

*Note: All questions are compulsory. Marks are indicated against each question in square brackets*

1. (a) For two sets  $A$  and  $B$ , find the dual of statement " $A \subseteq B$ ." Also, list all the fundamental products of three sets  $A$ ,  $B$  and  $C$  and show them with the help of Venn Diagram. [3] [CO3]

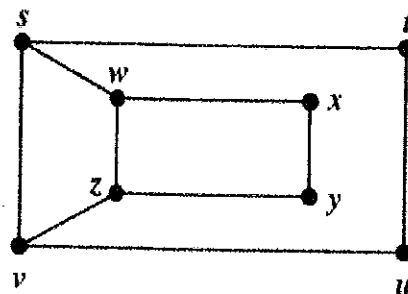
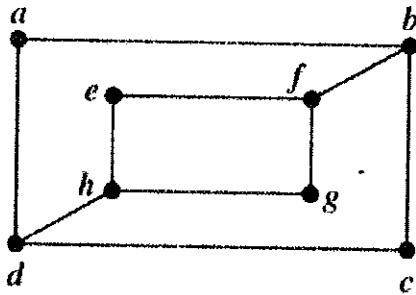
(b) Check the validity of the following argument with the help of truth table: [2] [CO3]  
*If it snows, then streets get slippery. If the streets get slippery, then accidents happen. Accidents do not happen. Therefore, it does not snow.*

2. (a) Using mathematical induction show that  $1^2 + 3^2 + \dots + (2n-1)^2 = \frac{(4n^3 - n)}{3}, \forall n \in \mathbb{Z}^+$ . [3] [CO2]

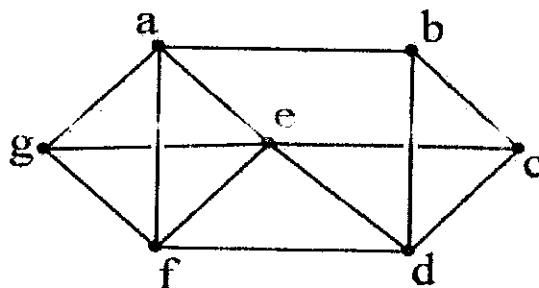
(b) Dirac's theorem states that "if  $G$  is a simple graph with  $n$  vertices with  $n \geq 3$  such that the degree of every vertex in  $G$  is at least  $n/2$  then  $G$  has a Hamilton circuit." Give an example to disprove the converse of the theorem. Write the contrapositive of the theorem. Also, give an example of a Eulerian graph which is not a Hamilton. graph. [3] [CO2]

3. (a) Draw wheel graph  $W_6$  on 6 vertices, and verify the Euler's formula for it. [4] [CO4]

(b) Using definition of graph Isomorphism prove or disprove that the following graphs are isomorphic: [2]



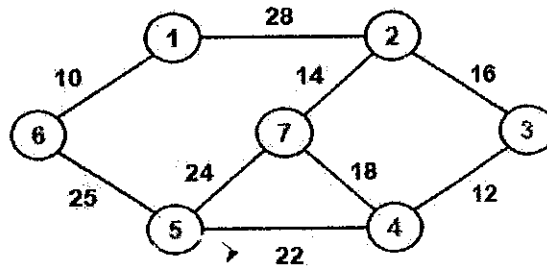
4. (a) Using the Welsh Powell algorithm color the following graph, and write its chromatic number: [4]



(b) Evaluate the prefix expression  $+ - * 2 3 5 / \uparrow 2 3 8$

[2] [CO4]

5. (a) Construct the minimum spanning tree (MST) for the given graph using Prim's Algorithm, also find the cost (or weight) of MST. [3] [CO6]



(b) Show that set  $G = \{1, w, w^2\}$ ; is an Abelian group under usual multiplication where  $w^3 = 1$ . [3]

6. Show that  $(\mathbb{Z}_5, +_5, \times_5)$  is a Ring with zero divisors where  $+_5, \times_5$  are addition and multiplication modulo 5 respectively. [6] [CO6]

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