Jaypee University of Information Technology, Waknaghat

Test-II Examination, October 2019

B.Tech (ECE/CSE/IT)

Course Title: Probability Theory and Random Processes
Course Code: 10B11MA411
Semester: III

Max. Marks: 25 marks

Max. Time: 90 minutes

Note: Answer all the questions. Describe random variables along with range where applicable.

1. Consider the following *cumulative distribution function* of **X**:   (3 Marks) [CO-2]

$$\mathbb{F}(x) = \begin{cases} 0 & , & x < 5 \\ 1 - \frac{1}{8}(7-x)^3 & , & 5 \le x \le 7 \\ 1 & , & x > 7 \end{cases}$$

   Determine the *density function* of **X** and compute $\mathbb{P}(6 \le X \le 8)$ only by using $\mathbb{F}(x)$.

2. Let **X** have a binomial distribution with n = 288 and p = 1/3. Use Chebyshev's inequality to determine a lower bound for $\mathbb{P}(76 < X < 116)$.   (2 Marks) [CO-2]

3. Consider the densities of random variables **X** and **Y**:   (4 Marks) [CO-2]

$$f_X(x) = \begin{cases} 5e^{-5x} & , & 0 < x < \infty \\ 0 & , & \text{else} \end{cases} \qquad f_Y(y) = \begin{cases} 5e^{-5y} & , & 0 < y < \infty \\ 0 & , & \text{else} \end{cases}$$

   Find the density of **X+Y** if the random variables **X** and **Y** are independent.

4. Calls to your cell phone are a Poisson process with mean of 2 per hour. (4 Marks) [CO-3]

   (a) What is the probability that, if you forget to turn your phone off in a 1.5 hour movie, your phone rings during that time?

   (b) How many phone calls do you expect to get during the movie?

5. A box contains 10 chips from supplier A, 16 chips from supplier B, and 14 chips from supplier C. Assume that we perform the following experiment 20 times: *draw one chip from the box, note the supplier from where it came, and put it back.*   (4 Marks) [CO-3]

   (a) Define three random variables of interest along with range.

   (b) What is the probability that a chip from vendor A is drawn 5 times and a chip from vendor C is drawn 6 times?

6. Consider fitting a Poisson distribution to the following data: (4 Marks) [CO-4]

x	0	1	2	3	4
f	46	38	22	9	1

   (a) Calculate the mean and variance of the data.

   (b) Determine the expected frequencies.

7. Consider the following data relating trunk diameter and tree height: (4 Marks) [CO-4]

Diameter x	8	9	7	6	13
Height y	35	49	27	33	60

   Determine the *correlation coefficient* and interpret your result.