waste=50;
end

%MATERIALREQUIREMENT FOR PCC

   sum=cement+sand+aggregate;cement_nowaste=(amt_conc/sum);
   a=(cement_nowaste*(1+(waste/100)));cement_reqd1=a*num_blocks;
   sand_reqd1=cement_reqd1*sand;aggregate_reqd1=cement_reqd1*aggregate;

   disp('MATERIAL REQUIREMENT FOR PLAIN CEMENT CONCRETE IS:');
   disp('THE AMOUNT OF CEMENT REQUIRED IN KILLOGRAMS IS ');
   cement_reqd1
   disp('THE AMOUNT OF SAND REQUIRED IN KILLOGRAMS IS ');
   sand_reqd1
   disp('THE AMOUNT OF AGGREGATE REQUIRED  IN KILLOGRAMS IS ');
   aggregate_reqd1
   disp('');
    disp('');
     disp('');
      disp('');

%MATERIALREQUIREMENT FOR PCC WITH FLYASH
  cho=menu('ENTER THE PERCENTAGE OF FLYASH IN THE MIX','10','15','20','25');

if choice==1
   flyash=10;
end
if choice==2
   flyash=15;
end
if choice==3
   flyash=20;
```

```
end
if choice==4
   flyash=25;
end
   cement_noflyash=num_blocks*a;flyash_reqd=(flyash*cement_noflyash/100);
   sand_reqd2=cement_noflyash*sand;
   aggregate_reqd2=cement_noflyash*aggregate;
   cement_reqd2=cement_noflyash-flyash_reqd;

   disp('MATERIAL REQUIREMENT FOR PLAIN CEMENT CONCRETE WITH FLYASH IS:');
   disp('THE AMOUNT OF CEMENT REQUIRED IN KILLOGRAMS IS ');
   cement_reqd2
   disp('THE AMOUNT OF CEMENT REQUIRED IN KILLOGRAMS IS ');
   flyash_reqd
   disp('THE AMOUNT OF SAND REQUIRED IN KILLOGRAMS IS ');
   sand_reqd2
   disp('THE AMOUNT OF AGGREGATE REQUIRED IN KILLOGRAMS IS ');
   aggregate_reqd2
    disp('');
     disp('');
      disp('');
       disp('');


%MATERIALREQUIREMENT FOR SFRC
fibre=menu('ENTER THE AMOUNT OF FIBRES YOU WANT IN YOUR MIX','40','60','80','100','120','140','160','180','200');
  if fibre==1
     fibre_content=40;
  end
   if fibre==2
     fibre_content=60;
   end
   if fibre==3
     fibre_content=80;
   end
   if fibre==4
     fibre_content=100;
   end
   if fibre==5
     fibre_content=120;
   end
   if fibre==6
     fibre_content=140;
   end
   if fibre==7
     fibre_content=160;
   end
   if fibre==8
     fibre_content=180;
   end
   if fibre==9
     fibre_content=200;
  end

   fib=(fibre_content*size*size*size/1000000);
   choice=menu('ENTER THE PERCENTAGE OF WASTE U WANT TO KEEP WITH FIBRES','10','20','30','40','50');
if choice==1
   fibre_waste=10;
end
if choice==2
   fibre_waste=20;
end
```

```
if choice==3
   fibre_waste=30;
end
if choice==4
   fibre_waste=40;
end
if choice==5
   fibre_waste=50;
end
   fib_wthwaste=(fib*(1+(fibre_waste/100)));
   fibre_reqd=fib_wthwaste*num_blocks;

   cement_reqd3=cement_reqd1;
   sand_reqd3=sand_reqd1;
   aggregate_reqd3=aggregate_reqd1;
   disp('MATERIAL REQUIREMENT FOR STEEL FIBRE REINFORCED CONCRETE IS:');
   disp('THE AMOUNT OF CEMENT REQUIRED  IN KILLOGRAMS IS  ');
   cement_reqd3
   disp('THE AMOUNT OF FIBRES REQUIRED  IN KILLOGRAMS IS  ');
   fibre_reqd
   disp('THE AMOUNT OF SAND REQUIRED  IN KILLOGRAMS IS  ');
   sand_reqd3
   disp('THE AMOUNT OF AGGREGATE REQUIRED  IN KILLOGRAMS IS  ');
   aggregate_reqd3
  disp('');
   disp('');
   output={cement_reqd1 cement_reqd2 cement_reqd3;sand_reqd1 sand_reqd2 sand_reqd3;aggregate_reqd1 aggregate_reqd2
aggregate_reqd3;0 flyash_reqd 0;0 0 fibre_reqd};

% --- Executes on button press in pushbutton7 (economic analysis)
function varargout=pushbutton7_Callback(hObject, eventdata, handles,varargin)
global output;
global num;
num
s=xlswrite('new.xlsx', output, 'material requirements', 'B3')

for i=1:3
   v(i)=num(3,i);
end
s=xlswrite('new.xlsx',v,'material requirements','B16')
   n = xlsread('new.xlsx', -1)
   hold off
  bar(n,.5)
  ylabel('COST (*10000) Rs.','FontSize',10)
  title('                        "COST COMPARISON" OF PCC,PCC WITH FLYASH AND SFRC','FONTSIZE',10);
ylim([10000 60000]);

g = xlsread('new.xlsx', -1)
 hold off
  bar(g,.5)
  title('"STRENGTH COMPARISON" OF PCC,PCC WITH FLYASH AND SFRC','FONTSIZE',10);
  ylabel('COMPRESSIVE STRENGTH(N/mm2) AFTER 28 DAYS','FontSize',10)
ylim([10 100]);
```

APPENDIX II

The results of the FEM analysis are shown below:

<table>
<tr><th>Node no.</th><th>Displacement (mm)</th><th>Stresses (N/mm^2) 1.0e+010 *</th><th>Strains 1.0e+005 *</th></tr>
<tr><td rowspan="4">1</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.7926</td><td>0</td><td>0</td></tr>
<tr><td>0.0000</td><td>0.0000</td></tr>
<tr><td rowspan="4">2</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">2.5902</td><td>0</td><td>0</td></tr>
<tr><td>0.0442</td><td>0.1484</td></tr>
<tr><td rowspan="4">3</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.7944</td><td>0</td><td>0</td></tr>
<tr><td>0.3232</td><td>1.0579</td></tr>
<tr><td rowspan="4">4</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">2.5960</td><td>0</td><td>0</td></tr>
<tr><td>0.3319</td><td>1.0532</td></tr>
<tr><td rowspan="4">5</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>1.3978</td><td>4.6884</td></tr>
<tr><td rowspan="4">6</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">2.5931</td><td>0</td><td>0</td></tr>
<tr><td>1.2628</td><td>4.4022</td></tr>
<tr><td rowspan="4">7</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.7932</td><td>0</td><td>0</td></tr>
<tr><td>2.0651</td><td>7.4048</td></tr>
<tr><td rowspan="4">8</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.5571</td><td>2.4057</td></tr>
<tr><td rowspan="4">9</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.9906</td><td>4.2786</td></tr>
<tr><td rowspan="4">10</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.0000</td><td>0.0001</td></tr>
<tr><td rowspan="4">11</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.0001</td><td>0.0005</td></tr>
<tr><td rowspan="4">12</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.0001</td><td>0.0004</td></tr>
<tr><td rowspan="4">13</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.0001</td><td>0.0004</td></tr>
<tr><td rowspan="4">14</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.0000</td><td>0.0000</td></tr>
<tr><td rowspan="4">15</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0.3232</td><td>1.0579</td></tr>
<tr><td rowspan="3">16</td><td rowspan="2">0.0000</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td></tr>
<tr><td>0.0000</td><td>0</td><td>0</td></tr>
</table>

17	0.0000	1.3978	4.6884
		0	0
	0.0000	0	0
		0	0
18	0.0000	2.0651	7.4048
		0	0
	0.0000	0	0
		0	0
19	0.0000	0.9906	4.2786
		0	0
	0.0000	0	0
		0	0
20	0.0000	0.0001	0.0004
		0	0
	0.0000	0	0
		0	0
21	0.0000	0.0002	0.0006
		0	0
	0.0000	0	0
		0	0
22	0.0000	0.0000	0.0000
		0	0
	0.0000	0	0
		0	0
23	0.0000	0.0442	0.1484
		0	0
	0.0000	0	0
		0	0
24	0.0000	0.3232	1.0579
		0	0
	0.0000	0	0
		0	0
25	0.0000	0.3319	1.0532
		0	0
	0.0000	0	0
		0	0
26	0.0000	1.3978	4.6884
		0	0
	0.0000	0	0
		0	0
27	0.0000	1.2628	4.4022
		0	0
	0.0000	0	0
		0	0
28	0.0000	2.0652	7.4048
		0	0
	0.0000	0	0
		0	0
29	0.0000	0.5571	2.4058
		0	0
	0.0000	0	0
		0	0
30	0.0000	0.9907	4.2787
		0	0
	0.0000	0	0
		0	0
31	0.0000	0.0000	0.0002
		0	0
	0.0000	0	0
		0	0
32	0.0000	0.0003	0.0011
		0	0
	0.0000	0	0
		0	0
33	0.0000	0.0002	0.0009
		0	0
	0.0000	0	0
		0	0
34	0.0000	0.0003	0.0012
		0	0
	0.0000	0	0
		0	0
35	0.0000	0.0000	0.0000
		0	0
		0	0

	0.0000	0	0
36	0.0000	0.0000	0.0000
		0	0
	0.0000	0	0
		0	0
37	0.0000	0.0000	0.0000
		0	0
	0.0000	0	0
		0	0
38	0.0000	0.0000	0.0000
		0	0
	0.0000	0	0
		0	0
39	0.0000	0.0000	0.0001
		0	0
	0.0000	0	0
		0	0
40	0.0000	0.0001	0.0004
		0	0
	0.0046	0	0
		0	0
41	0.0009	0.0002	0.0006
		0	0
	0.0046	0	0
		0	0
42	0.0009	0.0000	0.0001
		0	0
	0.0046	0	0
		0	0
43	0.0009	0.0000	0.0002
		0	0
	0.0046	0	0
		0	0
44	0.0009	0.0000	0.0011
		0	0
	0.0046	0	0
		0	0
45	0.0009	0.0000	0.0009
		0	0
	0.0046	0	0
		0	0
46	0.0009	0.0000	0.0030
		0	0
	0.0139	0	0
		0	0
47	0.0027	0.0000	0.0021
		0	0
	0.0139	0	0
		0	0
48	0.0027	0.0001	0.0255
		0	0
	0.0138	0	0
		0	0
49	0.0027	0.0000	0.0249
		0	0
	0.0138	0	0
		0	0
50	0.0027	0.0002	0.0797
		0	0
	0.0277	0	0
		0	0
51	0.0054	0.0001	0.0559
		0	0
	0.0277	0	0
		0	0
52	0.0054	0.0008	0.3581
		0	0
	0.0277	0	0
		0	0
53	0.0054	0.0004	0.3112
		0	0
	0.0055	0	0
		0	0
54	0.0022	0.0005	0.3008
		0	0

		0	0
	0.0055	0	0
		0.0000	0.0000
55	0.0023	0	0
		0	0
	0.0055	0	0
		0.0000	0.0010
56	0.0023	0	0
		0	0
	0.0164	0	0
		0.0000	0.0028
57	0.0068	0	0
		0	0
	0.0164	0	0
		0.0000	0.0166
58	0.0068	0	0
		0	0
	0.0327	0	0
		0.0001	0.0760
59	0.0135	0	0
		0	0
	0.0327	0	0
		0.0003	0.2729
60	0.0135	0	0
		0	0
	0.0046	0	0
		0.0006	0.4326
61	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0001
62	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0004
63	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0023
64	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0018
65	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0072
66	0.0139	0	0
		0	0
	0.0027	0	0
		0.0000	0.0041
67	0.0139	0	0
		0	0
	0.0027	0	0
		0.0001	0.0454
68	0.0138	0	0
		0	0
	0.0027	0	0
		0.0001	0.0480
69	0.0138	0	0
		0	0
	0.0009	0	0
		0.0003	0.1939
70	0.0027	0	0
		0	0
	0.0277	0	0
		0.0001	0.1099
71	0.0054	0	0
		0	0
	0.0277	0	0
		0.0013	0.7466
72	0.0054	0	0
		0	0
	0.0277	0	0
		0.0007	0.6074

73	0.0054	0	0
		0	0
	0.0055	0	0
		0.0012	0.8513
74	0.0022	0	0
		0	0
	0.0055	0	0
		0.0000	0.0000
75	0.0023	0	0
		0	0
	0.0055	0	0
		0.0000	0.0010
76	0.0023	0	0
		0	0
	0.0164	0	0
		0.0000	0.0028
77	0.0068	0	0
		0	0
	0.0164	0	0
		0.0000	0.0166
78	0.0068	0	0
		0	0
	0.0327	0	0
		0.0001	0.0760
79	0.0135	0	0
		0	0
	0.0327	0	0
		0.0003	0.2729
80	0.0135	0	0
		0	0
	0.0046	0	0
		0.0006	0.4326
81	0.0009	0	0
		0	0
	0.0046	0	0
		0.0000	0.0001
82	0.0009	0	0
		0	0
	0.0009	0	0
		0.0000	0.0004
83	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0023
84	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0018
85	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0072
86	0.0046	0	0
		0	0
	0.0009	0	0
		0.0000	0.0041
87	0.0139	0	0
		0	0
	0.0027	0	0
		0.0001	0.0454
88	0.0139	0	0
		0	0
	0.0027	0	0
		0.0001	0.0480
89	0.0138	0	0
		0	0
	0.0027	0	0
		0.0003	0.1939
90	0.0138	0	0
		0	0
	0.0027	0	0
		0.0001	0.1099
91	0.0277	0	0
		0	0
	0.0054	0	0

		0.0013	0.7466
92	0.0277	0	0
		0	0
	0.0054	0	0
		0.0007	0.6074
93	0.0277	0	0
		0	0
	0.0054	0	0
		0.0012	0.8513
94	0.0055	0	0
		0	0
	0.0022	0	0
		0.0000	0.0000
95	0.0055	0	0
		0	0
	0.0023	0	0
		0.0000	0.0010
96	0.0055	0	0
		0	0
	0.0023	0	0
		0.0000	0.0028
97	0.0164	0	0
		0	0
	0.0068	0	0
		0.0000	0.0166
98	0.0164	0	0
		0	0
	0.0068	0	0
		0.0001	0.0760
99	0.0327	0	0
		0	0
	0.0135	0	0
		0.0003	0.2729
101	0.0327	0	0
		0	0
	0.0135	0	0
		0.0006	0.4326
102	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0003
103	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0006
104	0.0093	0	0
		0	0
	0.0018	0	0
		0.0000	0.0034
105	0.0093	0	0
		0	0
	0.0018	0	0
		0.0000	0.0027
106	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0102
107	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0062
108	0.0277	0	0
		0	0
	0.0054	0	0
		0.0001	0.0709
109	0.0277	0	0
		0	0
	0.0054	0	0
		0.0001	0.0729
110	0.0277	0	0
		0	0
	0.0054	0	0
		0.0004	0.2734
111	0.0277	0	0
		0	0

	0.0054	0	0
		0.0002	0.1658
112	0.0553	0	0
		0	0
	0.0108	0	0
		0.0019	1.1041
113	0.0553	0	0
		0	0
	0.0108	0	0
		0.0010	0.9181
114	0.0553	0	0
		0	0
	0.0108	0	0
		0.0015	1.1512
115	0.0109	0	0
		0	0
	0.0045	0	0
		0.0000	0.0001
116	0.0110	0	0
		0	0
	0.0045	0	0
		0.0000	0.0019
117	0.0109	0	0
		0	0
	0.0045	0	0
		0.0000	0.0056
118	0.0328	0	0
		0	0
	0.0135	0	0
		0.0000	0.0332
119	0.0327	0	0
		0	0
	0.0135	0	0
		0.0002	0.1520
120	0.0655	0	0
		0	0
	0.0270	0	0
		0.0007	0.5457
121	0.0655	0	0
		0	0
	0.0270	0	0
		0.0012	0.8651
122	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0003
123	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0007
124	0.0093	0	0
		0	0
	0.0018	0	0
		0.0000	0.0046
125	0.0093	0	0
		0	0
	0.0018	0	0
		0.0000	0.0036
126	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0144
127	0.0092	0	0
		0	0
	0.0018	0	0
		0.0000	0.0082
128	0.0277	0	0
		0	0
	0.0054	0	0
		0.0002	0.0908
129	0.0277	0	0
		0	0
	0.0054	0	0
		0.0001	0.0960
130	0.0277	0	0

		0	0
	0.0054	0	0
		0.0007	0.3878
131	0.0277	0	0
		0	0
	0.0054	0	0
		0.0002	0.2199
132	0.0553	0	0
		0	0
	0.0108	0	0
		0.0026	1.4931
133	0.0553	0	0
		0	0
	0.0108	0	0
		0.0015	1.2148
134	0.0553	0	0
		0	0
	0.0108	0	0
		0.0024	1.7025
135	0.0109	0	0
		0	0
	0.0045	0	0
		0.0000	0.0001
136	0.0110	0	0
		0	0
	0.0045	0	0
		0.0000	0.0019
137	0.0109	0	0
		0	0
	0.0045	0	0
		0.0000	0.0056
138	0.0328	0	0
		0	0
	0.0135	0	0
		0.0000	0.0332
139	0.0327	0	0
		0	0
	0.0135	0	0
		0.0002	0.1520
140	0.0655	0	0
		0	0
	0.0270	0	0
		0.0007	0.5457
141	0.0655	0	0
		0	0
	0.0270	0	0
		0.0012	0.8651
142	0.0138	0	0
		0	0
	0.0027	0	0
		0.0000	0.0004
143	0.0138	0	0
		0	0
	0.0027	0	0
		0.0000	0.0010
144	0.0139	0	0
		0	0
	0.0027	0	0
		0.0000	0.0057
145	0.0139	0	0
		0	0
	0.0027	0	0
		0.0000	0.0045
146	0.0139	0	0
		0	0
	0.0027	0	0
		0.0000	0.0174
147	0.0139	0	0
		0	0
	0.0027	0	0
		0.0000	0.0102
148	0.0416	0	0

		0	0
		0	0
	0.0081	0.0002	0.1163
149	0.0416	0	0
		0	0
	0.0081	0	0
		0.0002	0.1209
150	0.0415	0	0
		0	0
	0.0081	0	0
		0.0008	0.4673
151	0.0415	0	0
		0	0
	0.0081	0	0
		0.0003	0.2757
152	0.0830	0	0
		0	0
	0.0162	0	0
		0.0032	1.8507
153	0.0830	0	0
		0	0
	0.0162	0	0
		0.0018	1.5255
154	0.0830	0	0
		0	0
	0.0162	0	0
		0.0027	2.0025
155	0.0164	0	0
		0	0
	0.0067	0	0
		0.0000	0.0001
156	0.0165	0	0
		0	0
	0.0068	0	0
		0.0000	0.0029
157	0.0164	0	0
		0	0
	0.0068	0	0
		0.0000	0.0085
158	0.0492	0	0
		0	0
	0.0203	0	0
		0.0001	0.0499
159	0.0491	0	0
		0	0
	0.0203	0	0
		0.0003	0.2279
160	0.0982	0	0
		0	0
	0.0405	0	0
		0.0010	0.8186
161	0.0982	0	0
		0	0
	0.0405	0	0
		0.0019	1.2977
162	0.0155	0	0
		0	0
	0.0054	0	0
		0.0000	0.0000
163	0.0206	0	0
		0	0
	0.0135	0	0
		0.0000	0.0005
164	0.0156	0	0
		0	0
	0.0054	0	0
		0.0000	0.0037
165	0.0207	0	0
		0	0
	0.0136	0	0
		0.0000	0.0026
166	0.0156	0	0
		0	0
	0.0054	0	0
		0.0000	0.0127

167	0.0206	0	0
		0	0
	0.0135	0	0
		0.0000	0.0060
168	0.0466	0	0
		0	0
	0.0162	0	0
		0.0001	0.0598
169	0.0618	0	0
		0	0
	0.0406	0	0
		0.0001	0.0694
170	0.0466	0	0
		0	0
	0.0162	0	0
		0.0007	0.3433
171	0.0618	0	0
		0	0
	0.0405	0	0
		0.0002	0.1622
172	0.0931	0	0
		0	0
	0.0324	0	0
		0.0023	1.1670
173	0.1235	0	0
		0	0
	0.0324	0	0
		0.0014	0.8900