Jaypee University of Information Technology, Waknaghat

Test-3 Examinations, December 2022

B.Tech - VII Semester (ALL)

Course Code/Credits: 22B1WMA731/3 — Max. Marks: 35
Course Title: Linear Algebra for Machine Learning & Data Science
Course Instructor: RAD — Max. Time: 2 hours

Instructions: All questions are compulsory. Marks are indicated against each question. Use of scientific calculators is allowed.

1. Consider the following sets of vectors: (4 Marks) [CO-1]

$$\mathbf{E}_1 = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \end{pmatrix} \right\}; \quad \mathbf{E}_2 = \left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$$

   (a) Are the sets of vectors linearly independent?

   (b) Describe the span of each set.

2. Determine a basis from the following vectors for $\mathbb{R}^3$: (4 Marks) [CO-1]

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix} \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 6 \\ 8 \end{pmatrix} \quad \mathbf{v}_4 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}.$$

3. Let $\mathbf{W}$ be the subspace spanned by the given vectors: (5 Marks) [CO-2]

$$\mathbf{w}_1 = \begin{bmatrix} 0 \\ -6 \\ 3 \\ -3 \end{bmatrix} \text{ and } \mathbf{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 3 \end{bmatrix}.$$

   (a) Find the orthogonal complement of $\mathbf{W}$.

   (b) What is the dimension of $\mathbf{W}^{\perp}$?

4. Consider the monthly sales figures (in thousands of dollars) for a newly opened shoe store:

Month	1	2	3	4	5
Sales	9	16	14	15	21

   (a) Use *least-squares* approximation to find the straight line that best fits this data.

   (b) If a machine learns the data by creating a least-squares line, what outcome will it predict for the input 6 (i.e., sales revenue for month 6)? (5 Marks) [CO-3]

5. Consider the following 4×2 matrix: (5 Marks) [CO-3]

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -1 & 1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix}$$

(a) Apply Gram-Schmidt orthogonalization process to the column vectors of $\mathbf{A}$.

(b) Find the QR-decomposition of $\mathbf{A}$.

6. Let $\mathbf{B}$ be the given 2×3 matrix: (6 Marks) [CO-3]

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

(a) Determine the *singular values* of $\mathbf{B}$.

(b) Compute *singular value decomposition* (SVD) of $\mathbf{B}$.

7. Consider the measurements of *happiness* and *achievement* indexes from N=100 countries:

Features	Country 1	Country 2	$\cdots$	Country N
Happiness	x_{11}	x_{12}	$\cdots$	x_{1N}
Achievement	x_{21}	x_{22}	$\cdots$	x_{2N}

Correlation values obtained among the two variables in the dataset are given in the matrix

$$\mathbf{S} = \begin{bmatrix} 1.02 & 0.72 \\ 0.72 & 0.41 \end{bmatrix}.$$

Perform *principal component analysis* on the given dataset to answer: (6 Marks) [CO-4]

(a) Find the *eigenvalues* of $\mathbf{S}$.

(b) Obtain the *principal components* of the dataset.

(c) What fraction of variation is explained by the first *principal component*?

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