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De-noising of ultrasound image using Bayesian approached heavy-tailed Cauchy distribution

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Abstract Medical ultrasound images are used in clinical diagnosis and generally degraded by speckle noise. This makes difficulty in automatic interpretation of diseases in ultrasound images. This paper presents a speckle removal algorithm by modeling the wavelet coefficients. A Bayesian approach is implemented to find the noise free coefficients. Cauchy prior and Gaussian Probability Density Function (PDF) are used to model the true wavelet coefficients and noisy coefficients respectively. A Maximum a Posteriori (MAP) estimator is used to estimate the noise free wavelet coefficients. A Median Absolute Deviation (MAD) estimator is used to find the variance of affected wavelet coefficients in finest scale. The proposed method is compared with existing denoising methods. The experimental results show that the method offer up to 21.48% enhancement in Peak Signal to Noise Ratio (PSNR), 1.82% enhancement in Structural Similarity Index (SSIM), 1% enhancement in Correlation coefficient (ρ) and 7.68% enhancement in Edge Preserving Index (EPI) than best existing wavelet modeling method. The results indicate that the proposed method outperforms over existing methods, both in noise reduction and edge preservation.

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1 Introduction

Ultrasound image, a real time imaging system plays an important role in medical diagnosis. Among several medical imaging modalities, ultrasound imaging modality is a powerful imaging technique and has been widely used due to its safe, economic and portable nature. It provides useful information about different parts of human body. High frequency sound waves are used to image internal body organs like kidney, liver, joints, vessels and musculoskeletal. Speckle noise degrades the image quality of ultrasound image [17, 25]. Speckle noise is a multiplicative noise and it is generated due to imaging system. Fundamental structure of ultrasound image is too small. Ultrasound imaging system uses transducer, which produces ultrasonic waves. These ultrasonic waves are passed through the internal organs of human body. The transducer is vibrated by the returned sound waves and converts them into electrical pulses. An ultrasonic scanner is used to convert electrical pulses into digital images, known as ultrasound image. The air gap between the human body and probe of the transducer may produce noise in the image. Ultrasound images are acquired using narrow band detection systems which results speckle, an undesirable granular structure.

Despeckling is an important preprocessing step used in medical science for analysis, extraction and recognition of features from imagery measurements. Several despeckling methods have been proposed, those can be classified as algorithmic approached and transform domain approached filters. Algorithmic approached filters can be classified as spatially domain filters and frequency domain filters. Different spatially domain filters (median filter, adaptive weighted filters, average filter, wiener filter) and frequency domain filters are efficient to suppress additive noise, However, these filters are fail to suppress the multiplicative noise. Spatial adaptive filters like Kuan filter, Lee filter and Frost filter are widely used to reduce speckle noise. The algorithms used to de-noise the additive noise are ineffective to preserve the information. Diffusion filters [35], non-local mean filter [37] and bilateral filters [28] are proposed in literature to recover the noise free ultrasound image corrupted by speckle noise. Recently, transform domain techniques are the important tool to recover signal from noisy data. Application of wavelet transform and contourlet transform to the noisy data, it is possible to reduce the speckle noise more effectively than the spatial and frequency domain filters. Wavelet transform domain filtering techniques based on thresholding have been proposed to suppress the speckle noise [14–16, 22]. It is proved that wavelet based methods recover signals from noisy data more accurately than the spatially and frequency domain filters [20, 21] because of its multiresolution approach. Wavelet transform decompose the image into multiscale details and wavelet based filters use both frequency and location information to suppress noise. Denoising using Bayesian estimator can perform better in denoising than the wavelet thresholding techniques [6, 18]. Wavelet based methods are not accurately suppress the noise because it depends on the correct choice of threshold. So wavelet shrinkage techniques are now the area of interest. Wavelet shrinkage techniques apply wavelet transform that converts the affected image (data) into wavelet coefficients. Estimators are used to shrink the generated wavelet coefficients and the denoised signal is recovered by applying inverse wavelet transform. All the above methods work efficiently in case of additive noise. Homomorphic approach is used to convert multiplicative noise (speckle noise) into additive noise [4]. The unwanted random signal which gets multiplied with the original signal is called multiplicative noise and which gets added and independent to the original signal, is called additive noise. In case of speckle noise the noisy signal can be represented as the multiplication of input signal and noisy signal, where as in case of additive noise the noisy signal can be represented as the addition of original signal and noisy signal. Homomorphic approach uses the concept of logarithmic transformation to convert speckle noise into additive noise [1] and then additive noise suppression techniques are implemented to recover the data. Wavelet methods in homomorphic environment provide better suppression of speckle noise than other spatial domain filters [7]. It has been shown in [3, 9, 27, 38] that wavelet methods in Bayesian environment work more accurately than the thresholding methods. Several techniques have been proposed in literature, those make use of Bayesian estimators. The noise free wavelet coefficients are extracted by choosing a suitable estimator. An appropriate PDF is required to model statistical behavior of wavelet coefficients both for true and noisy data. Type of PDF and estimator has a great impact on the denoising performance.

Rabbani et al. [33] propose MAP and minimum mean square error (MMSE) estimator for Gaussian mixture prior and Laplacian Mixture prior, assuming Rayleigh distribution and Gaussian distribution for noise. Achim et al. [2] present a MMSE estimator for alpha-stable distribution. Sadreazami et al. [36] propose Cauchy prior to model contourlet coefficients and MAP estimator to denoise the speckle noise from ultrasound image. MMSE estimator utilizing the normal inverse Gaussian prior is proposed by Bhuiyan et al. [8]. Ranjani et al. [13] propose a Levy distribution for denoising ultrasound image. Bhuiyan et al. [7] propose a Minimum Mean Absolute Error (MMAE) estimator for Cauchy prior. Lu et al. [29] developed a despeckling technique using Laplace mixture prior and MAP estimator. The authors applied the prior, for modeling directionlet transform coefficients. Biban and Amindavar [10] designed a mixture prior using Cauchy and Rayleigh distribution and applied to mixture ratio estimator. Jafari and Ghofrani [24] approached Levy model and MAP estimator to recover the non subsampled shearlet transform coefficients from ultrasound image. Chang et al. [11] proposed a denoising method that works in Bayesian framework. The authors proposed a generalized Gaussian prior and used it on wavelet coefficients.

The denoising efficiency depends on the distribution prior and estimator. In this paper, to suppress the multiplicative noise homomorphic approach is implemented. A new spatially adaptive wavelet based despeckling method that uses Bayesian approach with Cauchy prior is presented. MAP estimator is applied to estimate true wavelet coefficients. Noisy coefficients are modeled using Gaussian PDF. The rest of the paper is organized as follows. Section 2 contains a brief preliminary on statistical modeling of wavelet coefficient, Maximum a Posteriori (MAP) Estimator and Estimation of noise variances. The proposed method is presented in Section 3. The results and analysis of the work is discussed in Section 4. Section 5 presents the conclusions of the proposed research.

2 Preliminaries

2.1 Modeling of wavelet coefficients

The application of wavelet transform converts the logarithmic image in to sub-bands. Wavelet decomposition to m level results sub-bands LLm, LHn, HLn and HHn for $n = 1$, 2… m [30]. Low frequency portion, LLm sub-band is taken as approximation. This sub-band restores all the information. Rest of the sub-bands, LHn, HLn and HHn gives the horizontal feature, vertical feature and diagonal feature of the input image respectively. Wavelet decomposition has the property of orthogonality. This preserves the statistical properties of spatial domain in frequency domain [26]. The distribution of wavelet coefficients is non-Gaussian in nature. Figure 1 shows the distribution of wavelet coefficients in ultrasound image. The distributed coefficients are conditionally independent. They are of zero mean with heavy tails [30]. Within sub-bands, the wavelet coefficients are dependent and are locally stationary [12]. The modeling of wavelet coefficients with suitable PDF is critical in the issue. The distributions of the wavelet coefficients are not normal. Figure 2 shows the normal probability plot for horizontal sub-band for first level decomposition of ultrasound image. This plot verifies that the data distribution is not normal. The circles show the empirical data versus probability value. The straight line in the plot is the Gaussian line and the circles do not follow it. Figure 3 shows the distribution fit of the vertical wavelet coefficients for both Cauchy and normal PDFs. It is seen from the plot that Cauchy PDF models the data more accurately than normal PDF.

The denoising issue is to recover the noise free wavelet coefficients from the observed wavelet coefficients. Assuming dependency between these wavelet coefficients, give better performance rather than the assumption of independency [19]. It is assumed that, the wavelet coefficients are random variables, distributed with suitable PDF, given their variances. Assumption of accurate distribution retrieves the noise free coefficients accurately. The employed PDF for noise free data and noise plays a significant role in the

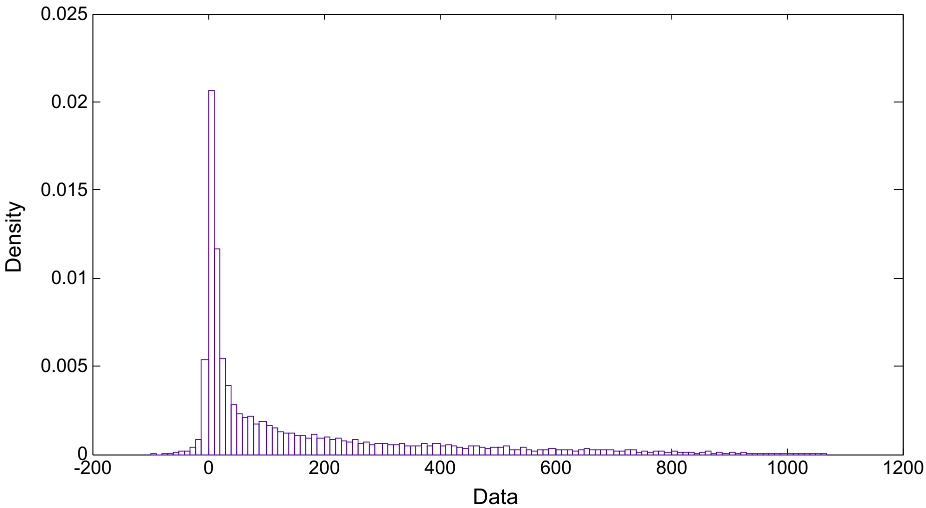

Fig. 1 Distribution of wavelet coefficients of ultrasound image

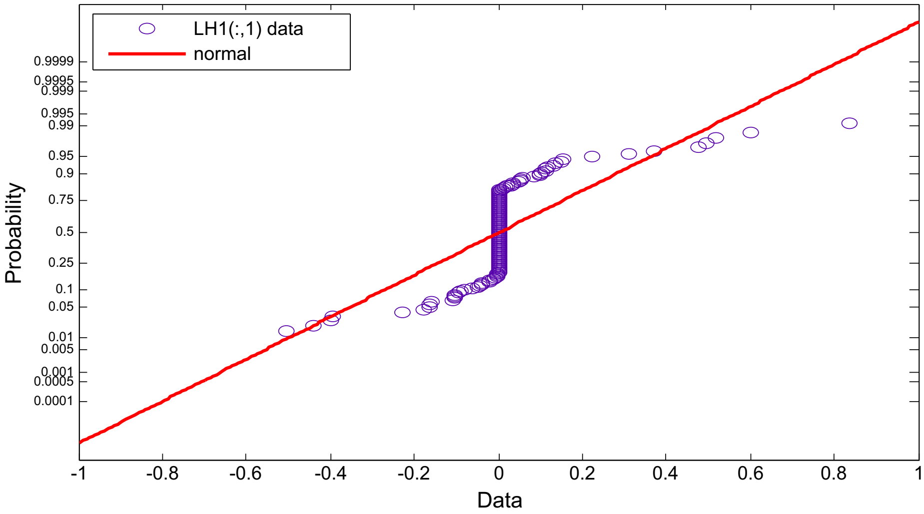

Fig. 2 Probability versus data curve for original ultrasound image

performance of the noise reduction process. Achim et al. [2] attempted alpha-stable PDF to model wavelet coefficients. However, it is not suitable to estimate due to non-existence of closed form expression of alpha-stable PDF. Bhuiyan et al. [8] worked with symmetric normal inverse Gaussian PDF. Gaussian and Cauchy PDFs have the closed-form expressions which makes it possible to be evaluated through finite number of operations. Cauchy PDF has symmetric and unimodal property that makes it suitable for noise free parameter estimation. Cauchy PDF is a stable distribution, as it is a linear combination of location parameter and scale parameter, so it can be expressed analytically. Bayesian estimator with Cauchy prior in wavelet domain has been shown a successful removal of speckle noise in Synthetic Aperture Radar (SAR) images [5]. Logarithmic transform converts multiplicative noise in to zero mean additive white Gaussian noise. The standard deviation of the

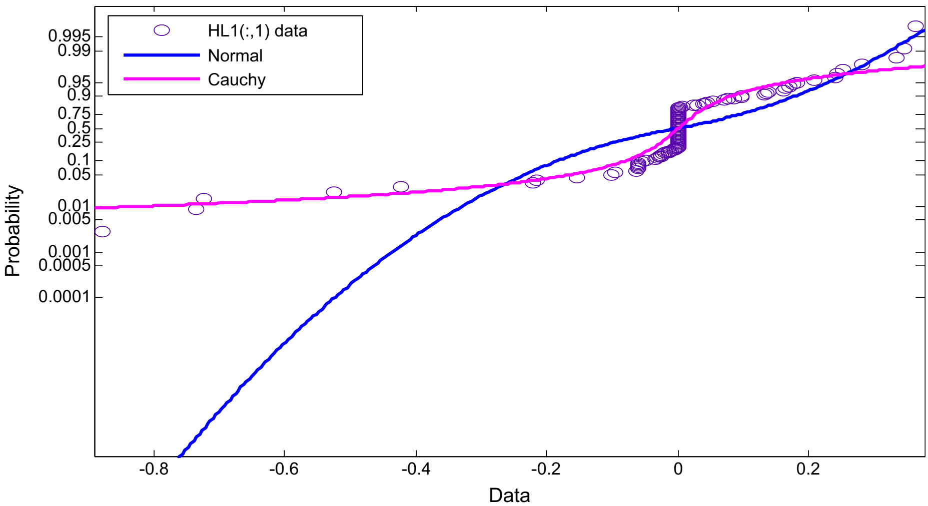

Fig. 3 Modeling of the ultrasound image wavelet coefficients with Cauchy and normal density function

wavelet transferred additive Gaussian noise is same as logarithmic transferred additive Gaussian noise due to the orthogonal property. Orthogonal wavelets have a decorrelation property. For orthonormal wavelet transform, the wavelet transform of the white noise is also white noise of same amplitude. White noise is spread out equally over all coefficients. The PDF of the wavelet transferred white Gaussian noise is given by

$$P_{\eta}(\eta) = \frac{e^{-\frac{\eta^2}{2\sigma_{\eta}^2}}}{\sqrt{2\pi\sigma_{\eta}^2}} \tag{1}$$

Where, σ_{η}^2 >0, is the variance of Gaussian PDF. $\eta \in \mathbb{R}$ and mean of the distribution is assumed to be zero.

The Cauchy PDF is given by

$$P_s(s) = \left(\frac{1}{\pi}\right)\left(\frac{\gamma}{s^2+\gamma^2}\right) \tag{2}$$

Where γ >0 (real), the scaling parameter, specifies half width at half maximum. $s \in (-\infty, +\infty)$. Location parameter is assumed as zero.

Cauchy PDF is a continuous probability distribution. The basic properties of Cauchy PDF include stability and heavy-tail feature. At lower scale parameters, Cauchy PDF shows highly impulsive behavior. The graph of Cauchy PDF for different values of scaling parameter is shown in the Fig. 4.

In the first step of the proposed method, the input speckled image is log transformed to convert speckle noise to additive noise. The generated image is then decomposed using discrete wavelet transform (DWT), which results wavelet coefficients. Wavelet decomposition is to decompose

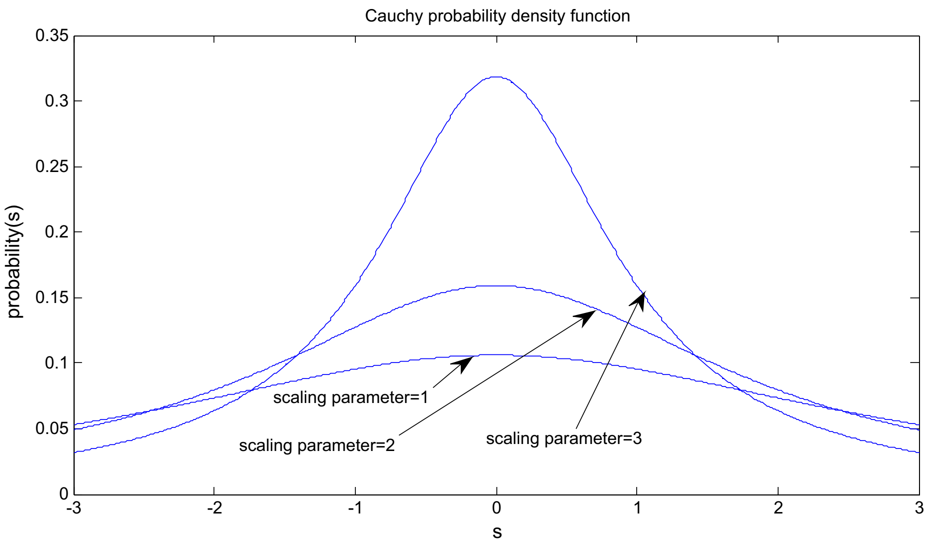

Fig. 4 Cauchy PDF for different values of scaling parameter (γ)

the input image into four low resolution sub-bands namely, LL, LH, HL and HH. Two low-pass filters are used to find the coefficients in LL sub-band and known as approximation coefficients. LH sub-band contains horizontal coefficients and can be found out by using low-pass filter and then high-pass filter. HL sub-band contains vertical coefficients and can be found out by using high-pass filter and then low-pass filter. Two high-pass filters are used to find the coefficients in HH sub-band and known as diagonal coefficients. Most of the noisy coefficients are available in HH sub-band. The complete process can be defined as follows:

If

$I_{x,y}$ = noisy ultrasound image
$S_{x,y}$ = a noisy free image.
$\eta_{x,y}$ = multiplicative noise (speckle noise) component.

The noisy ultrasound image can be defined as, whereas, ignoring the additive noise in the image

$$I_{x,y} = S_{x,y} \times \eta_{x,y} \tag{3}$$

Applying logarithmic function to eq. (3)

$$logI_{x,y} = logS_{x,y} + log\eta_{x,y} \tag{4}$$

Wavelet transform is linear in nature. By applying the wavelet transform to the eq. (2) a set of wavelet coefficients can be generated as eq. (5)

$$I^k_{m,n} = S^k_{m,n} + \eta^k_{m,n} \quad n = 0 \ldots 2^{i+m}-1 \tag{5}$$

where, m is the decomposition level and $-1< m <- i$ and k =1,2 mention the spatial orientations.

Figure 5 shows the three scale decomposition of the ultrasound image. The upper left portion of the image is the approximation sub-band at third level. Upper right portion of the image is the vertical sub-band at level one. Lower left shows the horizontal sub-band at level one and lower right shows the diagonal sub-band at level one.

2.2 Maximum a posteriori (MAP) estimator

The maximum a posteriori (MAP) estimator is used to retrieve the noise free coefficients, assuming the coefficients are distributed by a suitable prior. This estimator is applied to the generated wavelet coefficients to filter out the noisy coefficients. Bayesian theory has the requirement of a suitable prior distribution that accurately estimates the noise variance and signal parameters from the wavelet coefficients. By applying homomorphic approach to the image corrupted by speckle noise, the authors are able to convert the multiplicative noise in to additive noise. $log\eta_{x,y}$ and $logI_{x,y}$ are independent identical

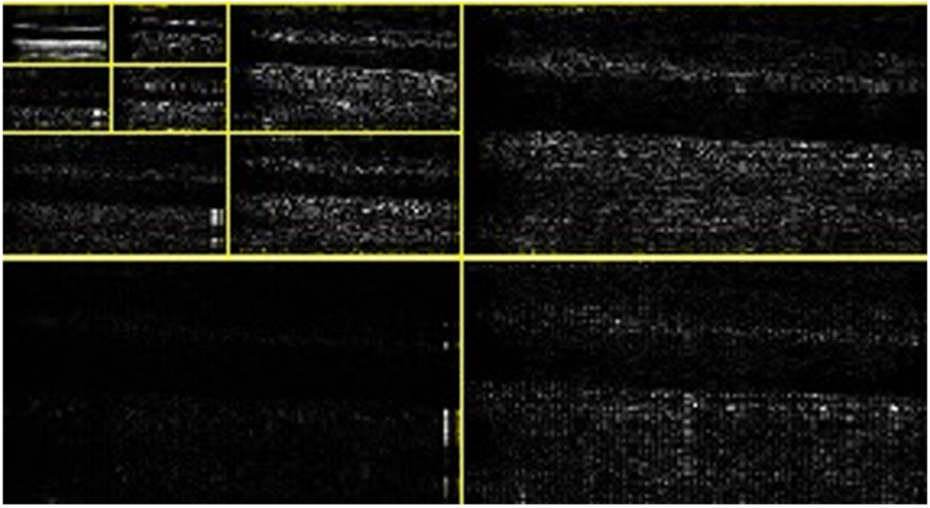

Fig. 5 Three level wavelet decomposition of ultrasound image

distributions. The problem is to retrieve the noise free coefficients from the log transformed image. Let the estimation of $logS_{x,y}$ is $\hat{log}S_{x,y}$. The problem is to minimize the Mean Squared Error (MSE) given by

$$MSE\left(\widehat{log}\left(S_{x,y}\right), \log\left(S_{X,Y}\right)\right) = \frac{1}{M^2}\left\|\widehat{log}\left(S_{x,y}\right) - log\left(S_{x,y}\right)\right\|^2$$
$$= \frac{1}{M^2}\sum_{x,y=1}^{M}\left(\widehat{log}\left(S_{x,y}\right) - log\left(S_{x,y}\right)\right)^2 \quad (6)$$

Where M is the size of the image. Due to the orthogonality property of DWT, the log transformed speckle noise can be approximated by Gaussian distribution with mean = 0 and variance σ_η^2. A suitable prior is added to the Maximum Likelihood (ML) estimate which results MAP estimator. MAP is defined as Likelihood multiplied by a prior. The problem is to find the Bayesian estimate of the noise free true wavelet coefficients. The Bayesian MAP estimate of the noise free coefficients are given by

$$\hat{S}(I) = arg\,\max_S \left[P_{I|S}(I|S)P_S(S)\right] \quad (7)$$

$$\hat{\mathrm{S}}(\mathrm{I}) = \arg\max_{\mathrm{S}} \mathrm{P}_\eta(\mathrm{I}-\mathrm{S})\mathrm{P}_\mathrm{S}(\mathrm{S}) \quad (8)$$

Where I, S and η are assumed as random variables referring to eq. (5). $P_S(S)$ is the prior of the noise free wavelet coefficients and defined by Cauchy distribution and defined by eq. (2).

$P_{I|S}(I|S)$ is the Likelihood function. $P_\eta(I-S) = P_\eta(\eta)$ is assumed as Gaussian distribution and defined as eq. (1).

The solution of eq. (8) can be simplified by applying shrinkage approach [23]

$$\hat{S}(I) = sign(I)\max\left(0, |I| - \sigma_\eta^2\left|\frac{2I}{\gamma^2 + I^2}\right|\right) \tag{9}$$

The scaling parameter γ can be found out by minimizing [5]

$$\int_{-\infty}^{+\infty}\left|\hat{\phi}_I(\omega) - \left(\phi_S(\omega)\phi_\eta(\omega)\right)\right| e^{(-\omega)^2} d\omega \tag{10}$$

Where $\hat{\phi}_I(\omega)$ is the empirical characteristics function corresponding to random variable I.

$\phi_S(\omega)$ and $\phi_\eta(\omega)$ are characteristics function of random variables S and η respectively and defined as $\phi_S(\omega) = e^{(-\gamma|\omega|)}$ and $\phi_\eta(\omega) = e^{\left(-\left(\sigma_\eta^2/2\right)|\omega|^2\right)}$

Equation (10) can be solved by applying Hermite Gauss quadrature rule [32] as

$$\int_{-\infty}^{+\infty} f(\omega) e^{(-\omega)^2} d\omega \approx \sum_{r=1}^{L} C_r f(\omega_r) \tag{11}$$

Where $f(\omega) = \hat{\phi}_I(\omega) - \left(\phi_S(\omega)\phi_\eta(\omega)\right)$

ω_r are the roots of the polynomial of order L and C_r are the weights of the Hermite quadrature polynomial.

2.3 Estimation of noise variances

The estimation of noise variance is the most important step in the de-noising technique, based on wavelet coefficients. MAP estimator depends upon the quality of the noise variance estimator. The estimation of true wavelet coefficients need information of $\hat{\sigma}_\eta^2$. To estimate the Gaussian noise variance $\hat{\sigma}_\eta^2$, a Median Absolute Deviation (MAD) estimator is used [34].

$$\hat{\sigma}_\eta^2 = \left(\frac{median(|l_i|)}{0.6745}\right)^2 \tag{12}$$

Where l_i € subband HH in finest scale.

3 Proposed method

In this method, a heavy tailed Cauchy prior is used to approximate the noise free coefficients using MAP estimator. Gaussian model is used to approximate the noisy wavelet coefficients following the orthogonal property of wavelet transform. The noisy wavelet coefficients are approximated in HH sub-band using MAD estimator. The detail of the proposed method is presented in Fig. 6. Further, algorithm steps for removing speckle noise from ultrasound image by the proposed method are follows:

STEP 1: Wavelet decomposition of logarithmic transformed ultrasound image.
STEP 2: Modeling of wavelet coefficients in Cauchy PDF.
STEP 3: Estimate the scaling parameter γ using eq. (10).
STEP 4: Estimate the variance $\hat{\sigma}_{\eta}^{2}$ using median estimator using eq. (12).
STEP 5: Using equation (9) and the values of γ and $\hat{\sigma}_{\eta}^{2}$, find the Bayesian MAP estimates of the true coefficients.
STEP 6: Perform the Inverse Discrete Wavelet Transform (IDWT) to recover the de-noised image.
STEP 7: Finding exponential transformation of the data obtained in STEP 6.

4 Simulation results and performance analysis

The proposed method is tested on ultrasound image and phantom image of size (538 × 340) [31] and (256 × 256) respectively, both qualitatively and quantitatively. MATLAB R2015a has been used to simulate the proposed method. Daubechies 8 (db8) wavelet of third level is applied for simulation. Figures 7 and 8 shows the qualitative comparison of the proposed despeckling method with various existing methods like median, wiener and wavelet thresholding (Hard & Soft) and state-of-the-art methods [11, 13]. The parameters of the standard filters, Levy shrink [13] and Bayes shrink [11] are assumed using trial and error methods to get optimum efficiency. Performance parameters are used to make the quantitative comparison of the proposed method with various methods. The performance parameters are Peak Signal to Noise Ratio (PSNR) and Signal to Mean Squared Error (S/mse) ratio. S/mse ratio is a good measure of noise suppression in case of multiplicative noise [20] and defined as the ratio of signal power to the mean squared error. A high value of PSNR and S/mse ratio is required for a good quality image. The performance parameters are defined as follows.

$$PSNR = 20\, log_{10} \frac{255}{\sqrt{MSE}} \tag{13}$$

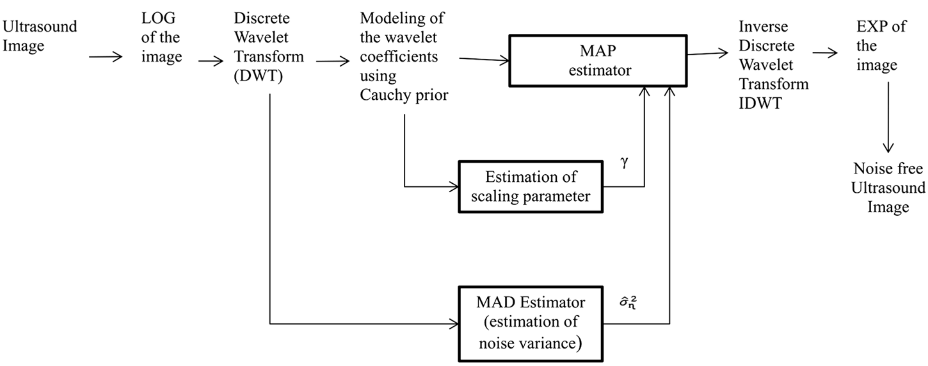

Fig. 6 Block diagram of the proposed method. (LOG: Logarithmic transformation and EXP: Exponential transformation)

MSE is the Mean Squared Error and defined as

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m \times n} \left(\hat{I}_{x,y} - I_{x,y} \right)^2 \tag{14}$$

Where $I_{x,y}$ and $\hat{I}_{x,y}$ are original and despeckled image respectively. $m \times n$ is the image size.

$$S/mse = 10log_{10} \left(\sum_{i=1}^{m \times n} I_{x,y}^2 / \sum_{i=1}^{m \times n} \left(\hat{I}_{x,y} - I_{x,y} \right)^2 \right) \tag{15}$$

Tables 1 and 2 show the PSNR and *S/mse* values obtained from various denoising methods for phantom image at different multiplicative noise variances (σ_η^2). This table indicates that the PSNR value of the proposed method is higher than existing methods. With reference to this table, PSNR value is obtained by the method is approximately 36 dB (σ_η =0.1). It is observed that the method offer 21.48%, 22.24% and 24.68% enhancement in PSNR than the Levy shrink [13], Bayes shrink [11] and Hard Thresholding methods, respectively. Further, in Table 2, it is noticed that S/mse

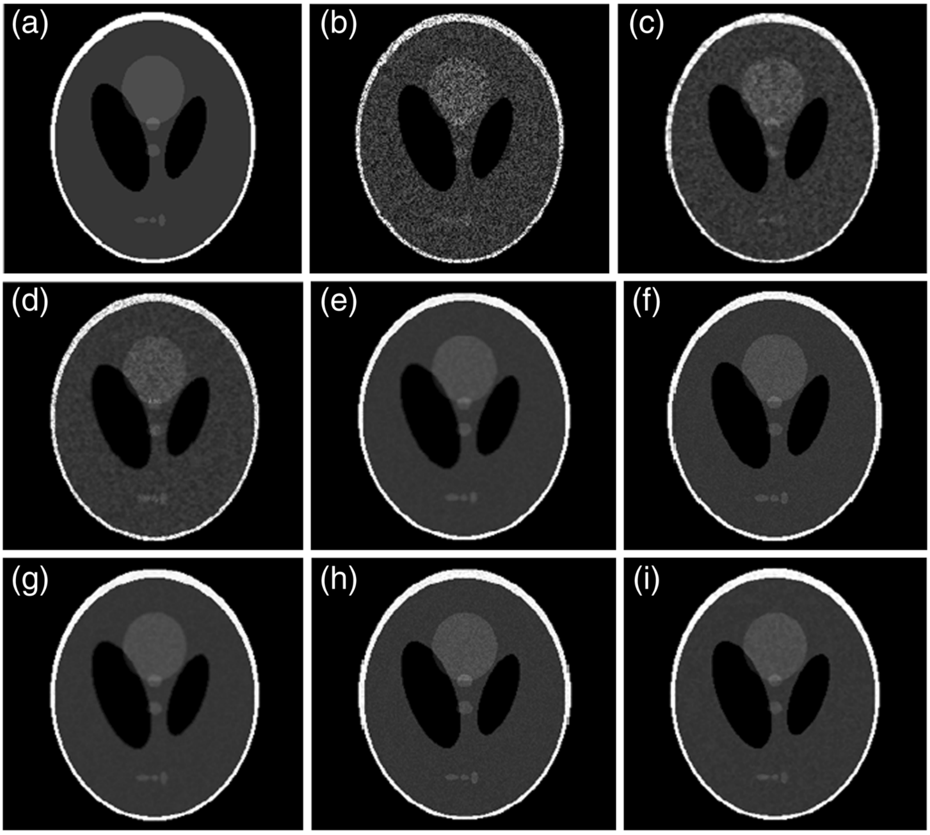

Fig. 7 De-noising performance on phantom image. (**a**) Original image (**b**) noisy image with standard deviation = 0.3 (**c**) denoising result with median filter (**d**) denoising result with wiener filter (**e**) denoising result with hard thresholding (**f**) denoising with soft thresholding (**g**) denoising result with Bayes shrink method (**h**) denoising with levy shrink method (**i**) denoising result with proposed method

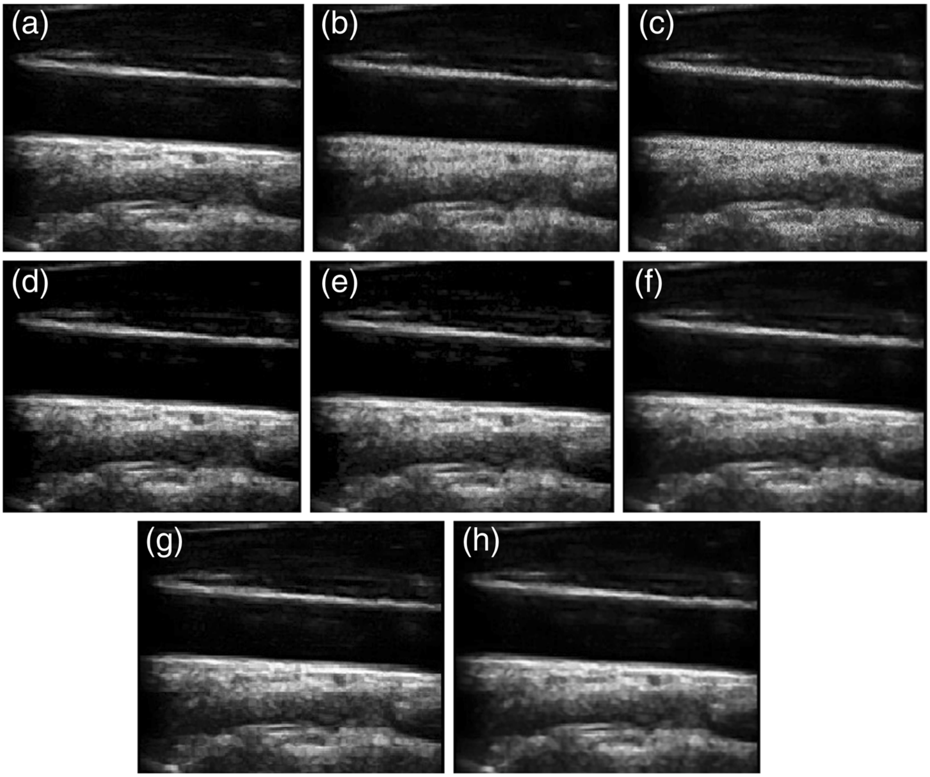

Fig. 8 De-noising performance on ultrasound image. **a** original ultrasound image. **b** Image denoising using median filter. **c** Image denoising using wiener filter. **d** image denoising using hard thresholding. **e** image denoising using soft thresholding. **f** image denoising using Bayes shrink method. **g** image denoising using Levy shrink method. **h** image denoising using proposed method

is highly dependent on signal content and noise level. Optimal S/mse is obtained for proposed method is approximately 23.55 dB (σ_{η} =0.1), which is better than the others compared methods.

It is observed that the proposed despeckling method performs better than the existing methods. As proposed in [33], three quality parameters are implemented, which will increase the effectiveness of the proposed method. The three parameters are (i) Correlation coefficient (ρ), (ii) Structural Similarity Index (SSIM) and (iii) Edge Preserving Index (EPI). These concepts are defined as follows:

(i) **Correlation coefficient** (ρ) determines the interdependence between the true image and denoised image. Unity value is required for perfect correlation. It is defined as

$$\rho = \frac{cov\left(I_{x,y}, \hat{I}_{x,y}\right)}{\sigma_{I_{x,y}}\sigma_{\hat{I}_{x,y}}} \tag{16}$$

Where $\sigma_{I_{x,y}}$ and $\sigma_{\hat{I}_{x,y}}$ are the standard deviation of noise free image and expected image respectively. cov is the covariance operation and defined as

Table 1 PSNR (dB) Performance of the proposed method for phantom image

(σ_η)	Noisy image	Median	Wiener	Hard Thresholding	Soft Thresholding	Levy shrink [13]	Bayes shrink [11]	Proposed Method
0.1	24.105485	26.057011	26.752311	27.108571	27.106715	28.263541	27.986614	35.995504
0.15	22.393041	24.353460	24.629063	25.398711	25.397173	26.935563	25.759335	34.178600
0.2	21.169219	23.055253	23.588899	24.041090	24.039745	26.011550	25.204379	32.956857
0.25	20.203715	21.676319	22.190628	23.188276	23.187032	25.589818	24.185837	31.736766
0.3	19.374075	21.267044	21.340280	22.444001	22.433250	24.603888	23.629846	31.131972
0.35	18.762251	20.724664	20.837599	21.863495	21.852856	23.702070	22.599344	30.668393
0.4	18.216625	20.615691	20.384248	21.185182	21.175583	22.641816	21.457412	30.006933
0.45	17.776322	19.603381	20.082733	20.584332	20.575548	22.203715	21.881681	29.250734
0.5	17.615885	19.704335	19.476300	20.150477	20.141891	21.154709	20.450637	28.851272

$$cov\left(I_{x,y},\hat{I}_{x,y}\right)=E\left[\left(I_{x,y}-E\left[I_{x,y}\right]\right)\left(\hat{I}_{x,y}-E\left[\hat{I}_{x,y}\right]\right)\right]$$

Where, E [.] is the expectation operation.

(ii) **Structural similarity index** (SSIM) is an index for image quality assessment. It is a measurement of similarity between true image and despeckled image. It is defined as

$$SSIM=\left(2\overline{I_{x,y}}\overline{\hat{I}_{x,y}}+2.55\right)\left(2\sigma_{I_{x,y}\hat{I}_{x,y}}+7.65\right)/\left(\overline{I_{x,y}}^2+\overline{\hat{I}_{x,y}}^2+2.55\right)\left(\sigma^2_{I_{x,y}}+\sigma^2_{\hat{I}_{x,y}}+7.65\right) \tag{17}$$

Where $\overline{I_{x,y}}$ and $\overline{\hat{I}_{x,y}}$ are the expectation of observed image and recovered image respectively.
$\sigma_{I_{x,y}\hat{I}_{x,y}}$ is the covariance between observed image and recovered image.
$\sigma^2_{I_{x,y}}$ and $\sigma^2_{\hat{I}_{x,y}}$ are the variance of observed image and recovered image respectively.
For good visual quality, SSIM is required to be unity.

(iii) **Edge Preserving Index** (EPI) is a quantitative measure of edge preservation. In medical science it is interested in despeckling while preserving the edges. For perfect edge preservation, EPI is required to be unity. Mathematically EPI is defined as

Table 2 S/MSE (dB) Performance of the proposed method for phantom image

(σ_η)	Noisy image	Median	Wiener	Hard Thresholding	Soft Thresholding	Levy shrink [13]	Bayes shrink [11]	Proposed Method
0.1	11.933051	13.100757	14.579878	14.936137	14.934282	17.510869	15.992145	23.547758
0.15	10.220608	11.275696	12.456629	13.226277	13.224740	16.819521	14.310755	21.664908
0.2	8.996786	9.881744	11.416466	11.852151	11.838726	15.990594	13.250734	20.399815
0.25	8.031282	8.228322	10.018194	10.961550	10.949369	14.212181	11.905424	19.113298
0.3	7.201641	7.811625	9.167846	10.271567	10.260817	13.385672	11.169219	18.403863
0.35	6.589818	7.327466	8.665165	9.691063	9.680423	13.141104	10.823969	18.020860
0.4	6.044192	7.223052	8.211815	9.012748	9.003149	12.450637	10.487036	17.310755
0.45	5.603888	6.197862	7.910300	8.411899	8.403115	11.918195	9.453521	16.482528
0.5	5.443452	5.616688	7.303866	7.978043	7.969458	11.166126	8.319766	16.095916

Table 3 Performance analysis of phantom image based on SSIM

(σ_η)	Noisy image	Median	Wiener	Hard Thresholding	Soft Thresholding	Levy shrink [13]	Bayes shrink [11]	Proposed Method
0.1	0.722503	0.832235	0.894371	0.966921	0.966900	0.971101	0.968796	0.989169
0.15	0.702070	0.794392	0.857520	0.955507	0.955474	0.965563	0.957273	0.975665
0.2	0.685542	0.766769	0.827964	0.942387	0.942033	0.953232	0.947523	0.963282
0.25	0.673119	0.740701	0.800952	0.931260	0.930880	0.947599	0.938628	0.950850
0.3	0.662635	0.726563	0.778803	0.921642	0.921239	0.931132	0.928718	0.941741
0.35	0.653195	0.708756	0.759335	0.911374	0.910417	0.924252	0.917282	0.934097
0.4	0.646487	0.703063	0.746607	0.908548	0.908056	0.911187	0.908956	0.925424
0.45	0.641816	0.683824	0.732328	0.894369	0.893839	0.909972	0.900345	0.916737
0.5	0.638712	0.679668	0.719400	0.883886	0.883314	0.892176	0.889696	0.909890

$$EPI=\sum\left(\Delta I_{x,y}-\overline{\Delta I_{x,y}}\right)\left(\Delta\hat{I}_{x,y}-\overline{\Delta\hat{I}_{x,y}}\right)\Big/\sqrt{\sum\left(\Delta I_{x,y}-\overline{\Delta I_{x,y}}\right)^2\sum\left(\Delta\hat{I}_{x,y}-\overline{\Delta\hat{I}_{x,y}}\right)^2} \quad (18)$$

Where $\Delta I_{x,y}$ is the filtered (high pass) output of $I_{x,y}$ using 3 × 3 pixel approximation of discrete Laplacian operator.

The performance comparison of the proposed algorithm in terms of SSIM and ρ for a phantom image is tabulated in Tables 3 and 4. It can be observed that proposed method yields SSIM and ρ, that are more closer to one than other existing methods. The best value of SSIM is 0.989 and the best value of ρ is 0.997 for σ_η =0.1. The proposed method offers 1.82%, 2.06% and2.25% enhancement in SSIM and 1%, 1.07% and 1.09% enhancement in correlation coefficient than the Levy shrink [13], Bayes shrink [11] and Hard threshold methods, respectively.

The efficiency of the proposed method is evaluated by simulating on ultrasound image at noise levels 0.4 and 0.5. The authors obtained quality parameters, SSIM, ρ and EPI from ultrasound image, which are tabulated in Table 5. It can be seen that the proposed method has achieved better result at higher noise levels. SSIM of the proposed method is 0.791 and 0.717 for σ_η =0.4 and σ_η =0.5 respectively. Correlation Coefficient of the proposed method is 0.978 and 0.976 for σ_η =0.4 and σ_η =0.5 respectively. EPI of the proposed method is 0.814 and 0.772 for σ_η =0.4 and σ_η =0.5 respectively. For a noise level σ_η =0.4, the proposed method offers 4.05%, 6.04% and 8.78% enhancement in SSIM index, 0.89%, 1.14% and 1.22% enhancement in correlation coefficient index and 7.68%, 10.24% and 11.47% enhancement in EPI index than the Levy shrink [13], Bayes shrink [11] and Hard threshold methods, respectively. It is noticed that, in Table 5, the soft thresholding filter results better in EPI parameter than hard

Table 4 Performance analysis of phantom image based on correlation coefficient (ρ)

(σ_η)	Noisy image	Median	Wiener	Hard Thresholding	Soft Thresholding	Levy shrink [13]	Bayes shrink [11]	Proposed Method
0.1	0.857389	0.975682	0.978695	0.986921	0.984469	0.987817	0.987138	0.997862
0.15	0.835346	0.962406	0.965681	0.968980	0.968901	0.978196	0.972130	0.996722
0.2	0.813160	0.947581	0.954528	0.956545	0.956519	0.971860	0.962921	0.995572
0.25	0.790888	0.927305	0.935563	0.946081	0.946035	0.962274	0.952044	0.994139
0.3	0.767274	0.918732	0.919820	0.936037	0.935960	0.958856	0.943385	0.993140
0.35	0.747767	0.905316	0.908023	0.926554	0.926455	0.948093	0.931376	0.992145
0.4	0.727354	0.902618	0.896396	0.913552	0.912654	0.932605	0.918974	0.990872
0.45	0.709555	0.874948	0.888000	0.900079	0.899926	0.922300	0.912588	0.989279
0.5	0.632670	0.817031	0.829666	0.871043	0.863188	0.909829	0.883495	0.980279

Table 5 Image quality metrics of ultrasound image

Methods	Noise standard deviation (σ_η =0.4)			Noise standard deviation (σ_η = 0.5)		
	SSIM	ρ	EPI	SSIM	ρ	EPI
Without filtering	0.479104	0.885983	0.206094	0.450648	0.868083	0.190201
Median filter	0.634488	0.952995	0.682729	0.609690	0.951709	0.641812
Wiener	0.522213	0.934151	0.376108	0.510119	0.923656	0.349549
Hard thresholding	0.721991	0.966686	0.749977	0.703437	0.967427	0.743443
Soft thresholding	0.721689	0.966368	0.750261	0.703135	0.957112	0.743606
Levy shrink [13]	0.759465	0.969915	0.761842	0.709355	0.970134	0.759156
Bayes shrink [11]	0.743717	0.967465	0.759006	0.705142	0.969833	0.752025
Proposed method	0.791560	0.978626	0.814411	0.717742	0.976187	0.772661

thresholding filter. For a noise level σ_η =0.5, the method offers 1.16%, 1.75% and 1.99% enhancement in SSIM index, 0.62%, 0.65% and 0.89% enhancement in correlation coefficient index and 1.74%, 2.67% and 3.78% enhancement in EPI index than the Levy shrink [13], Bayes shrink [11] and Hard threshold methods, respectively.

5 Conclusion

A new speckle reduction algorithm for ultrasound image is proposed in this paper. Homomorphic approach is used to convert the signal noise into signal independent noise. Orthogonalty property of wavelet is used to find the variance of this noise. By modeling the wavelet coefficients using Cauchy prior and estimating the true coefficient using Bayesian MAP estimator, it has proved that the proposed method removes speckle noise in ultrasound images effectively. Effectiveness of the proposed method is proved by comparing with existing methods through various performance and quality parameters. Statistical modeling of wavelet coefficients plays a vital role in the suppression of speckle noise. Suppression result may be improved by complex prior that correctly model the wavelet coefficients and noise. In future the proposed method may be applied to Optical Coherence Tomography (OCT) image. OCT is a digital retinal image which is affected by speckle noise and plays an important role in the detection of retinal diseases.

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