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Analysis on phase properties of a transmission line model with non-linear elements

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Meta-material

Abstract The theme of this paper is to analyze phase response of a conventional transmission line (Tx line) unit cell with the presence of a non-linear element. In this analysis, a non-linear inductor and non-linear capacitor are used in two Tx line models, firstly, forward wave supporting structure (low pass in nature) and secondly composite right hand left hand meta-material (CRLH-MTM) structure (bandpass in nature) which supports both forward and backward waves. For the non-linearity, a quadratic constitutive relation is considered between charge and voltage for a capacitor. For an inductor it is between the flux and the current. This analysis is to investigate the possibility of producing the negative group delay in the unit cell of Tx model in the presence of a non-linear element. This implies the opposite phase velocity and group velocities in the dispersion relationship for the Tx line model.

1. Introduction

Linear transmission line theory has been used very extensively in microwave engineering field in designing filters and phase shifters and in almost all passive and active components. Realization of composite right hand left hand meta-materials (CRLH-MTMs) using microstrip is a very active research area in electromagnetics and many devices have been developed so far [1–8]. The main reason of research for these novel circuits is to come up with negative refractive index material or to come up with the conclusion that the phase velocity and group velocity are in opposite direction for a forward travelling wave which is a different phenomenon from conventional positive refractive index material. But the main problem with these circuits when they are realized in the microstrip is that, the gain appears to be very small [9] when the transition is happening from same sign of phase velocity and group velocity to the opposite sign. In this paper it is shown that with the non-linearity in the unit cell, it is possible to avoid this difficulty thereby opening an opportunity for the non-linear transmission lines in a better perspective.

Almost all devices exhibit non-linearity depending on the conditions of operations. But non-linear circuits are very common in electronics field such as clippers, clampers and harmonic generators. In many cases, if there are any non-linear devices present in the circuit or system, they are linearized within its operating region designated as bias point or quiescent point using the Taylor's series expansion [10]. In some applications the circuits are supposed to be analyzed without

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any linearization to see the actual effects of non-linearity in the entire system. There are so many different reasons for the non-linearity for any given component. Capacitor with fringing field can be modelled as a non-linear capacitor, inductor on an iron core is a non-linear device and resistor in an environment with changing temperature is considered as non-linear resistance. Sometimes these non-linearities maybe very complicated functions of controlling parameters. Even though there are infinitely many possibilities for the non-linearity of a device, it is appropriate to fundamentally study the effect of a specific non-linearity for observing the systems behaviour to some extent.

This paper starts with modelling an inductor on iron core as a non-linear element with quadratic constitutive relation between its controlling parameters (magnetic flux and current) [11]. Two transmission line models are analyzed in time domain with the Gaussian pulse as its excitation. Then with the help of Fourier Transform theory [12], its phase response in frequency domain is obtained. First circuit is with the low pass structure and the second one is the composite right hand left hand meta-material. Along with this a five section LC transmission line model is also analyzed to see the response of this in circuit domain to see the modified low pass behaviour as a function on non-linearity.

2. Problem formulation

The aim of this paper is to analyze the effects of a non-linearity on the phase delay and group delay for forward wave supporting structure and forward–backward supporting structure. Fig. 1 represents the conventional transmission line model. For a forward wave supporting structure, the series branch is an inductor and shunt branch is a capacitor forming the well known LC ladder network. For forward and backward wave supporting structure, the series branch is a combination of capacitor and inductor connected in series while the shunt branch is the parallel combination of capacitor and inductor. Linear circuit analysis is very easy compared with the non-linear circuit analysis. A linear circuit or system's behaviour can be expressed completely in terms of unit impulse response, if it is assumed to be time invariant. This is not the case with non-linear systems. In analyzing the non-linear circuits or systems, there are problems such as frequency conversion, harmonics generations and hence it is not possible to come up the output calculation for a given circuit from its impulse response as it is not an LTI system. As far as finding the frequency response for a non-linear device is concerned, depending on the spectrum of input signal, it is possible to find the ratio of output to input, if both (input and output) the power spectral densities are confined within some reasonable frequency limits like as is defined in half power bandwidth for low pass or high pass filters.

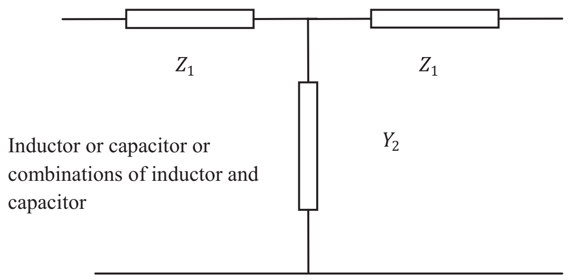

Figure 1 Transmission line model. (Unit Cell.)

For analyzing the non-linear circuit model, circuit theory has been used (KCL and KVL) [13] to come up with the relation between input and output quantities. At the input of the model, a Gaussian voltage pulse is applied and the output voltage is measured in time domain. Then these quantities are converted into frequency domain and along with the transmission line theory, the dispersion characteristics are obtained. Dispersion is the relation between phase constant and frequency. This can be calculated easily from the complex propagation constant as $\gamma = \sqrt{Z_1 Y_2} = \alpha + j\beta$, where α is attenuation constant and β is a phase constant which gives the dispersion relation.

2.1. Equations governing the system

For an inductor on iron core, the relationship between magnetic flux and current passing through the loop is related as

$$i(t) = \frac{N\phi(t)}{L} + A\phi(t)^3 \tag{1}$$

where N is the number of turns of the coil, A is the cross sectional area of the core and L is the self inductance. For a linear inductor the coefficient $A = 0$. Using the Faraday's law of induction along with Lenz's law, the induced voltage can be found out as

$$v(t) = -N\frac{d\phi(t)}{dt} \tag{2}$$

For a linear inductor the relation between induced voltage and current is related as

$$v(t) = L\frac{di}{dt} \tag{3}$$

For a non-linear device, it is little difficult to come up with a simple equation of $\phi(t)$ in terms of $i(t)$ not involving radical signs. If the values of A, N and L are known, then with the help of curve fitting it is easy to approximate the same function and $\phi(t)$ can be expressed as some function of $i(t)$. This is illustrated with the following numbers.

Fig. 2 represents this non-linear relation between flux and current with $N = 5$; $L = 1H$; $A = 0.1\ \mathrm{m}^2$ when flux is changing from 0 to 20 T. This non-linearity can be converted into an equation for $\phi(t)$ in terms of $i(t)$ with the help of curve fitting technique or interpolation. As though these numbers seem to be not practical at the present moment, there is no mistake in analyzing this model to study the behaviour of the circuits with non-linearities. From interpolation it is possible to get the following equation

$$\phi(t) = p_0 + p_1 i(t) + p_2 i(t)^2 + p_3 i(t)^3 + p_4 i(t)^4 \tag{4}$$

where $p_4 = -0.1776 \times 10^{-9}$, $p_3 = 0.3623 \times 10^{-6}$, $p_2 = -0.262 \times 10^{-3}$, $p_1 = 0.09147$ and $p_0 = 1.309$.

Using the above equation, it is easy to find the induced voltage using the equation $v(t) = -N\frac{d\phi(t)}{dt}$. This is considered in detail in next section to represent a close approximation to the practical application.

After observing that non-linearity, it is a reasonable choice to analyze the non-linear inductor with the following relationship between voltage and current as

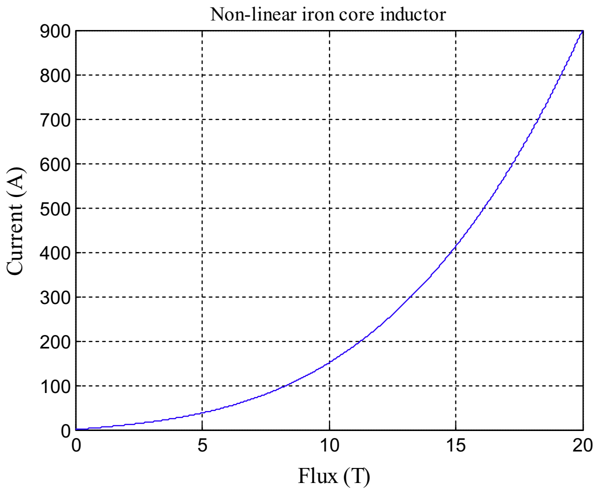

Figure 2 Relation between flux and current for a non-linear inductor.

$$\phi(t) = L_0 + L_1 i(t) + L_2 i^2(t) \tag{5}$$

which leads to

$$v(t) = \frac{d\phi}{dt} = \dot{\phi} = L_1 \frac{di(t)}{dt} + L_2 2i(t) \frac{di(t)}{dt} \tag{6}$$

Using the same arguments to the non-linear capacitor, the same non-linearity can be assumed (even though the cause of it may be attributed to different factors such as fringing field and material deviations), the effects of this can be analyzed to understand the behaviour of non-linear devices in circuits as a starting point. For a non-linear capacitor, these equations can be taken as

$$q(t) = C_0 + C_1 v(t) + C_2 v^2(t) \tag{7}$$

And

$$i(t) = \frac{dq(t)}{dt} = \dot{q} = C_1 \frac{dv(t)}{dt} + C_2 2v(t) \frac{dv(t)}{dt} \tag{8}$$

It is now required to study the effects of non-linearities in time domain and frequency domains to understand the behaviour to some extent.

2.1.1. Methodology used

The governing equations for these circuits are higher order non-linear differential equations and with the help of state space representation they are converted into first order differential equations. These equations are solved using the Runge–Kutta fourth order method and different responses are obtained for different non-linearities. All these are coded in Matlab software [14].

For a low pass circuit model Z_1 is an inductor and Y_2 is a capacitor. For composite right hand left hand meta-material structure Z_1 is a series combination of inductor and capacitor while Y_2 is a parallel combination of inductor and capacitor.

3. Simulations and results

Simulations are carried out for three circuits. Firstly the conventional transmission line model which acts like a low pass filter is analyzed. The values for elements can be scaled to any frequencies and hence the choice of the values does not alter the conclusions. For all the simulations $L_1 = 0.1H$ and $C_1 = 0.1F$ unless otherwise specified. Fig. 3 represents the phase characteristics of the unit cell for a non-linear capacitor with different values. From this, it is observed that the non-linearity can produce the negative group delay in the circuit even though it is relatively small.

Fig. 4 represents the response of the low pass structure with both the elements as non-linear, that is the series inductor is non-linear with different values of L_1 and L_2 along with different values of C_1 and C_2. From this it is observed that the phase response can be altered very easily if there are more non-linearities are present in the circuit which can be controlled without the pushing the system into instability.

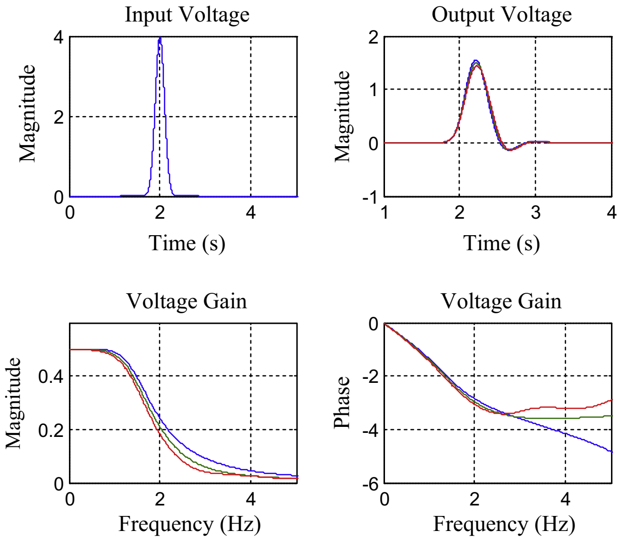

Figure 3 Forward wave supporting structure with non-linear shunt capacitor, Blue: $C_2 = -0.01F$, Green: $C_2 = 0.01F$, Red: $C_2 = 0.03F$.

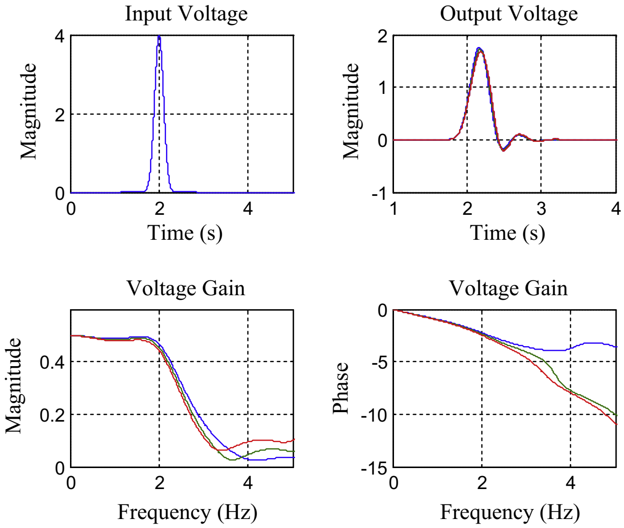

Figure 4 Low pass response for non-linear capacitor and inductor: Blue: $L_2 = 0.01H$, Green: $L_2 = 0.02H$, Red: $L_2 = 0.03H$ with $C_2 = -0.005F$.

Along with the low pass structure which supports the forward waves, the most extensively used structure for the negative refractive index meta-material realization is the composite right hand left hand meta-material structure which acts a bandpass equivalent in the frequency domain. In this case both the elements are considered as non-linear and compared the response of the non-linear unit cell with that of the linear model. The phase response and gain are shown in Fig. 5. After observing this, this structure has slightly better advantage than the previous circuit as far as the gain of the circuit is considered. Compared with the low pass circuit, the phase reversal is happening at the frequencies where the gain is reasonably high. Hence these models can be used in a much better way for meta-material realizations. Blue is with linear elements and red is with non-linear capacitor and inductor from Figs. 5–7.

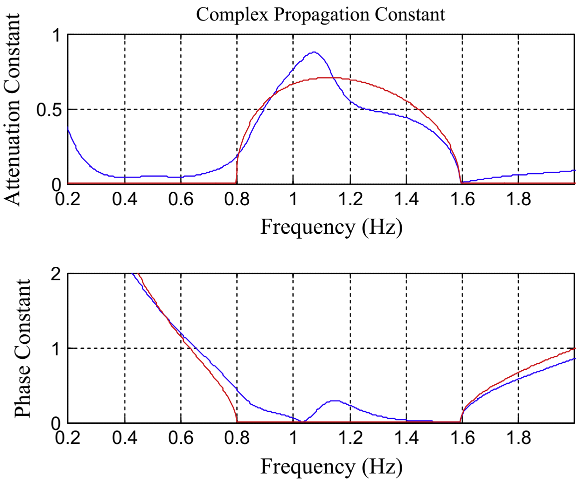

Figure 7 Complex propagation constant.

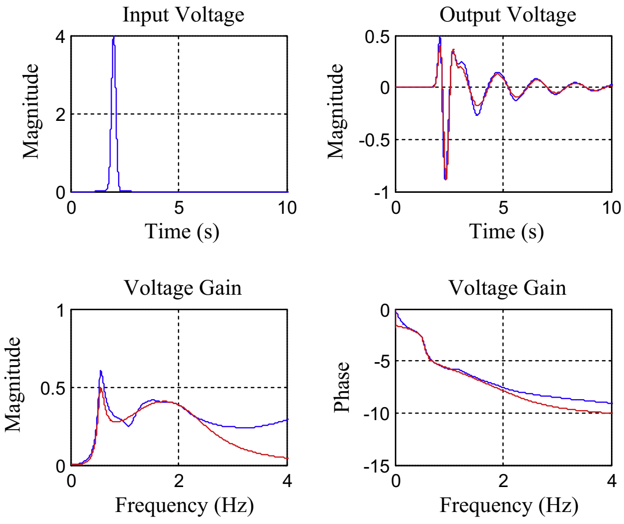

Figure 5 Transfer characteristic for the unit cell.

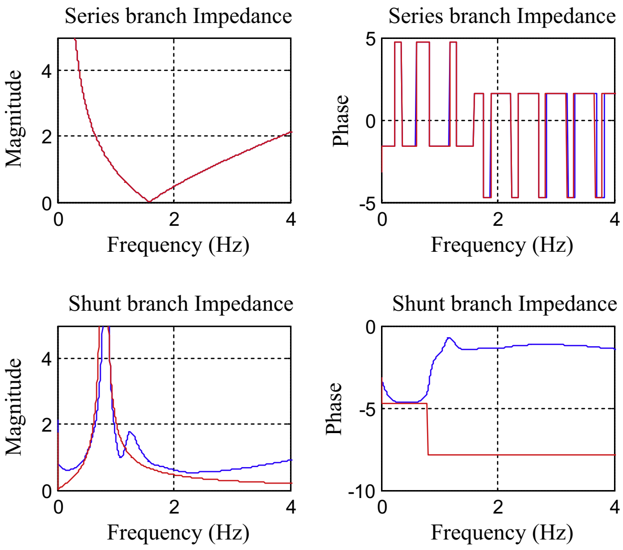

Figure 6 Impedance of series branch and shunt branch.

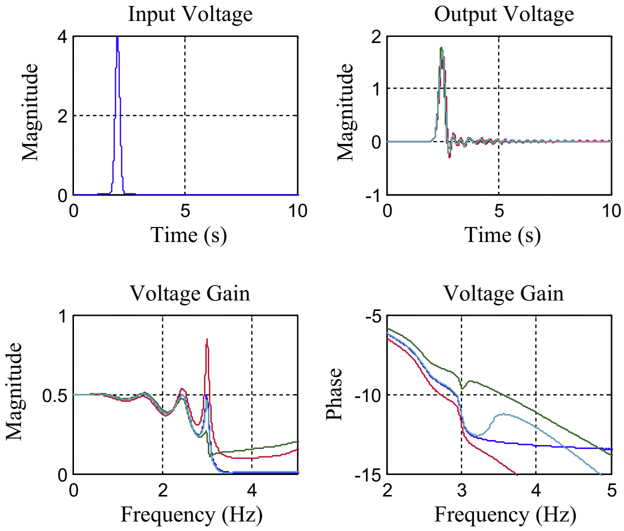

Figure 8 Five section low pass structure: Blue: $C_2 = -0.01F$, Green: $C_2 = -0.01F$, Red: $C_2 = 0.01F$ and Cyan: $C_2 = -0.0009F$.

To find the attenuation constant for the unit cell, transmission line theory has been used which acts a bridge between circuit theory and wave theory. With the help of that series impedance and shunt impedances are plotted which indicates the deviation from normal characteristics of normal circuits.

Finally to study the effect of the non-linearity on the distributed transmission line model, five sections of low pass structure containing (nine elements, each inductor is $0.1H$ and capacitor is $0.1F$) are considered. In this model capacitor is considered as non-linear with different values. Fig. 8 represents the voltage gain and the phase response. From the characteristic, it is observed that the non-linearity is producing more ripples in the passband gain as compared with the linear model. The phase response as a function of frequency having more deviation from the linear model with an increase and decrease in the phase of as a function of frequency, which indicates the negative group delay for the same model. This can be translated into group velocity and the requirement that the phase velocity and group velocity are in opposite directions has been met. The only problem with these types of circuits is the instability. If the device non-linearity can be controlled, then these devices can be used in different applications such as phase shifters in microwave engineering.

4. Conclusion

After the analysis is carried out, it is observed that even a small non-linearity in the circuit or system can cause deviations in the phase response considerably. The problem with the non-linear devices is that they produce harmonics and other effects which can lead to instability in the given circuit. For the low pass and bandpass filter characteristics, there is maxima/minima in the phase characteristics for the gain of the unit cell thereby indicating the opposite phase delay and group delay. These delays can be translated into phase velocity and group velocities in the wave theory thereby giving a possibility of negative refractive index materials with the non-linear elements. In case if, this non-linearity can be controlled in a precise manner, these non-linear models can be used in many electronic circuits.

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