Enrollment No.:

JAYPEE UNIVERSITY OF INFORMATION TECHNOLOGY, WAKNAGHAT

TEST -1 EXAMINATIONS-2022

Ph.D.-I Semester (Mathematics)

COURSE CODE (CREDITS): 17P1WMA231 (3) MAX. MARKS: 15

COURSE NAME: ADVANCED LINEAR ALGEBRA

COURSE INSTRUCTOR: Pradeep Kumar Pandey MAX. TIME: 1 Hour

Note: *All questions are compulsory. Marks are indicated against each question in square brackets. Mobile Phones, smart watches, ear phones, calculators, and any other electronic gadgets etc. during the Examination is prohibited.*

Q1. (Under the usual matrix operations) prove or disprove that: [CO-1] [1+1]

(i) $V = \left\{ \begin{pmatrix} a & 1 \\ b & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$ is a vector space.

(ii) $V = \{(x, y, z)^t : (x, y, z)^t \in \mathbb{R}^3;\ x + y + z = 1\}$ is a vector space.

Q2. Consider a vector $\boldsymbol{v} = (a, b, c)^t \in \mathbb{R}^3$ with respect to the basis $B = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$, where $\boldsymbol{v_1} = \boldsymbol{e_3}, \boldsymbol{v_2} = \boldsymbol{e_1}, \boldsymbol{v_3} = \boldsymbol{e_2}$. Compute the coordinate vector $[\boldsymbol{v}]_B$. [CO-1] [2]

Q3. Let $B = \{\boldsymbol{e_1}, \boldsymbol{e_2}\}$, and $C = \{(1,1)^t, (-1,1)^t\}$ are two bases of $\mathbb{R}^2$. Compute: [CO2] [2+2]

(i) the change of coordinate matrix from B to C.

(ii) the change of coordinate matrix from C to B.

Q4: Find the eigenvalues of matrix $A = \begin{pmatrix} 2 & 0 & 0 \\ 4 & 2 & 0 \\ 6 & 0 & 2 \end{pmatrix}$, and investigate their algebraic multiplicity and geometric multiplicity. [CO-2] [4]

Q5. Compute the characteristic polynomial, and minimal polynomial for the following matrix:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ [CO-2] [3]

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