Some Constructions of $\mathscr{T}$-*Direct* Codes over $GF(2^n)$

R.S. Raja Durai and Meenakshi Devi

Abstract. The class of $\mathscr{T}$-*Direct* codes are an extension to the class of *linear codes with complementary duals*. In this paper, a construction procedure that constructs an n^2-*Direct* code from an *n-Direct* code is described. Further, the construction procedure is employed recursively to construct $n^{2^{m+1}}$-*Direct* codes for $m \geq 0$. Finally, $\mathscr{T}^2$-*Direct* codes are obtained from arbitrary $\mathscr{T}$-*Direct* codes with *constituent* codes of variable rates. The proposed construction procedure, when employed on an existing $\mathscr{T}$-*Direct* code, in fact increases the number of *constituent* codes (users), thereby supporting more users in a multi-user environment.

1 Introduction

A $\mathscr{T}$-*Direct* code $(\Gamma_1, \Gamma_2, \ldots, \Gamma_{\mathscr{T}})$ is constituted by a set of $\mathscr{T}$ F-ary linear codes $\Gamma_1, \Gamma_2, \ldots, \Gamma_{\mathscr{T}}$ with $\Gamma_i \cap \Gamma_i^{\perp} = \{\mathbf{0}\}$, where $\Gamma_i^{\perp} = \Gamma_1 \oplus \Gamma_2 \oplus \cdots \oplus \Gamma_{i-1} \oplus \Gamma_{i+1} \oplus \cdots \oplus \Gamma_{\mathscr{T}}$ is the dual of Γ_i with respect to the direct sum $\Lambda = \Gamma_1 \oplus \Gamma_2 \oplus \cdots \oplus \Gamma_{\mathscr{T}}$ for each $i = 1, 2, \ldots, \mathscr{T}$. The class of $\mathscr{T}$-*Direct* codes are in fact an extension to the class of *linear codes with complementary duals* (*LCD* codes) [2]. An interesting algebraic characterization of the class of $\mathscr{T}$-*Direct* codes is that $G_i G_i^{\mathbf{T}}$ is non-singular [5, Theorem 1], where G_i is the generator matrix of Γ_i for $i = 1, 2, \ldots, \mathscr{T}$. These class of multi-user codes can be used in a multiple access channel environment for encoding and decoding (or separating the user codewords). A coding problem for the $\mathscr{T}$-user Binary Adder Channel and $\mathscr{T}$-user F-Adder Channel is addressed via the class

R.S. Raja Durai
Department of Mathematics, Jaypee University of Information Technology Waknaghat,
Solan-173 234, Himachal Pradesh, India
e-mail: `rsraja.durai@juit.ac.in`

Meenakshi Devi
Department of Mathematics, Bahra University, Shimla Hills, Waknaghat,
Solan - 173 234, India
e-mail: `meenakshi_juit@yahoo.co.in`

S.M. Thampi et al. (eds.), *Recent Advances in Intelligent Informatics*,
Advances in Intelligent Systems and Computing 235,
DOI: 10.1007/978-3-319-01778-5_13,

of $\mathscr{T}$-*Direct* codes in [5, 6], where the F-Adder Channel is a non-binary channel model (analogues to the so-called binary adder channel) introduced by Urbanke and Rimoldi [3, 4] to model transmission of message symbols, possibly from a finite field F, of non-binary type. Unlike the *binary adder channels* which are real-number adder channels, arithmetic operations on the channel alphabets in F-Adder Channel are performed under the operations defined in the underlying field F.

A construction method (termed as the *distance* construction) for the class of $\mathscr{T}$-*Direct* codes (defined over $GF(2^n)$) that can increase the minimum distance of the *constituent* codes is proposed in [8], where it is shown that the proposed construction method also increases the number of *constituent* codes from n to $2n-1$ [7], and further to $\frac{n(n+1)}{2}$. This paper generalizes the constructions given in [7, 8] to the constructions of n^2-*Direct* codes and $\mathscr{T}^2$-*Direct* codes. While the former class of n^2-*Direct* codes comprises of constant-rate *constituent* codes, the later class of codes are equipped with variable-rate *constituent* codes. In all the constructions, Kronecker product represented by the symbol $\otimes$ is used as a basic tool. The class of Maximum Rank Distance (MRD) codes introduced by Gabidulin [1] are used to facilitate the results obtained in this paper.

The remaining part of the paper is organized as follows. By making use of the non-commutative nature of Kronecker product, section 2 constructs the class of n^2-*Direct* codes from the class of *n-Direct* codes. Section 3 generalizes the construction procedure outlined in section 2 and obtains $n^{2^{m+1}}$-*Direct* codes from n^{2^m}-*Direct* codes for $m \geq 0$. A construction of $\mathscr{T}^2$-*Direct* codes which are equipped with variable-rate *constituent* codes is presented in section 4. Final section draws the conclusion based on the results.

Notation and abbreviation: We use the notation $(\{n_1,n_2,\ldots,n_{\mathscr{T}}\},\{k_1,k_2,\ldots,k_{\mathscr{T}}\},\{d_1,d_2,\ldots,d_{\mathscr{T}}\})$ to denote a $\mathscr{T}$-*Direct* code constituted by the *constituent* codes (n_1,k_1,d_1), (n_2,k_2,d_2), …, $(n_{\mathscr{T}},k_{\mathscr{T}},d_{\mathscr{T}})$ and is abbreviated as $(\{n_i\},\{k_i\},\{d_i\})$. In particular, a $\mathscr{T}$-*Direct* code with $k_1 = k_2 = \cdots = k_{\mathscr{T}} = k$ (say) is denoted by $(\{n_i\},\{k\},\{d_i\})$ rather than $(\{n_i\},k,\{d_i\})$ - to distinguish a $\mathscr{T}$-*Direct* code from a conventional single user code, namely (n,k,d).

2 Construction of n^2-*Direct* Codes

Consider an $(\{n\},\{1\},\{n\})$ *n-Direct* code $(\Gamma_1,\Gamma_2,\ldots,\Gamma_n)$ along with the generator matrices of the *constituent* codes:

$$G_1 = \left[\, \alpha_1^{[1]}\ \alpha_2^{[1]}\ \cdots\ \alpha_n^{[1]} \,\right] \tag{1}$$

$$G_2 = \left[\, \alpha_1^{[2]}\ \alpha_2^{[2]}\ \cdots\ \alpha_n^{[2]} \,\right] \tag{2}$$

$$\vdots \qquad\qquad\qquad \vdots$$

$$\text{and}\quad G_n = \left[\, \alpha_1^{[n]}\ \alpha_2^{[n]}\ \cdots\ \alpha_n^{[n]} \,\right] \tag{n}$$