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Complex Neutrosophic Matrix with Some Algebraic Operations and Matrix Norm Convergence

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Abstract: In this article, a new concept of the Complex Neutrosophic Matrix is introduced to solve different problems related to uncertainties. Based on the proposed matrix, we have provided various algebraic operations like addition, subtraction, union many others which will be of great help in estabalishing the fundamental concepts. The results obtained through the above operations will be consequently utilized in solving the higher-dimensional problems due to their matrix properties. This novel concept lays the foundation of various research solutions in the field of the complex neutrosophic matrix, which are yet to be considered by researchers.The matrix norm convergence of the proposed matrix has also beenstudied for the necessary foundation of the complex neutrosophic matrix. Also, the future studies of the proposed concept have been proposed.

Keywords: Complex neutrosophic matrix, Complex neutrosophic set, Matrix Norm, Convergence, Matrix operations

1. Introduction

Zadeh [1] in 1965 introduced the concept of the fuzzy set to deal with the problems created by the uncertainty components. Atanassov [2] in 1993 further extended the theory of the fuzzy set to the intuitionistic fuzzy set (IFS) in which the non-membership function was added to the membership function. IFS played a significant role in solving the unsolved problems, but the indeterminacy/neutrality function was the dependent concept in this case which started creating the problem andwas later resolved by Samarandache in 1995 by introducing the theory of neutrosophic set. *Neutrosophic set is the branch of philosophy that deals with neutrality and its interaction with the different philosophical spectra.* Samarandache [3] defined the concept of neutrosophic set in the form of degree of truth, indeterminacy and falsity membership functions. Further, the notion of the neutrosophic set has been extended to a single-valued neutrosophic set [4], complex neutrosophic set [5], neutrosophic hypergraph [6] etc. Wang et al. [4] extended the theory of single valued neutrosophic sets by introducing the various properties and set-theoretic operators. Many difficulties related to the various fields of medicine [7-9], decision-making[10-12] and pattern recognition in the range [0,1] have been solved using these theories. Later, Ramot et al. [13] presented the novel theory of a complex fuzzy set (CFS) in 2010, which extended the range of fuzzy set to the unit circle in the complex plane. "*A complex fuzzy set preserves the* *concept of uncertainty through amplitude in [0,1] with the addition of the phase term in* $[0,2\pi]$. *Further, the properties of union, intersection and complement with its application in the case of solar activity and signals have been defined. Alkouri and Salleh [14] introduced the concept of a complex intuitionistic fuzzy set (CIFS), which added the complex non-membership function to the complex membership function. Along with the various properties related to its complement, union, T-norm and S-norm have been described. Later, Ali and Samarandache [5] proposed the concept of a complex neutrosophic set (CNS) which added the degree of complex neutrality function to membership and non-membership functions followed by various properties like union, intersection, a product with their application.*"

Further, the concept was extended in direction of matrices. Thomson [15] initiated by introducing the concept of the fuzzy matrix in 1977 and defined the convergence of fuzzy matrix. Kim and Roush [16] contributed by defining the fuzzy matrices concept as an extension of Boolean algebra. Later, the determinant of the intuitionistic fuzzy matrix was proposed by Pal [17] and interval-valued intuitionistic fuzzy matrices were introduced by Khan and Pal [18]. In 2014, Dhar et al. [19] gave the concept of neutrosophic fuzzy matrices, which was extended by Kandasamy et al. [20] who proposed the concept of neutrosophic interval matrices with its application.

Various researchers have extended their study in the direction of extension of fuzzy theories, which later turned to complex fuzzy matrices by Zhao and Ma [21] in 2016. They defined the complex fuzzy matrices in the form of $C = \left(A_{ij}(x) + \mathrm{i}B_{ij}(x)\right)$ and also explained the norm convergence. Khan et al. [22] extended the concept of complex fuzzy matrices to complex fuzzy soft matrices in 2020 and also proposed some theorems, which have been explained with the help of its application in DMP (Decision-Making Problems).

Various extensions have been done in the theory of the neutrosophic sets and information in literature. This has been explained with the help of the following figure:

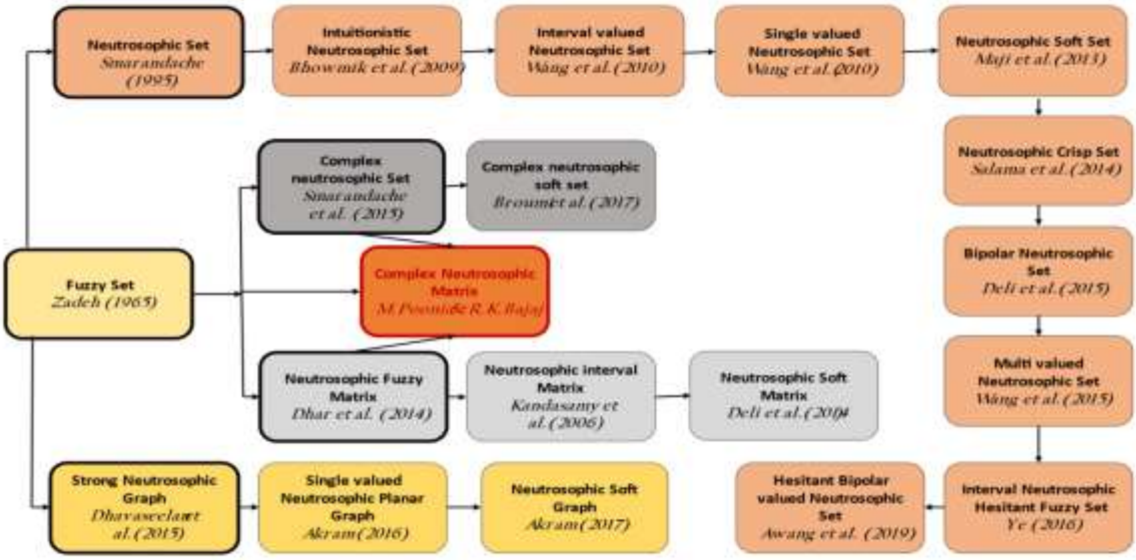

Fig 1: Literature Survey of Neutrosophic Theory.

Now, after considering the importance of matrix form in solving a large number as well as higher dimension problems in a single interval of time motivated us to extend these advantages of the matrix form from the real plane to the complex pane of unit range. Thus, we extended the theory of neutrosophic matrices to the complex plane and introduced the novel concept of a complex neutrosophic matrix. In this article, we have defined the concept of complex neutrosophic matrices and explained it with the help of algebraic operations. In continuation, the concepts like union, intersection have been also explained for a better understanding of the concept. Also, the concept of matrix norm and power convergence have been provided along with various results.

The manuscript has been organized as follows. Section 2 provides the basic and fundamental preliminaries related to the proposed concept. The formal notion of the complex neutrosophic matrix has been detailed and studied in Section 3. Different algebraic operations and properties related to the proposed matrix along with norm convergence have been included in Sections 4 and 5. Finally, the conclusion and scope for future work have been presented in Section 6.

2. Preliminaries

In this section, we briefly discuss the basic preliminaries and definitions related to the complex neutrosophic matrix present in the literature.

Definition 2.1. ***(Neutrosophic set) [4]*** "Let U be a space of points and $x \epsilon U$. A neutrosophic set S in U is characterized by a truth membership function $\Gamma_S(x)$, an indeterminacy membership function $\mathrm{I}_S(x)$ and falsity membership function $\Pi_S(x)$. $\Gamma_S(x)$, $\mathrm{I}_S(x)$ and $\Pi_S(x)$ are a real-valued subset of $]0^-, 1^+[$. The neutrosophic set can be represented as

$$S = \{(x, \Gamma_S(x) = a_\Gamma, \mathrm{I}_S(x) = a_I, \Pi_S(x) = a_\Pi) : x \epsilon U\}$$

where $^-0 \leq \Gamma_S(x) + \mathrm{I}_S(x) + \Pi_S(x) \leq 3^+$."

Definition 2.2. ***(Complex fuzzy set) [13]*** "A complex fuzzy set S, defined on a universe of discourse U, is characterized by a membership function $\mu_S(x)$ that assigns any element $x \epsilon U$ a complex-valued grade of membership in S. By definition, the values $\mu_S(x)$ may receive all values lie within in the unit circle in the complex plane, and are thus of form $\mathrm{R}_S(x) e^{\mathrm{i}\gamma_S(x)}$, where $\mathrm{i} = \sqrt{-1}$, $\mathrm{R}_S(x)$ and $\gamma_S(x)$ are both real-valued, and $\mathrm{R}_S(x) \epsilon [0,1]$. The complex fuzzy set S may be represented as the set of the ordered pairs

$$S = \{(x, \mu_S(x)) : x \epsilon U\}."$$

Definition 2.3. ***(Complex Neutrosophic set) [5]*** "A complex neutrosophic set S defined on the universe of discourse U, which is characterized by a truth membership function $\Gamma_S(x)$, an indeterminacy membership function $\mathrm{I}_S(x)$ and falsity membership function $\Pi_S(x)$ that assigns a complex-valued grade of $\Gamma_S(x)$, $\mathrm{I}_S(x)$ and $\Pi_S(x)$ in S for any $x \epsilon U$. The values $\Gamma_S(x), \mathrm{I}_S(x), \Pi_S(x)$ and their sum may all lie within the unit circle in the complex plane and so is of the following form,

$\Gamma_S(x) = P_S(x)e^{i\alpha_S(x)}, \mathrm{I}_S(x) = \mathrm{Q}_S(x)e^{i\beta_S(x)}$and $\Pi_S(x) = \mathrm{R}_S(x)e^{i\gamma_S(x)}$
where $P_S(x)$, $Q_S(x)$, $R_S(x)$ and $\alpha_S(x)$, $\beta_S(x)$, $\gamma_S(x)$ are respectively, real-valued and $P_S(x), Q_S(x), R_S(x)\epsilon[0,1]$ such that $^-0 \leq P_S + Q_S + R_S \leq 3^+$.

$$S = \{(x, \Gamma_S(x) = a_\Gamma, \mathrm{I}_S(x) = a_I, \Pi_S(x) = a_\Pi): x\epsilon U\},$$

where $|a_\Gamma| \leq 1, |a_\mathrm{I}| \leq 1, |a_\Pi| \leq 1$ &$|a_\Gamma + a_\mathrm{I} + a_\Pi| \leq 3$."

Example 1: Let $U = \{u_1, u_2, u_3\}$be the universe of discourse. Then, consider a complex neutrosophic set S in U which is denoted by:

$$S = \frac{\left(\frac{3}{5}e^{i0.8}, \frac{2}{5}e^{i\frac{\pi}{4}}, \frac{1}{2}e^{i\frac{3\pi}{4}}\right)}{u_1} + \frac{\left(\frac{3}{10}e^{i0.1}, \frac{2}{5}e^{i\frac{3\pi}{4}}, \frac{1}{10}e^{i\frac{\pi}{4}}\right)}{u_2} + \frac{\left(\frac{1}{5}e^{i0.7}, \frac{1}{10}e^{i\frac{5\pi}{4}}, \frac{2}{5}e^{i\frac{\pi}{4}}\right)}{u_3}$$

Definition 2.4. ***(Complex fuzzy matrix) [21]*** "Suppose$S = \left(\Gamma_{ij}(x) + \mathrm{i}\,\Pi_{ij}(x)\right)_{m\times n}$ is the matrix, all of the $\Gamma_{ij}(x) + \mathrm{i}\Pi_{ij}(x)$ is a complex fuzzy number for $i, j(1 \leq i \leq n, 1 \leq j \leq n)$, then called S is a complex fuzzy matrix."

3. Notion of Complex Neutrosophic Matrix

In this segment of the current article, we proposed a new kind of a complex neutrosophic matrix. The formal definition and an example of the complex neutrosophic matrix have been provided along with the operation of union and intersection for a better understanding of the concept.

Definition 3.1. "A complex neutrosophic fuzzy matrix$S_{m\times n}$ defined on a universe of discourse U, which can be characterized by a truth membership function $\Gamma_S(x_{ij})$, an indeterminacy membership function $\mathrm{I}_S(x_{ij})$ and a falsity membership function $\Pi_S(x_{ij})$ that assign complex value functions of the form,

$$\Gamma_S(x_{ij}) = P_S(x_{ij})e^{i\alpha_S(x_{ij})},$$

$$\mathrm{I}_S(x_{ij}) = \mathrm{Q}_S(x_{ij})e^{i\beta_S(x_{ij})}$$

and

$$\Pi_S(x_{ij}) = \mathrm{R}_S(x_{ij})e^{i\gamma_S(x_{ij})}$$

in $S_{m\times n}$ for any $x_{ij}\epsilon U$, where $P_S, Q_S, R_S \in [0,1]$ s.t. $^-0 \leq P_S + Q_S + R_S \leq 3^+$. The values and the sum of Γ_S, I_S and Π_S may always lie within the unit circle in the complex plane."
Then, the complex neutrosophic fuzzy matrix $S_{m\times n}$is represented as

$$S_{m\times n} = \left\{\left[\Gamma_S(x_{ij}), \mathrm{I}_S(x_{ij}), \Pi_S(x_{ij})\right]_{m\times n} | x_{ij} \in U\right\}$$

where $|\Gamma_S| \leq 1, |\mathrm{I}_S| \leq 1, |\Pi_S| \leq 1$ &$|\Gamma_S + \mathrm{I}_S + \Pi_S| \leq 3$.

Example 2. We could represent example 1 in matrix form or order 3×1 i.e.

$$S_{3\times1} = \begin{bmatrix} \left(\frac{3}{5}e^{\mathrm{i}0.8}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right) \\ \left(\frac{3}{10}e^{\mathrm{i}0.1}, \frac{2}{5}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{10}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}$$

Definition 3.2. ***(Complement)***

The complement of the complex neutrosophic matrix can be written in the form of

$$(S_{m\times n})^c = \left(\left[\Gamma_S(x_{ij}), \mathrm{I}_S(x_{ij}), \Pi_S(x_{ij})\right]_{m\times n}\right)^c = \left[\Gamma_S{}^c(x_{ij}), \mathrm{I}_S{}^c(x_{ij}), \Pi_S{}^c(x_{ij})\right]_{m\times n}$$

$$= \left[\left(P_S(x_{ij})e^{\mathrm{i}\alpha_S(x_{ij})}\right)^c, \left(\mathrm{Q}_S(x_{ij})e^{i\beta_S(x_{ij})}\right)^c, \left(\mathrm{R}_S(x_{ij})e^{\mathrm{i}\gamma_S(x_{ij})}\right)^c\right]_{m\times n}$$

where $\left(P_S(x_{ij})\right)^c = \mathrm{R}_S(x_{ij})$and $\left(e^{\mathrm{i}\alpha_S(x_{ij})}\right)^c = e^{\mathrm{i}(2\pi-\alpha_S(x_{ij}))}$.

Similarly, $\left(R_S(x_{ij})\right)^c = \mathrm{P}_S(x_{ij})$ and $\left(e^{\mathrm{i}\gamma_S(x_{ij})}\right)^c = e^{\mathrm{i}(2\pi-\gamma_S(x_{ij}))}$ and

Finally, $\left(Q_S(x_{ij})\right)^c = 1 - Q_S(x_{ij})$ and $\left(e^{\mathrm{i}\beta_S(x_{ij})}\right)^c = e^{\mathrm{i}(2\pi-\beta_S(x_{ij}))}$.

Example3. Suppose $S_{3\times1}$be a complex neutrosophic matrix. Then, the complement of $(S_{3\times1})^c$ will be given by

$$S_{3\times1} = \begin{bmatrix} \left(\frac{3}{5}e^{\mathrm{i}0.8}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right) \\ \left(\frac{3}{10}e^{\mathrm{i}0.1}, \frac{2}{5}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{10}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}, \quad (S_{3\times1})^c = \begin{bmatrix} \left(\frac{2}{5}e^{\mathrm{i}\left(2\pi-\frac{\pi}{225}\right)}, \frac{3}{5}e^{\mathrm{i}\frac{7\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{5\pi}{4}}\right) \\ \left(\frac{7}{10}e^{\mathrm{i}\left(2\pi-\frac{\pi}{1800}\right)}, \frac{3}{5}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{9}{10}e^{\mathrm{i}\frac{7\pi}{4}}\right) \\ \left(\frac{4}{5}e^{\mathrm{i}\left(2\pi-\frac{7\pi}{1800}\right)}, \frac{9}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{3}{5}e^{\mathrm{i}\frac{7\pi}{4}}\right) \end{bmatrix}$$

Definition 3.3. ***(Union of the complex neutrosophic matrix)*** Consider two complex neutrosophic matrices $S^1_{m\times n} = \left[\Gamma^1_S(x_{ij}), \mathrm{I}^1_S(x_{ij}), \Pi^1_S(x_{ij})\right]_{m\times n}$ and

$S^2_{m\times n} = \left[\Gamma^2_S(x_{ij}), \mathrm{I}^2_S(x_{ij}), \Pi^2_S(x_{ij})\right]_{m\times n}$ respectively. Then the union of these two matrices will be given by

$$S^1_{m\times n} \cup S^2_{m\times n} = \left\{\max\{\Gamma^1_S(x_{ij}), \Gamma^2_S(x_{ij})\}, \min\{\mathrm{I}^1_S(x_{ij}), \mathrm{I}^2_S(x_{ij})\}, \min\{\Pi^1_S(x_{ij}), \Pi^2_S(x_{ij})\}\right\}_{m\times n}$$

where

$$\max\{\Gamma^1_S(x_{ij}), \Gamma^2_S(x_{ij})\} = \max\{\mathrm{P}^1_S(x_{ij}), \mathrm{P}^2_S(x_{ij})\}\, e^{\mathrm{i}\max\{\alpha^1_S(x_{ij}), \alpha^2_S(x_{ij})\}},$$

$\min\{\mathrm{I}^1_S(x_{ij}), \mathrm{I}^2_S(x_{ij})\} = \min\{\mathrm{Q}^1_S(x_{ij}), \mathrm{Q}^2_S(x_{ij})\}e^{\mathrm{i}\min\{\beta^1_S(x_{ij}), \beta^2_S(x_{ij})\}}$ and

$$\min\{\Pi_S^1(x_{ij}), \Pi_S^2(x_{ij})\} = \min\{\mathrm{R}_S^1(x_{ij}), \mathrm{R}_S^2(x_{ij})\}e^{\mathrm{i}\min\{\gamma_S^1(x_{ij}),\gamma_S^2(x_{ij})\}}$$

Example 4. Consider two complex neutrosophic matrices

$$S_{3\times1}^1 = \begin{bmatrix} \left(\frac{3}{5}e^{\mathrm{i}0.8}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right) \\ \left(\frac{1}{10}e^{\mathrm{i}0.7}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{9}{10}e^{\mathrm{i}\frac{5\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{10}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}, \quad S_{3\times1}^2 = \begin{bmatrix} \left(\frac{1}{10}e^{\mathrm{i}0.2}, \frac{3}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.5}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}, \frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{3}{5}e^{\mathrm{i}0.7}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}$$

$$S_{3\times1}^1 \cup S_{3\times1}^2 = \begin{bmatrix} \left(\frac{3}{5}e^{\mathrm{i}0.8}, \frac{2}{5}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{7}{10}e^{\mathrm{i}\frac{3\pi}{4}}\right) \\ \left(\frac{1}{10}e^{\mathrm{i}0.5}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}$$

Definition 3.4. ***(Intersection of the complex neutrosophic matrix)*** Consider two complex neutrosophic matrices $S_{m\times n}^1 = \left[\Gamma_S^1(x_{ij}), \mathrm{I}_S^1(x_{ij}), \Pi_S^1(x_{ij})\right]_{m\times n}$ and $S_{m\times n}^2 = \left[\Gamma_S^2(x_{ij}), \mathrm{I}_S^2(x_{ij}), \Pi_S^2(x_{ij})\right]_{m\times n}$ respectively. Then, the intersection of these two matrices will be given by

$$S_{m\times n}^1 \cap S_{m\times n}^2 = \left\{\min\{\Gamma_S^1(x_{ij}), \Gamma_S^2(x_{ij})\}, \max\{\mathrm{I}_S^1(x_{ij}), \mathrm{I}_S^2(x_{ij})\}, \max\{\Pi_S^1(x_{ij}), \Pi_S^2(x_{ij})\}\right\}_{m\times n}$$

where,

$$\min\{\Gamma_S^1(x_{ij}), \Gamma_S^2(x_{ij})\} = \min\{\mathrm{P}_S^1(x_{ij}), \mathrm{P}_S^2(x_{ij})\}\, e^{\mathrm{i}\min\{\alpha_S^1(x_{ij}),\alpha_S^2(x_{ij})\}},$$

$\max\{\mathrm{I}_S^1(x_{ij}), \mathrm{I}_S^2(x_{ij})\} = \max\{\mathrm{Q}_S^1(x_{ij}), \mathrm{Q}_S^2(x_{ij})\}e^{\mathrm{i}\max\{\beta_S^1(x_{ij}),\beta_S^2(x_{ij})\}}$ and

$$\max\{\Pi_S^1(x_{ij}), \Pi_S^2(x_{ij})\} = \max\{\mathrm{R}_S^1(x_{ij}), \mathrm{R}_S^2(x_{ij})\}e^{\mathrm{i}\max\{\gamma_S^1(x_{ij}),\gamma_S^2(x_{ij})\}}$$

Example 5. Consider two complex neutrosophic matrices

$$S_{3\times1}^1 = \begin{bmatrix} \left(\frac{3}{5}e^{\mathrm{i}0.8}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right) \\ \left(\frac{1}{10}e^{\mathrm{i}0.7}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{9}{10}e^{\mathrm{i}\frac{5\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{10}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}, \quad S_{3\times1}^2 = \begin{bmatrix} \left(\frac{1}{10}e^{\mathrm{i}0.2}, \frac{3}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.5}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}, \frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{3}{5}e^{\mathrm{i}0.7}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}$$

$$S_{3\times1}^1 \cap S_{3\times1}^2 = \begin{bmatrix} \left(\frac{1}{10}e^{\mathrm{i}0.2}, \frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}, \frac{9}{10}e^{\mathrm{i}\frac{5\pi}{4}}\right) \\ \left(\frac{3}{5}e^{\mathrm{i}0.7}, \frac{1}{5}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}$$

4. Algebraic Operations On Complex Neutrosophic Matrices

In this segment of the current article, we have discussed the theoretical operations on the complex neutrosophic matrices. This section begins with the basic definition related to the concept and is followed by the theorem, multiplication and additive identity.

Definition 4.1. A be a $m \times n$neutrosophic matrix. If all of its entries are $\langle 0,0,1e^{i0}\rangle$, then $A$is called zero complex neutrosophic matrices and denoted by 0.
If all of its entries are $\langle 1e^{i0}, 1e^{i0}, 0\rangle$, then A is called universal complex neutrosophic matrix and denoted by J.

Theorem 1. The matrix $S_{m\times n}$ are a complex neutrosophic fuzzy algebra under the component-wise addition and multiplication operations $(+,\odot)$ represented as:

For $S_1 = [\Gamma_{S_1}(x_{ij}), \mathrm{I}_{S_1}(x_{ij}), \Pi_{S_1}(x_{ij})]_{m\times n}$and $S_2 = [\Gamma_{S_2}(x_{ij}), \mathrm{I}_{S_2}(x_{ij}), \Pi_{S_2}(x_{ij})]_{m\times n}$ in $S_{m\times n}$,

$$S_1 + S_2 = (\sup\{S_1, S_2\})$$
$$= (\sup\{\Gamma_{S_1}(x_{ij}), \Gamma_{S_2}(x_{ij})\}, \sup\{\mathrm{I}_{S_1}(x_{ij}), \mathrm{I}_{S_2}(x_{ij})\}, \inf\{\Pi_{S_1}(x_{ij}), \Pi_{S_2}(x_{ij})\})$$
$$S_1 \odot S_2 = (\inf\{S_1, S_2\}) = (\inf\{\Gamma_{S_1}(x_{ij}), \Gamma_{S_2}(x_{ij})\}, \inf\{\mathrm{I}_{S_1}(x_{ij}), \mathrm{I}_{S_2}(x_{ij})\}, \sup\{\Pi_{S_1}(x_{ij}), \Pi_{S_2}(x_{ij})\})$$

where $S_1 = [\Gamma_{S_1}(x_{ij}), \mathrm{I}_{S_1}(x_{ij}), \Pi_{S_1}(x_{ij})]_{m\times n}$or

$$S_1 = \left[P_{S_1}(x_{ij})e^{\mathrm{i}\alpha_{S_1}(x_{ij})}, \mathrm{Q}_{S_1}(x_{ij})e^{\mathrm{i}\beta_{S_1}(x_{ij})}, \mathrm{R}_{S_1}(x_{ij})e^{\mathrm{i}\gamma_{S_1}(x_{ij})}\right]_{m\times n} \text{and}$$

$$S_2 = \left[P_{S_2}(x_{ij})e^{\mathrm{i}\alpha_{S_2}(x_{ij})}, \mathrm{Q}_{S_2}(x_{ij})e^{\mathrm{i}\beta_{S_2}(x_{ij})}, \mathrm{R}_{S_2}(x_{ij})e^{\mathrm{i}\gamma_{S_2}(x_{ij})}\right]_{m\times n}$$

Proof. Every matrix in complex neutrosophic algebra should also satisfy the properties of fuzzy algebra. Therefore, $S_1 + O = S_1$ and $S_1 \odot J = S_1 \forall\, S_1 \epsilon S_{m\times n}$, hence in this case the addition and multiplication identities are denoted by the zero matrix O and the universal matrix J respectively. Thus, the identity element relative to the operations $(+,\odot)$ exist. Further, $S_1 + J = J$ and $S_1 \odot O = O$. This proves that Universal bound holds.
Similarly, we can prove for Idempotence, Commutativity, Associative and Absorption properties.
Now, in the case of Distributivity property, we have to prove
$S_1 \odot (S_2 + S_3) = (S_1 \odot S_2) + (S_1 \odot S_3)$
where $S_1 = \left[P_{S_1}(x_{ij})e^{\mathrm{i}\alpha_{S_1}(x_{ij})}, \mathrm{Q}_{S_1}(x_{ij})e^{\mathrm{i}\beta_{S_1}(x_{ij})}, \mathrm{R}_{S_1}(x_{ij})e^{\mathrm{i}\gamma_{S_1}(x_{ij})}\right]_{m\times n}$,

$$S_2 = \left[P_{S_2}(x_{ij})e^{\mathrm{i}\alpha_{S_2}(x_{ij})}, \mathrm{Q}_{S_2}(x_{ij})e^{\mathrm{i}\beta_{S_2}(x_{ij})}, \mathrm{R}_{S_2}(x_{ij})e^{\mathrm{i}\gamma_{S_2}(x_{ij})}\right]_{m\times n} \text{ and}$$

$$S_3 = \left[P_{S_3}(x_{ij})e^{\mathrm{i}\alpha_{S_3}(x_{ij})}, \mathrm{Q}_{S_3}(x_{ij})e^{\mathrm{i}\beta_{S_3}(x_{ij})}, \mathrm{R}_{S_3}(x_{ij})e^{\mathrm{i}\gamma_{S_3}(x_{ij})}\right]_{m\times n}.$$

Next, if $S_1 \le S_2 (or) S_3$ i.e. $\Gamma_{S_1}(x_{ij}) \le \Gamma_{S_2}(x_{ij})\ or\ \Gamma_{S_3}(x_{ij}), \mathrm{I}_{S_1}(x_{ij}) \le \mathrm{I}_{S_2}(x_{ij})\ or\ \mathrm{I}_{S_3}(x_{ij}), \Pi_{S_1}(x_{ij}) \ge \Pi_{S_2}(x_{ij})\ or\ \Pi_{S_3}(x_{ij})$ then in both cases

$\inf\{S_1, \sup\{S_2, S_3\}\} = S_1$ and $sup\ \{\inf\{S_1, S_2\}, \inf\{S_1, S_3\}\} = S_1$.

In a similar manner,

$S_1 + (S_2 \odot S_3) = (S_1 + S_2) \odot (S_1 + S_3)$

Next, if $S_1 \geq S_2 (or) S_3$ then in both cases

$\sup\{S_1, \inf\{S_2, S_3\}\} = S_1$ and $inf\{\sup\{S_1, S_2\}, \sup\{S_1, S_3\}\} = S_1$.

Hence, all the properties are proved.

Definition 4.2. Multiplication of two complex neutrosophic matrices

Consider two complex neutrosophic matrices given by $S^1_{3\times2}$ and $S^2_{2\times1}$ on the unit circle in complex plane i.e.

$$S^1_{3\times2} = \begin{bmatrix} \left(a_1 e^{\mathrm{i}\theta_1}, a_2 e^{\mathrm{i}\theta_2}, a_3 e^{\mathrm{i}\theta_3}\right) \left(a_4 e^{\mathrm{i}\theta_4}, a_5 e^{\mathrm{i}\theta_5}, a_6 e^{\mathrm{i}\theta_6}\right) \\ \left(b_1 e^{\mathrm{i}\sigma_1}, b_2 e^{\mathrm{i}\sigma_2}, b_3 e^{\mathrm{i}\sigma_3}\right) \left(b_4 e^{\mathrm{i}\sigma_4}, b_5 e^{\mathrm{i}\sigma_5}, b_6 e^{\mathrm{i}\sigma_6}\right) \\ \left(c_1 e^{\mathrm{i}\rho_1}, c_2 e^{\mathrm{i}\rho_2}, c_3 e^{\mathrm{i}\rho_3}\right) \left(c_4 e^{\mathrm{i}\rho_4}, c_5 e^{\mathrm{i}\rho_5}, c_6 e^{\mathrm{i}\rho_6}\right) \end{bmatrix},$$

$$S^2_{2\times1} = \begin{bmatrix} \left(p_1 e^{\mathrm{i}\alpha_1}, p_2 e^{\mathrm{i}\alpha_2}, p_3 e^{\mathrm{i}\alpha_3}\right) \\ \left(q_1 e^{\mathrm{i}\beta_1}, q_2 e^{\mathrm{i}\beta_2}, q_3 e^{\mathrm{i}\beta_3}\right) \end{bmatrix}$$

Now the product of two matrices is given by

$$S^1_{3\times2} . S^2_{2\times1} = \begin{bmatrix} d_{11} \\ d_{21} \\ d_{31} \end{bmatrix}$$

where,

$$"d_{11} = \left(\sup\left\{\inf\left(a_1 e^{\mathrm{i}\theta_1}, p_1 e^{\mathrm{i}\alpha_1}\right), \inf\left(a_4 e^{\mathrm{i}\theta_4}, q_1 e^{\mathrm{i}\beta_1}\right)\right\}, \sup\left\{\inf\left(a_2 e^{\mathrm{i}\theta_2}, p_2 e^{\mathrm{i}\alpha_2}\right), \inf\left(a_5 e^{\mathrm{i}\theta_5}, q_2 e^{\mathrm{i}\beta_2}\right)\right\},\right.$$
$$\left.\inf\left\{\sup\left(a_3 e^{\mathrm{i}\theta_3}, p_3 e^{\mathrm{i}\alpha_3}\right), \sup\left(a_6 e^{\mathrm{i}\theta_6}, q_3 e^{\mathrm{i}\beta_3}\right)\right\}\right)$$
$$d_{21} = \left(\sup\left\{\inf\left(b_1 e^{\mathrm{i}\sigma_1}, p_1 e^{\mathrm{i}\alpha_1}\right), \inf\left(b_4 e^{\mathrm{i}\sigma_4}, q_1 e^{\mathrm{i}\beta_1}\right)\right\}, \sup\left\{\inf\left(b_2 e^{\mathrm{i}\sigma_2}, p_2 e^{\mathrm{i}\alpha_2}\right), \inf\left(b_5 e^{\mathrm{i}\sigma_5}, q_2 e^{\mathrm{i}\beta_2}\right)\right\},\right.$$
$$\left.\inf\left\{\sup\left(b_3 e^{\mathrm{i}\sigma_3}, p_3 e^{\mathrm{i}\alpha_3}\right), \sup\left(b_6 e^{\mathrm{i}\sigma_6}, q_3 e^{\mathrm{i}\beta_3}\right)\right\}\right)$$
$$d_{31} = \left(\sup\left\{\inf\left(c_1 e^{\mathrm{i}\rho_1}, p_1 e^{\mathrm{i}\alpha_1}\right), \inf\left(c_4 e^{\mathrm{i}\rho_4}, q_1 e^{\mathrm{i}\beta_1}\right)\right\}, \sup\left\{\inf\left(c_2 e^{\mathrm{i}\rho_2}, p_2 e^{\mathrm{i}\alpha_2}\right), \inf\left(c_5 e^{\mathrm{i}\rho_5}, q_2 e^{\mathrm{i}\beta_2}\right)\right\},\right.$$
$$\left.\inf\left\{\sup\left(c_3 e^{\mathrm{i}\rho_3}, p_3 e^{\mathrm{i}\alpha_3}\right), \sup\left(c_6 e^{\mathrm{i}\rho_6}, q_3 e^{\mathrm{i}\beta_3}\right)\right\}\right)."$$

Example 6. Let us consider two matrices given below

$$S^1_{3\times2} = \begin{bmatrix} \left(\frac{3}{5}e^{\mathrm{i}0.8}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right) & \left(\frac{1}{2}e^{\mathrm{i}0.4}, \frac{1}{5}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{1}{10}e^{\mathrm{i}\frac{5\pi}{4}}\right) \\ \left(\frac{1}{10}e^{\mathrm{i}0.7}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{9}{10}e^{\mathrm{i}\frac{5\pi}{4}}\right) & \left(\frac{7}{10}e^{\mathrm{i}0.3}, \frac{1}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{1}{2}\ e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.7}, \frac{1}{10}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{2}{5}e^{\mathrm{i}\frac{\pi}{4}}\right) & \left(\frac{7}{10}e^{\mathrm{i}0.1}, \frac{9}{10}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{5}e^{\mathrm{i}\frac{3\pi}{4}}\right) \end{bmatrix}, \qquad S^2_{2\times1} = \begin{bmatrix} \left(\frac{1}{10}e^{\mathrm{i}0.2}, \frac{3}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \\ \left(\frac{1}{5}e^{\mathrm{i}0.5}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}, \frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}\right) \end{bmatrix}$$

$$S^1_{3\times2} . S^2_{2\times1} = \begin{bmatrix} \left(\sup\left\{\frac{1}{10}e^{\mathrm{i}0.2}, \frac{1}{5}e^{\mathrm{i}0.4}\right\}, \sup\left\{\frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}\right\}, \inf\left\{\frac{7}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{3}{10}e^{\mathrm{i}\frac{5\pi}{4}}\right\}\right) \\ \left(\sup\left\{\frac{1}{10}e^{\mathrm{i}0.2}, \frac{1}{5}e^{\mathrm{i}0.3}\right\}, \sup\left\{\frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}}, \frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}}\right\}, \inf\left\{\frac{9}{10}e^{\mathrm{i}\frac{5\pi}{4}}, \frac{1}{2}\ e^{\mathrm{i}\frac{\pi}{4}}\right\}\right) \\ \left(\sup\left\{\frac{1}{10}e^{\mathrm{i}0.2}, \frac{1}{5}e^{\mathrm{i}0.1}\right\}, \sup\left\{\frac{1}{10}e^{\mathrm{i}\frac{3\pi}{4}}, \frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}\right\}, \inf\left\{\frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}}, \frac{3}{10}e^{\mathrm{i}\frac{3\pi}{4}}\right\}\right) \end{bmatrix}$$

$$S_{3\times2}^{1}.S_{2\times1}^{2}=\begin{bmatrix}\left(\frac{1}{5}e^{\mathrm{i}0.4},\frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}},\frac{3}{10}e^{\mathrm{i}\frac{3\pi}{4}}\right)\\\left(\frac{1}{5}e^{\mathrm{i}0.3},\frac{1}{5}e^{\mathrm{i}\frac{\pi}{4}},\frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}}\right)\\\left(\frac{1}{5}e^{\mathrm{i}0.2},\frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}},\frac{3}{10}e^{\mathrm{i}\frac{\pi}{4}}\right)\end{bmatrix}$$

Definition 4.3. The identity element for addition

Consider two neutrosophic matrices $S_{2\times2}$ and $I_{2\times2}$ respectively, where $I_{2\times2}$ is an identity matrix. Then,

$$S_{2\times2}=\begin{bmatrix}\left(\frac{1}{10}e^{\mathrm{i}0.3},\frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}},\frac{1}{5}e^{\mathrm{i}\frac{5\pi}{4}}\right) & \left(\frac{7}{10}e^{\mathrm{i}0.4},\frac{3}{5}e^{\mathrm{i}\frac{5\pi}{4}},\frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}}\right)\\\left(\frac{1}{5}e^{\mathrm{i}0.2},\frac{4}{5}e^{\mathrm{i}\frac{5\pi}{4}},\frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right) & \left(\frac{3}{5}e^{\mathrm{i}0.7},\frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}},\frac{2}{5}e^{\mathrm{i}\frac{3\pi}{4}}\right)\end{bmatrix}$$

$$I_{2\times2}=\begin{bmatrix}\left(0,0,1e^{\mathrm{i}0}\right) & \left(0,0,1e^{\mathrm{i}0}\right)\\\left(0,0,1e^{\mathrm{i}0}\right) & \left(0,0,1e^{\mathrm{i}0}\right)\end{bmatrix}$$

$$S_{2\times2}+I_{2\times2}=\begin{bmatrix}d_{11} & d_{12}\\d_{21} & d_{22}\end{bmatrix}=S_{2\times2}$$

where,

$$d_{11}=\left(\sup\left(\frac{1}{10}e^{\mathrm{i}0.3},0\right),\sup\left(\frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}},0\right),\inf\left(\frac{1}{5}e^{\mathrm{i}\frac{5\pi}{4}},1\right)\right)=\left(\frac{1}{10}e^{\mathrm{i}0.3},\frac{7}{10}e^{\mathrm{i}\frac{\pi}{4}},\frac{1}{5}e^{\mathrm{i}\frac{5\pi}{4}}\right)$$

$$d_{12}=\left(\sup\left(\frac{7}{10}e^{\mathrm{i}0.4},0\right),\sup\left(\frac{3}{5}e^{\mathrm{i}\frac{5\pi}{4}},0\right),\inf\left(\frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}},1\right)\right)=\left(\frac{7}{10}e^{\mathrm{i}0.4},\frac{3}{5}e^{\mathrm{i}\frac{5\pi}{4}},\frac{1}{10}e^{\mathrm{i}\frac{\pi}{4}}\right)$$

$$d_{21}=\left(\sup\left(\frac{1}{5}e^{\mathrm{i}0.2},0\right),\sup\left(\frac{4}{5}e^{\mathrm{i}\frac{5\pi}{4}},0\right),\inf\left(\frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}},1\right)\right)=\left(\frac{1}{5}e^{\mathrm{i}0.2},\frac{4}{5}e^{\mathrm{i}\frac{5\pi}{4}},\frac{1}{2}e^{\mathrm{i}\frac{3\pi}{4}}\right)$$

$$d_{22}=\left(\sup\left(\frac{3}{5}e^{\mathrm{i}0.7},0\right),\sup\left(\frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}},0\right),\inf\left(\frac{2}{5}e^{\mathrm{i}\frac{3\pi}{4}},1\right)\right)=\left(\frac{3}{5}e^{\mathrm{i}0.7},\frac{1}{2}e^{\mathrm{i}\frac{\pi}{4}},\frac{2}{5}e^{\mathrm{i}\frac{3\pi}{4}}\right)$$

5. Norm Convergence for Complex Neutrosophic Matrix

This section includes the norm convergence of the complex neutrosophic matrix, followed by some basic properties, definitions and theorem.

Definition 5.1. [35] "Suppose $F = R$ or C, V in linear space over F. If the real vector function $\|*\|$ on Vverify the properties given below:

a) For arbitrary $u \in V, \|u\| \geq 0,$ and $\|u\| = 0 \Leftrightarrow u = 0.$
b) For arbitrary $a \in F, u \in V$ get $\|au\| = \|a\|\|u\|,$
c) For arbitrary $u, v \in V,$ get $\|u + v\| \leq \|u\| + \|v\|,$

Then $\|u\|$ is called the vector norm of X in V."

Definition 5.2. Consider $\|*\|$ is a non-negative real function on $F^{n\times n}$, if

$$\left\|C_1C_2R\left(\Gamma_S(x_{ij})\right)\right\| \leq \left\|C_1R\left(\Gamma_S(x_{ij})\right)\right\| \left\|C_2R\left(\Gamma_S(x_{ij})\right)\right\|$$

$$\left\|C_1C_2\tau\left(\Gamma_S(x_{ij})\right)\right\| \leq \left\|C_1\tau\left(\Gamma_S(x_{ij})\right)\right\| \left\|C_2\tau\left(\Gamma_S(x_{ij})\right)\right\|$$

where $R\left(\Gamma_S(x_{ij})\right)\&\tau\left(\Gamma_S(x_{ij})\right)$ is the real and imaginary part of the complex neutrosophic matrix.

Similarly, for $\mathrm{I}_S(x_{ij})\&\Pi_S(x_{ij})$.Then, this known as $\|*\|$ is $CNFM(n,m)$.

Definition 5.3. "Suppose $(V,\|*\|)$ is a n-dimensional normed linear space, $p_1,p_2,\dots,p_k,\dots$ is a vector sequence of V,δ is a fixed vector V, if

$$\lim_{k\to\infty}\|p_k-\delta\|=0$$

Then called vector sequence convergence in the norm, $\mathcal{M}$ is a limit of a sequence, note as: $\lim_{k\to\infty} p_k=\delta$ or $p_k\to\delta$

The vector sequence does not converge called divergence."

Definition 5.4. "Suppose $(V,\|*\|)$ is a n-dimensional normed linear space, $p_1,p_2,\dots,p_k,\dots$ where $p_1=\left(\Gamma_p^1(x_{ij}),\mathrm{I}_p^1(x_{ij}),\Pi_p^1(x_{ij})\right), p_2=\left(\Gamma_p^2(x_{ij}),\mathrm{I}_p^2(x_{ij}),\Pi_p^2(x_{ij})\right),\dots$ is a complex neurotrophic matrix sequence of V, $p(k)=\left(\Gamma_p^k(x_{ij}),\mathrm{I}_p^k(x_{ij}),\Pi_p^k(x_{ij})\right)$ constitutes a complex neutrosophic function $\delta=\left(\Gamma_\delta(x_{ij}),\mathrm{I}_\delta(x_{ij}),\Pi_\delta(x_{ij})\right)$ is a fixed complex neutrosophic matrix of V, if

$$\lim_{k\to\infty}\left\|pR\left(\Gamma_p^k(x_{ij})\right)-\delta R\left(\Gamma_\delta(x_{ij})\right)\right\|=0, \lim_{k\to\infty}\left\|p\tau\left(\Gamma_p^k(x_{ij})\right)-\delta\tau\left(\Gamma_\delta(x_{ij})\right)\right\|=0$$

where $R\left(\Gamma_p^k(x_{ij})\right), R\left(\Gamma_\delta(x_{ij})\right)\&\tau\left(\Gamma_p^k(x_{ij})\right),\tau\left(\Gamma_\delta(x_{ij})\right)$ is the real and imaginary part of the complex neutrosophic matrix and function respectively."

Similarly, for the case of indeterminacy and falsity components of the matrix i.e.

$$\lim_{k\to\infty}\left\|pR\left(\mathrm{I}_p^k(x_{ij})\right)-\delta R\left(\mathrm{I}_\delta(x_{ij})\right)\right\|=0, \lim_{k\to\infty}\left\|p\tau\left(\mathrm{I}_p^k(x_{ij})\right)-\delta\tau\left(\mathrm{I}_\delta(x_{ij})\right)\right\|=0$$

$$\lim_{k\to\infty}\left\|pR\left(\Pi_p^k(x_{ij})\right)-\delta R\left(\Pi_\delta(x_{ij})\right)\right\|=0, \lim_{k\to\infty}\left\|p\tau\left(\Pi_p^k(x_{ij})\right)-\delta\tau\left(\Pi_\delta(x_{ij})\right)\right\|=0$$

Then, it is known as complex neutrosophic matrix sequence, $p_1,p_2,\dots,p_k,\dots$ converges in the norm, $\delta=\left(\Gamma_\delta(x_{ij}),\mathrm{I}_\delta(x_{ij}),\Pi_\delta(x_{ij})\right)$ is the limit of the sequence.

5.1. The Convergence of Power of Complex Neutrosophic Matrix

Definition 5.1.1. Consider $\mathcal{M}\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right) \in CNFM(n,n)$ power K of $\mathcal{M}$ is defined as $\mathcal{M}^k$, among them $\mathcal{M}^1 = \mathcal{M}$, $\mathcal{M}^k = \mathcal{M}^{k-1}\mathcal{M}$.

Theorem. Consider $\mathcal{M}\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right) \in CNFM(n,n)$, there exists a positive integer a and K, such that $\forall k \geq K$ has $\mathcal{M}^{k+a} = \mathcal{M}^k$.

Proof. Suppose $\forall k \geq 1$,

$$\mathcal{M}\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right)$$

$$= \left(R\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right) + \mathrm{i}\left(\tau\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right)\right)\right)_{n\times n}$$

$$\mathcal{M}^k = \left(R\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right) + \mathrm{i}\left(\tau\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right)\right)\right)^k_{n\times n}$$

$$= \left(R^k\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right) + \mathrm{i}\left(\tau^k\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right)\right)\right)_{n\times n}$$

$$R\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right)$$

$$= \bigvee_{1\leq p_1,\dots,p_{k-1}\leq n}\left(R\left(\Gamma_{\mathcal{M}}(x_{ip_1}), \mathrm{I}_{\mathcal{M}}(x_{ip_1}), \Pi_{\mathcal{M}}(x_{ip_1})\right) \wedge \dots \wedge R\left(\Gamma_{\mathcal{M}}(x_{i,p_{k-1}}), \mathrm{I}_{\mathcal{M}}(x_{i,p_{k-1}}), \Pi_{\mathcal{M}}(x_{i,p_{k-1}})\right)\right)$$

$$\tau\left(\Gamma_{\mathcal{M}}(x_{ij}), \mathrm{I}_{\mathcal{M}}(x_{ij}), \Pi_{\mathcal{M}}(x_{ij})\right)$$

$$= \bigwedge_{1\leq p_1,\dots,p_{k-1}\leq n}\left(\tau\left(\Gamma_{\mathcal{M}}(x_{ip_1}), \mathrm{I}_{\mathcal{M}}(x_{ip_1}), \Pi_{\mathcal{M}}(x_{ip_1})\right) \vee \dots \vee \tau\left(\Gamma_{\mathcal{M}}(x_{i,p_{k-1}}), \mathrm{I}_{\mathcal{M}}(x_{i,p_{k-1}}), \Pi_{\mathcal{M}}(x_{i,p_{k-1}})\right)\right).$$

It is known that $\vee$ & $\wedge$ are closed, therefore, the number of the elements of $\{\mathcal{M}^k, k \geq 1\}$ will not be greater than $(n^{4n})^n$.

Then there exists a positive integer a and K, s.t. $\mathcal{M}^{k+a} = \mathcal{M}^k$, thus $k \geq K$ has

$$\mathcal{M}^{k+a} = \mathcal{M}^{(k-k)+k+a} = \mathcal{M}^{k-k}\mathcal{M}^{k+a} = \mathcal{M}^{k-k}\mathcal{M}^k = \mathcal{M}^k.$$

6. Conclusions & Future Work

In the current study, a novel concept of complex neutrosophic matrices is presented and explained with the help of a few algebraic operations and properties, which will be of great help for the researchers to understand the basics of the concept. The outcomes of these operations will lay the foundation of the fundamental rules to solve or design methodology to solve a complex problem. The numerical examples presented in the current manuscript will be of great help to understand the process more clearly. This could be the initial research in the direction of the novel concept. Further, the matrix norm convergence and power convergence of the complex neutrosophic matrix is discussed thoroughly. These results can be applied in further study of the complex neutrosophic theory. In future, the complex neutrosophic matrices concept may be applied to various applications related to pattern recognition, decision-making, medical diagnosis etc.

Declarations & Compliance with ethical standards

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